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AI for Large Imperfect-Information Games: Beating Top Humans in No-Limit Poker

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Joint work with my advisor Tuomas Sandholm

Imperfect-Information Games

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Heads-Up No-Limit Texas Hold'em

- Has become the main ***benchmark and challenge problem*** in AI for imperfect-information games
- No-Limit Betting = Continuous Action Space
  - Technically, 10^{161} situations since bets must be integers
- The most popular variant of poker in the world
  - Played in the World Series of Poker Main Event
  - Featured in *Casino Royale* and *Rounders*
- "Purest form of poker"
- No prior AI has been able to beat top humans

2017 Brains vs AI

- Libratus (our 2017 AI) against four of the ***best*** heads-up no-limit Texas Hold'em specialist pros



- 120,000 hands over 20 days in January 2017
- $200,000 divided among the pros based on performance
- Conservative experiment design

How good are these pros?

How good are the pros that are playing against libratus? self.poker

Submitted 1 year ago by CaptainRonSwanson

I don't follow pro poker like I once did, my hero's still being Ivey, Hellmuth(hero?), Ferguson, Dwan, Doyle... the guys you could find on every poker show from the mid 2000's.

Who can enlighten me on how good these pros are in comparison, to say, the megastars of poker? If Ivey and Hellmuth were to go head to head agains this AI, do you think they would have a better shot?

Or, are the players playing against the AI a little less than stellar? Poker Pro seems kind of subjective these days.

[–] **dabsindenver** 11 points 1 year ago

These player would absolutely trounce all of the 2000s heros in heads up poker. The hero's from the 2000s would be division III college players while these guys are all-star callibre pros.

These guys are top pros in the heads up world and make their living playing it and similar games.

Final Result

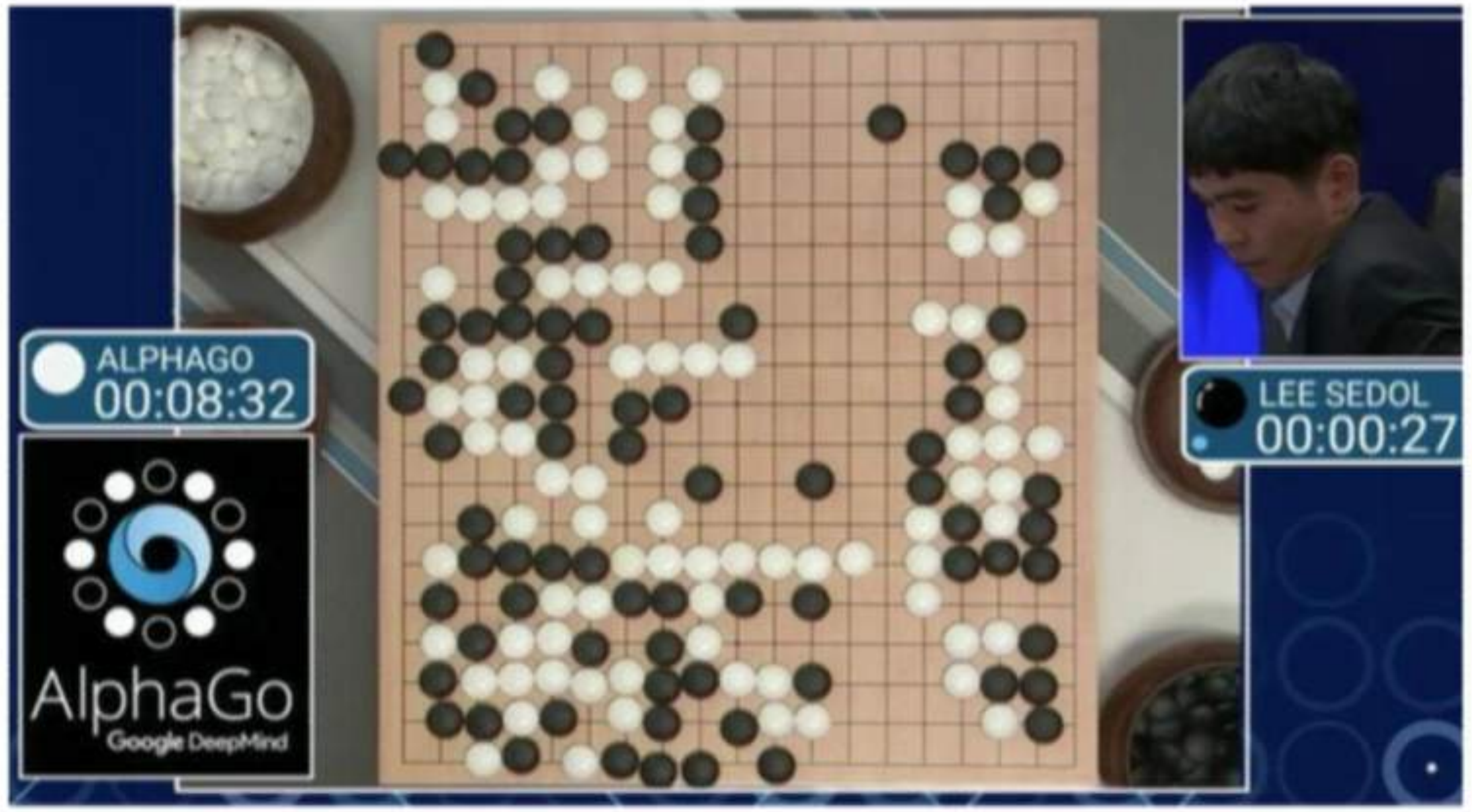
- Libratus beat the top humans in this game by a lot
  - 147 mbb/game
  - Statistical significance 99.98%, i.e., p-value of 0.0002
  - Each human lost to Libratus

Lengpudashi vs humans event

- 36,000 hands against 6 Chinese poker players
  - WSOP bracelet winner
  - Expertise in computer science & ML
  - Studied Libratus's hand histories in advance

- Lengudashi won by 220 mbb/game
  - Won each of the 9 sessions
  - Also beat each human individually
  - Watched live by **millions** of people

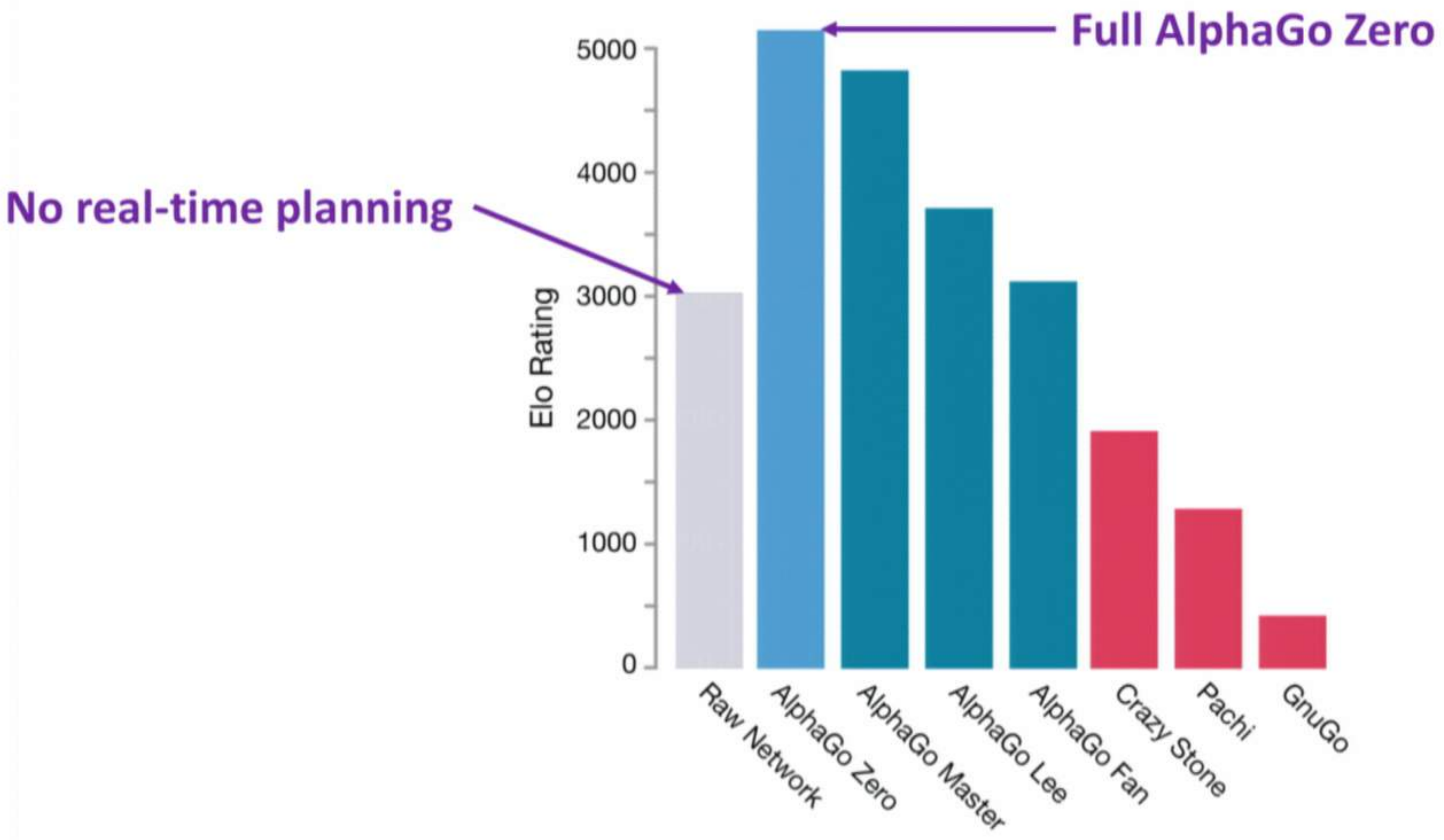
Why are imperfect-information games hard?



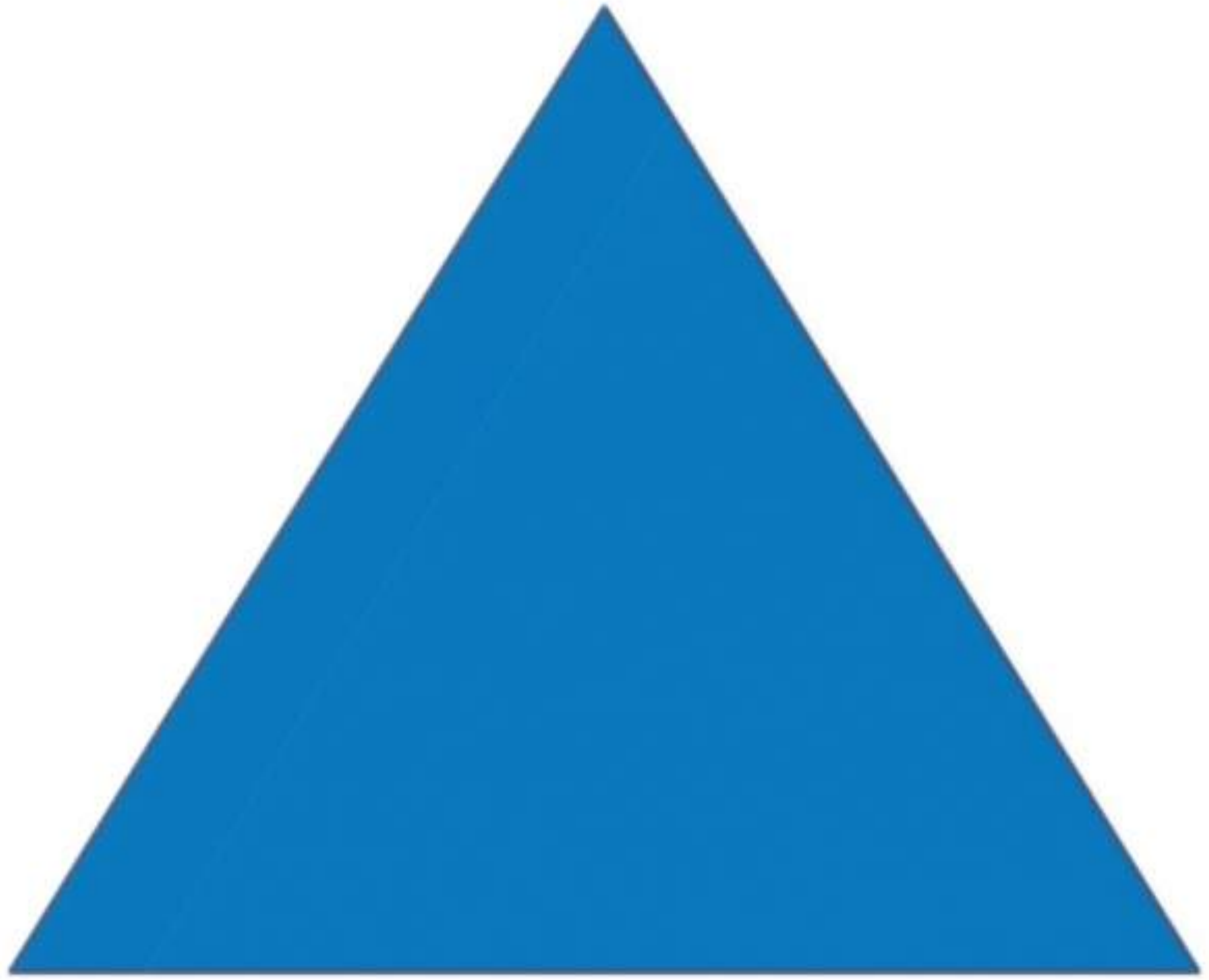
Why are imperfect-information games hard?

Because an optimal strategy for a subgame cannot be determined from that subgame alone

Real-time planning is important



Perfect-Information Games



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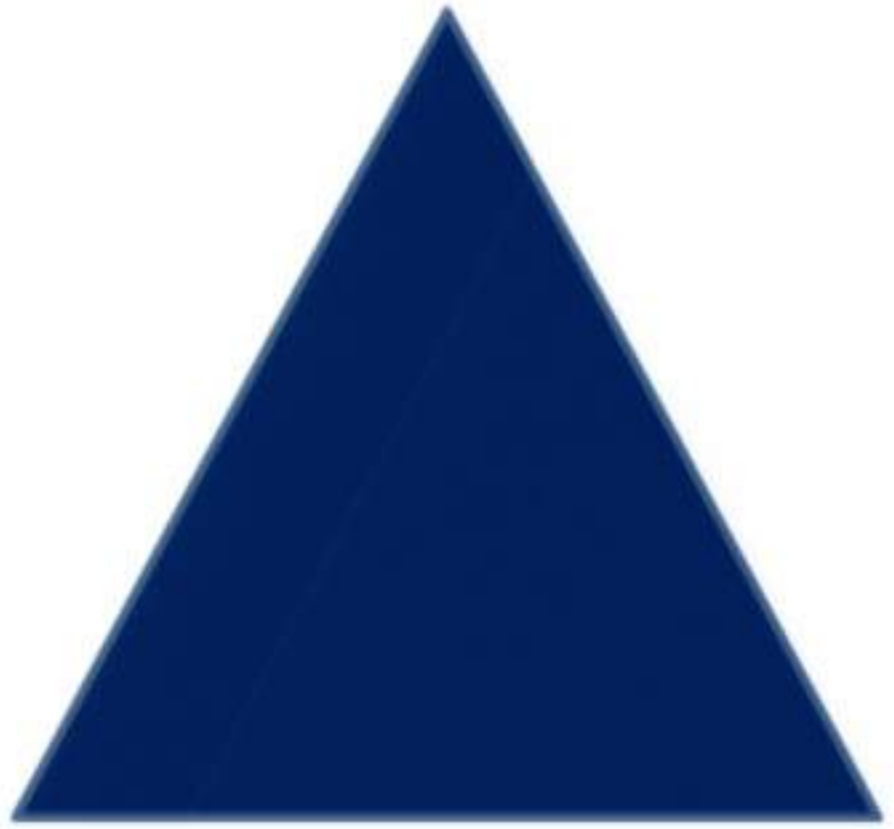

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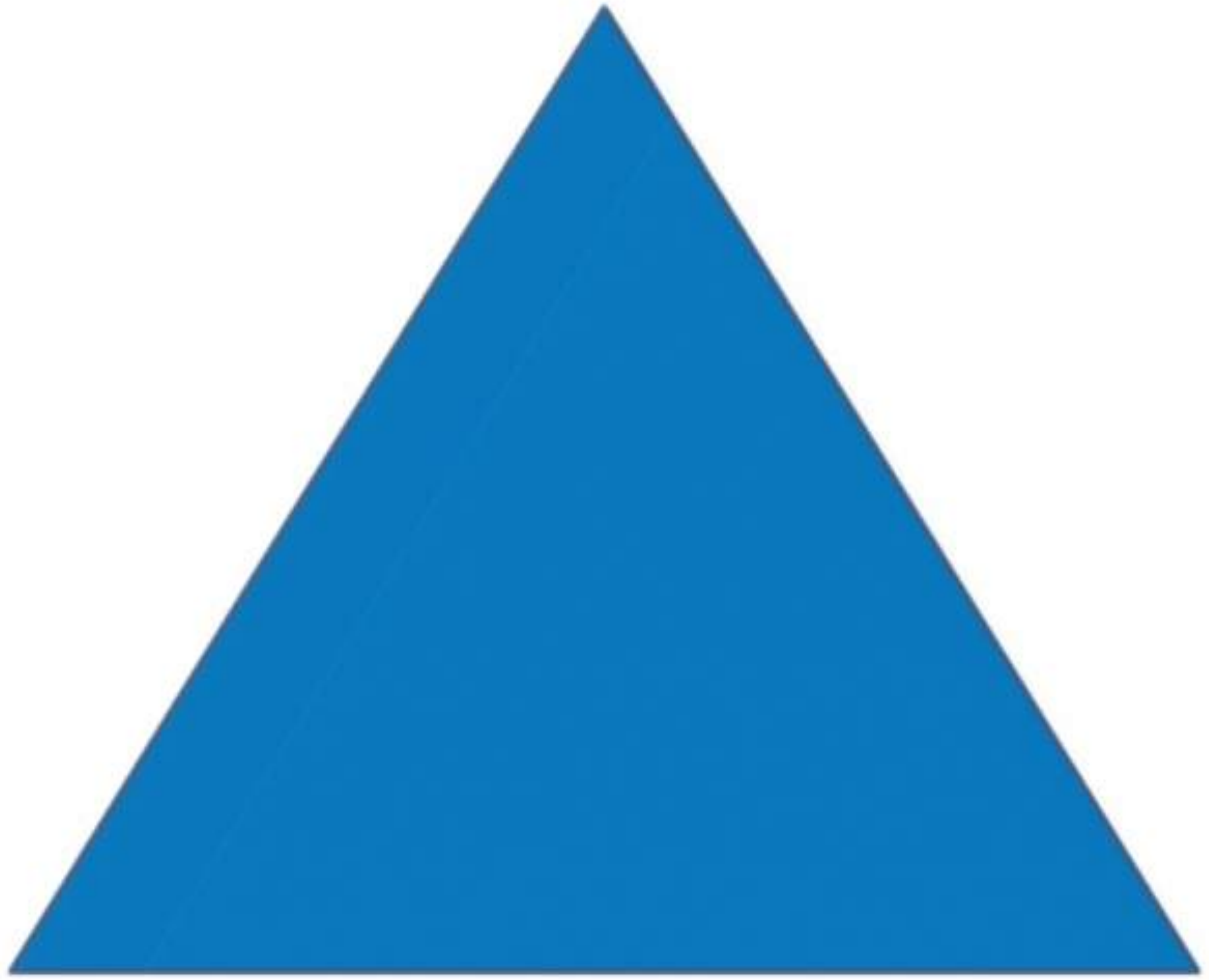

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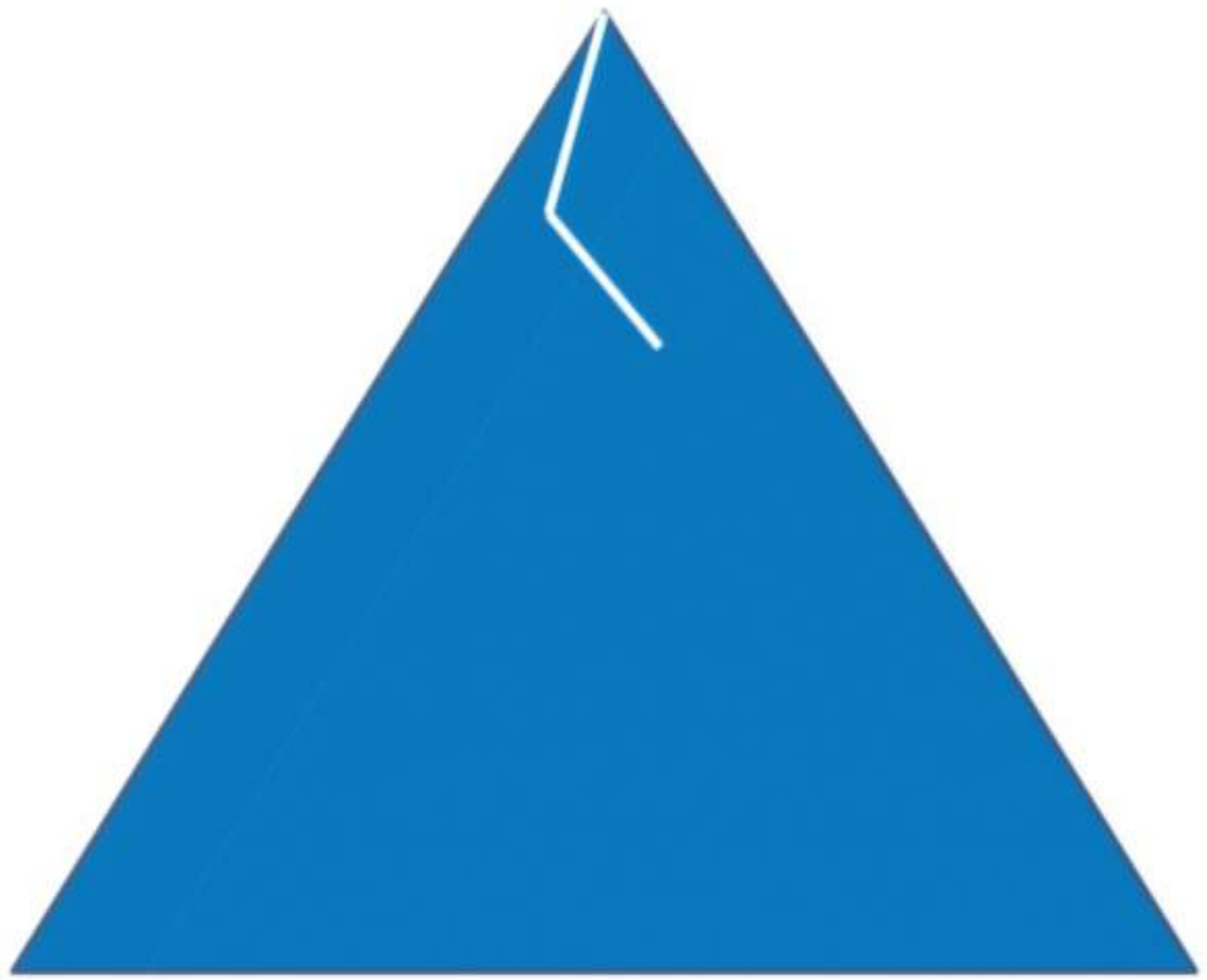

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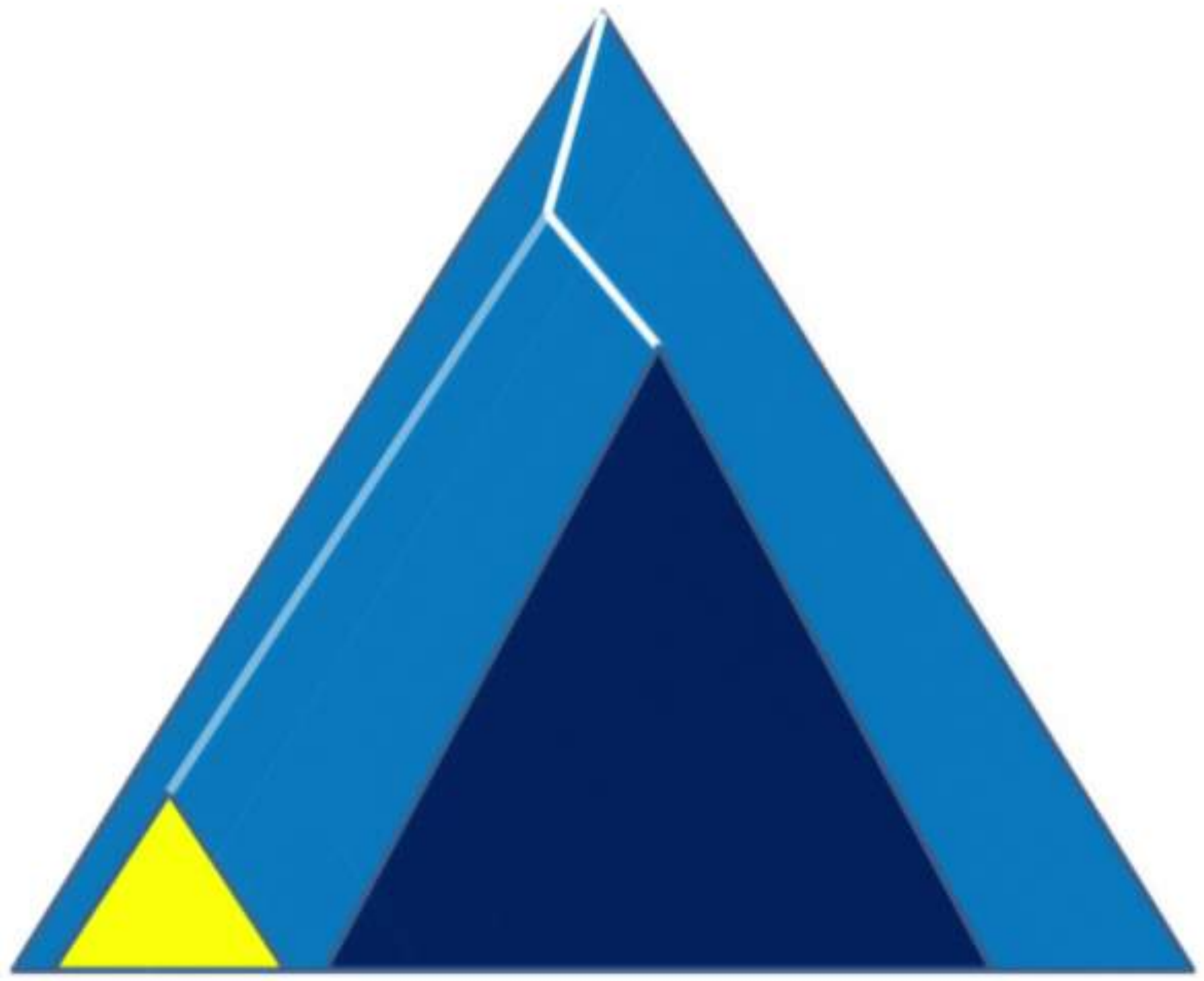

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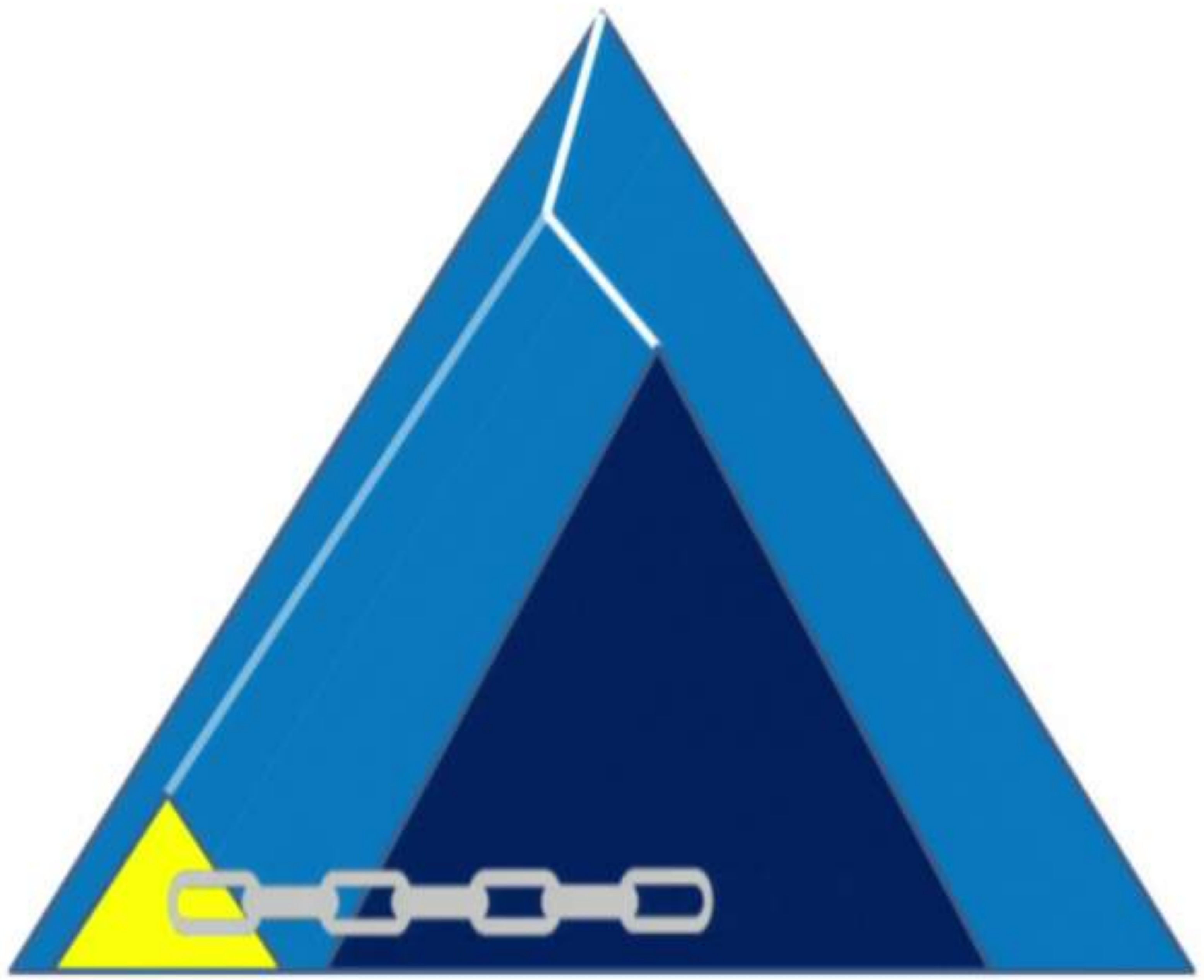


Nash Equilibrium

- **Nash Equilibrium:** a profile of strategies in which no player can improve by deviating

- In two-player zero-sum games, playing a Nash equilibrium ensures you will not lose in expectation

- **Exploitability**: Worst-case performance relative to Nash

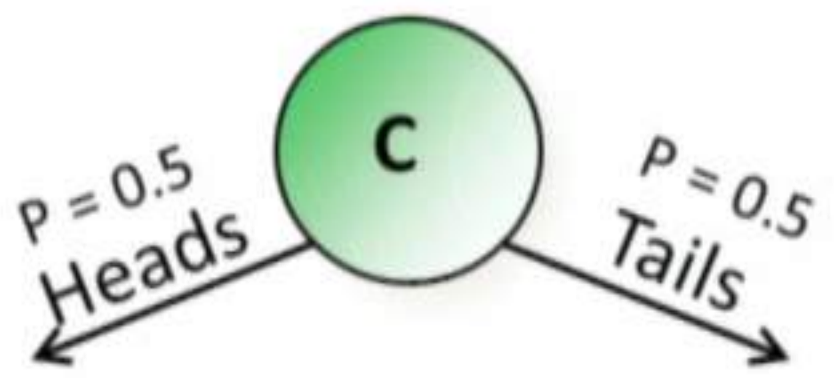
Why care about exploitability?

Real-world AI must be **robust** to adversarial adaptation and exploitation

Fan Hui vs AlphaGo

OpenAI 1v1 Dota2 Matches

Example game: Coin Toss



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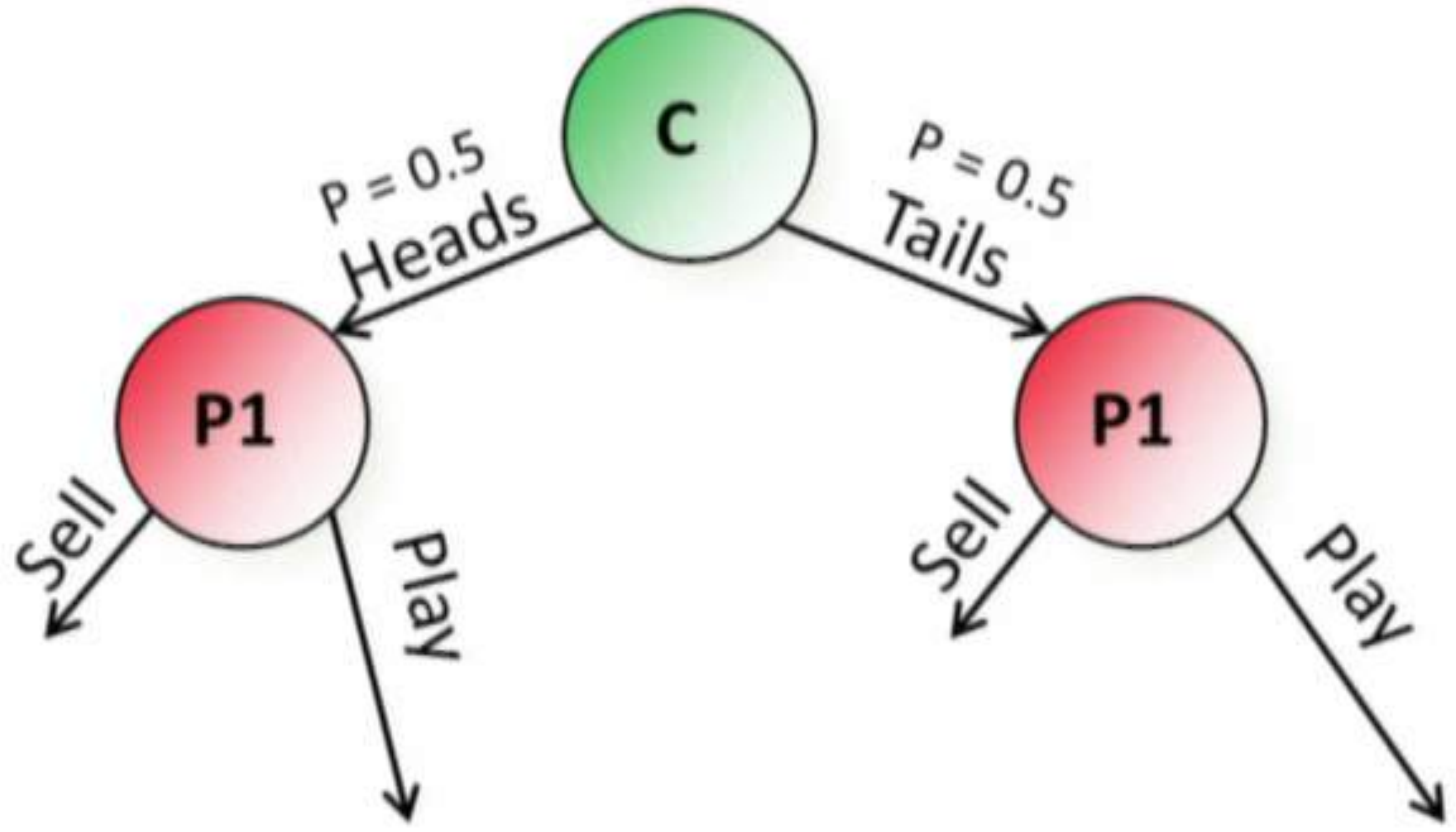

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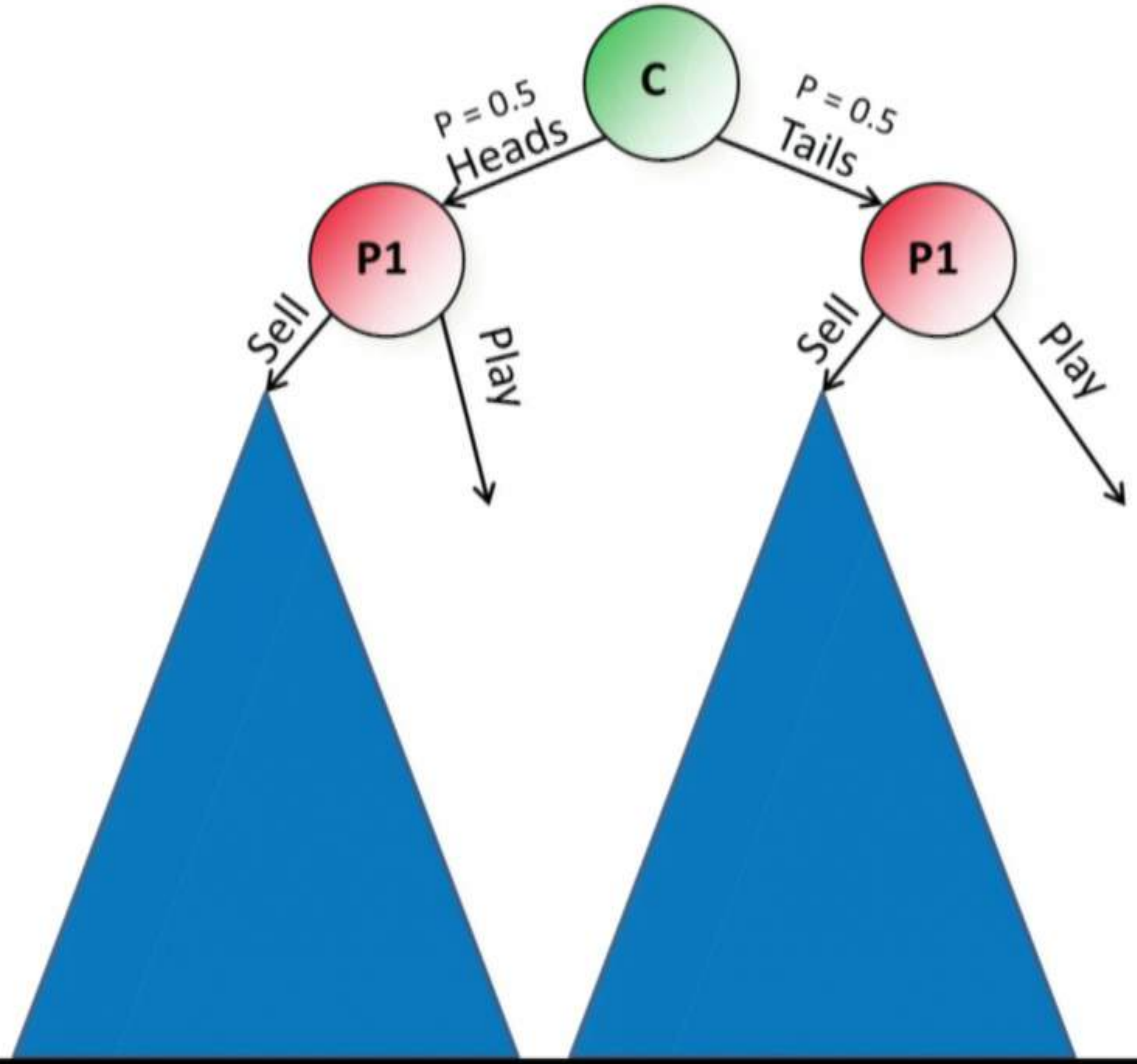

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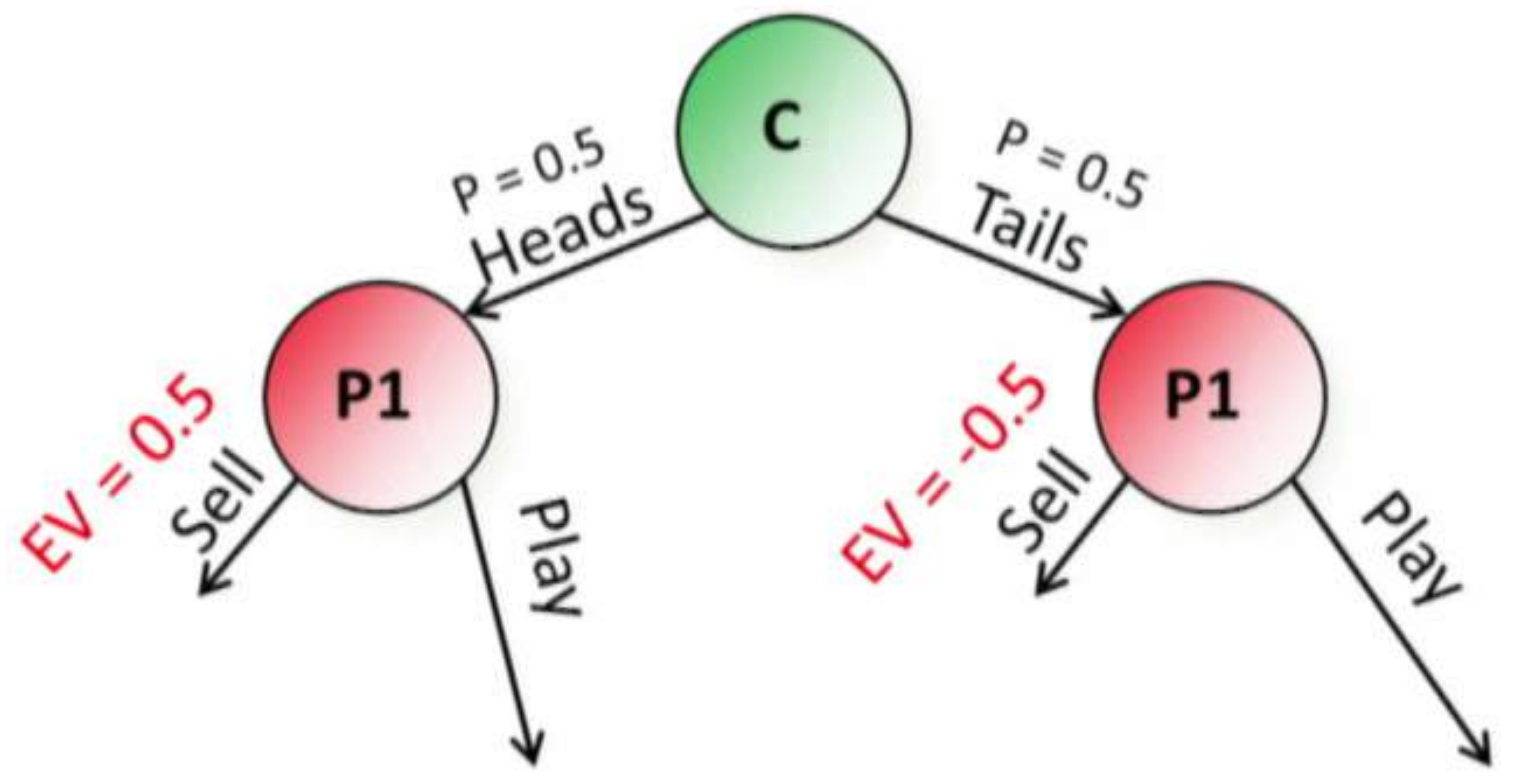

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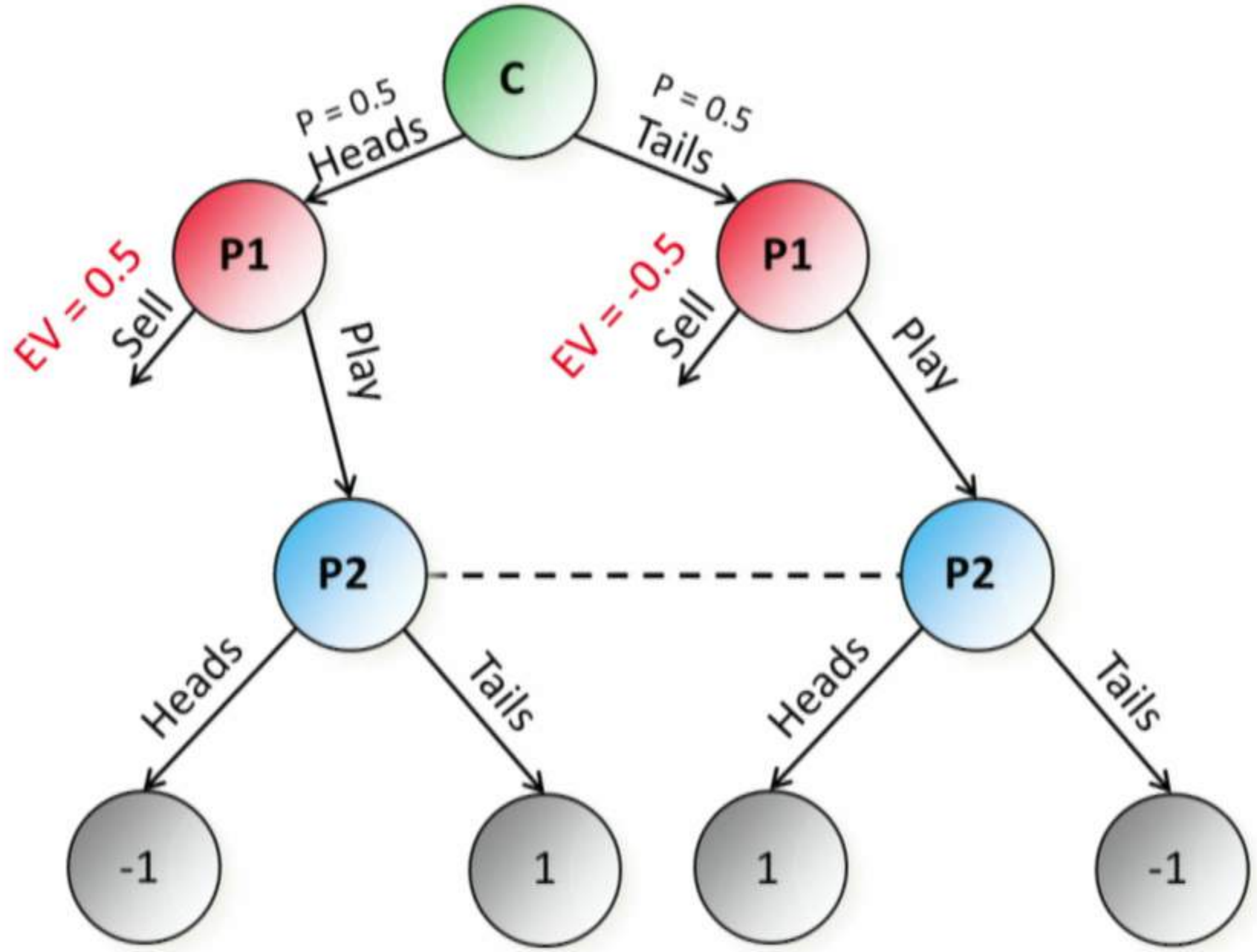

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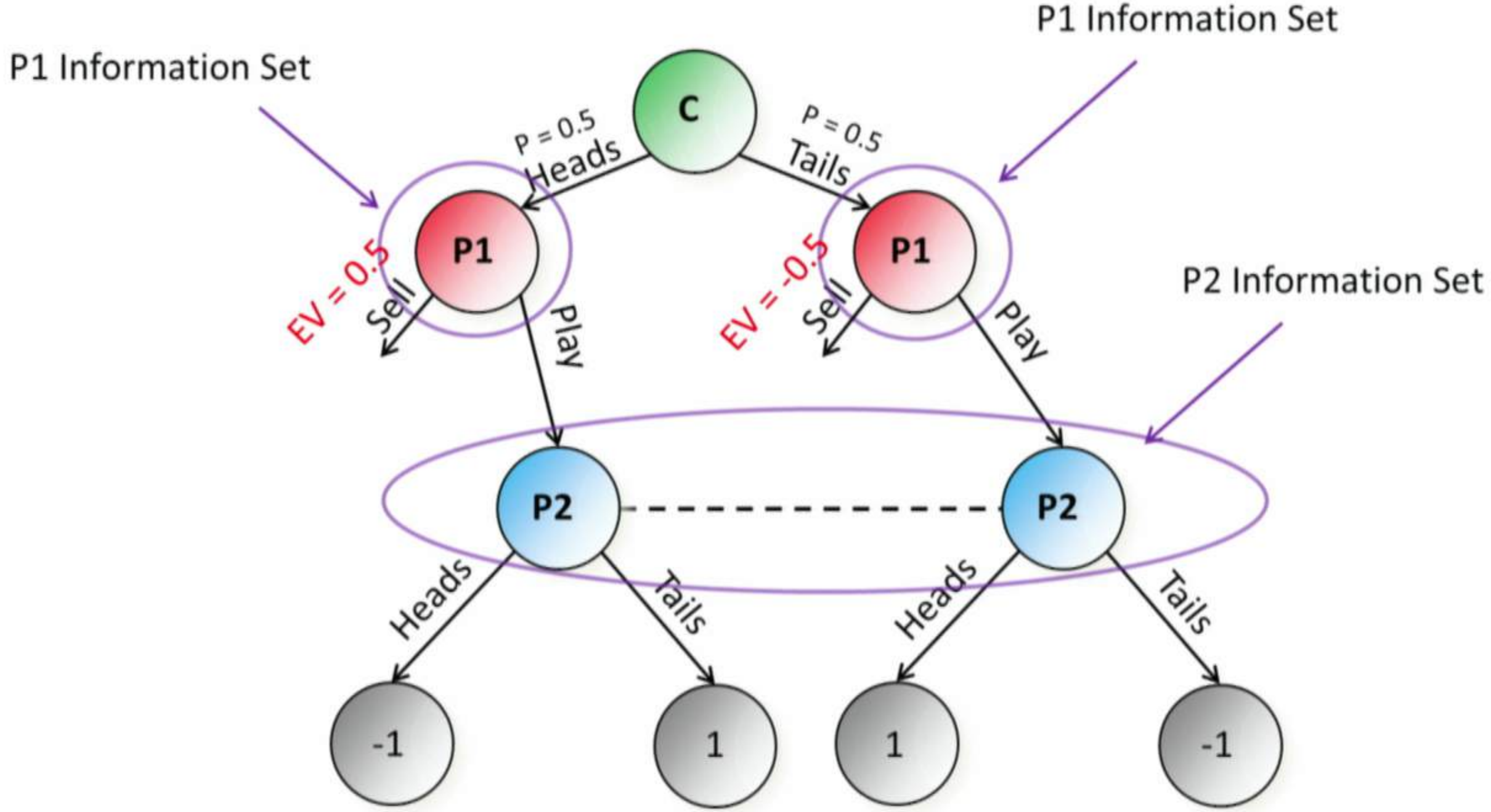

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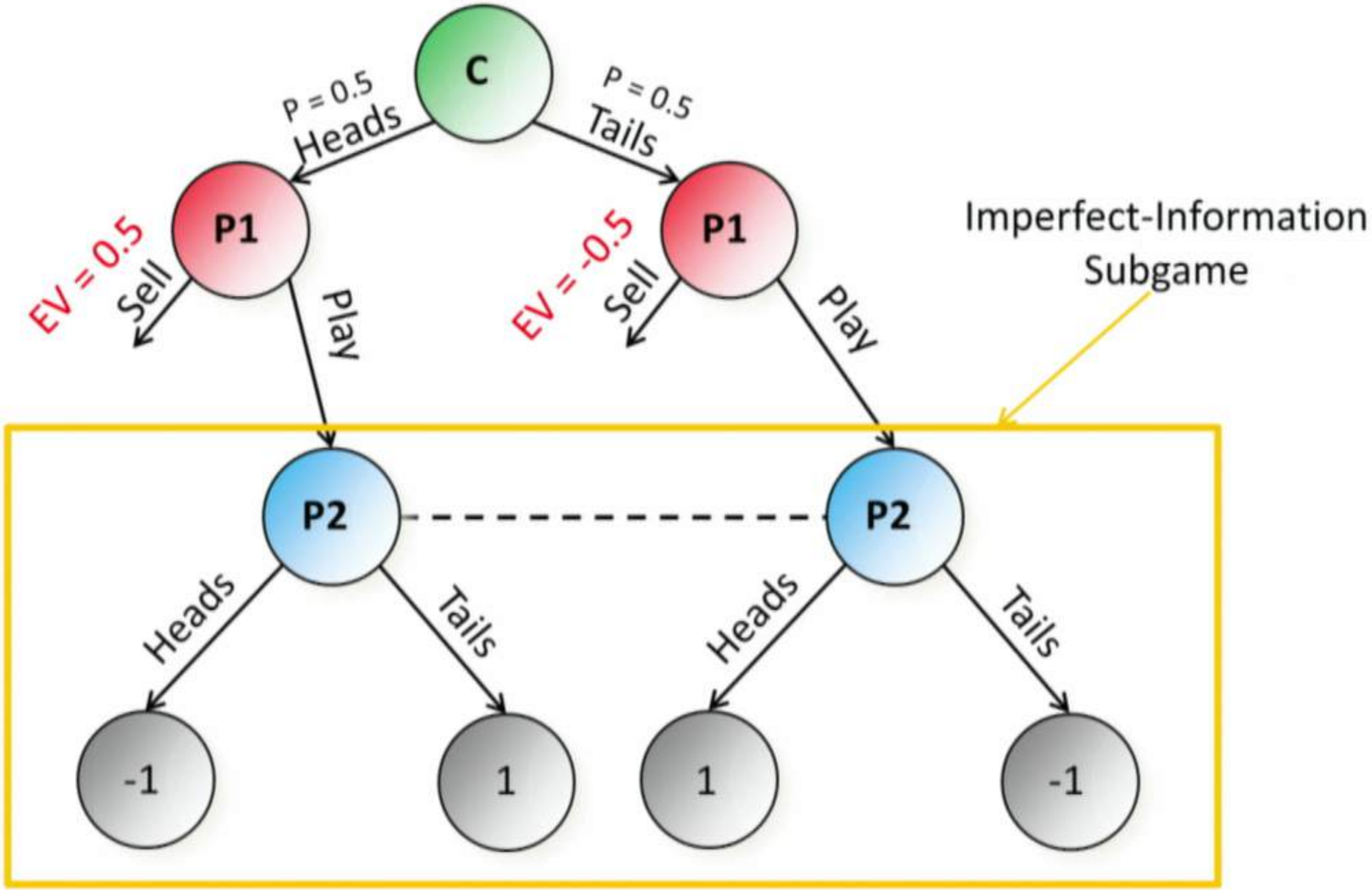

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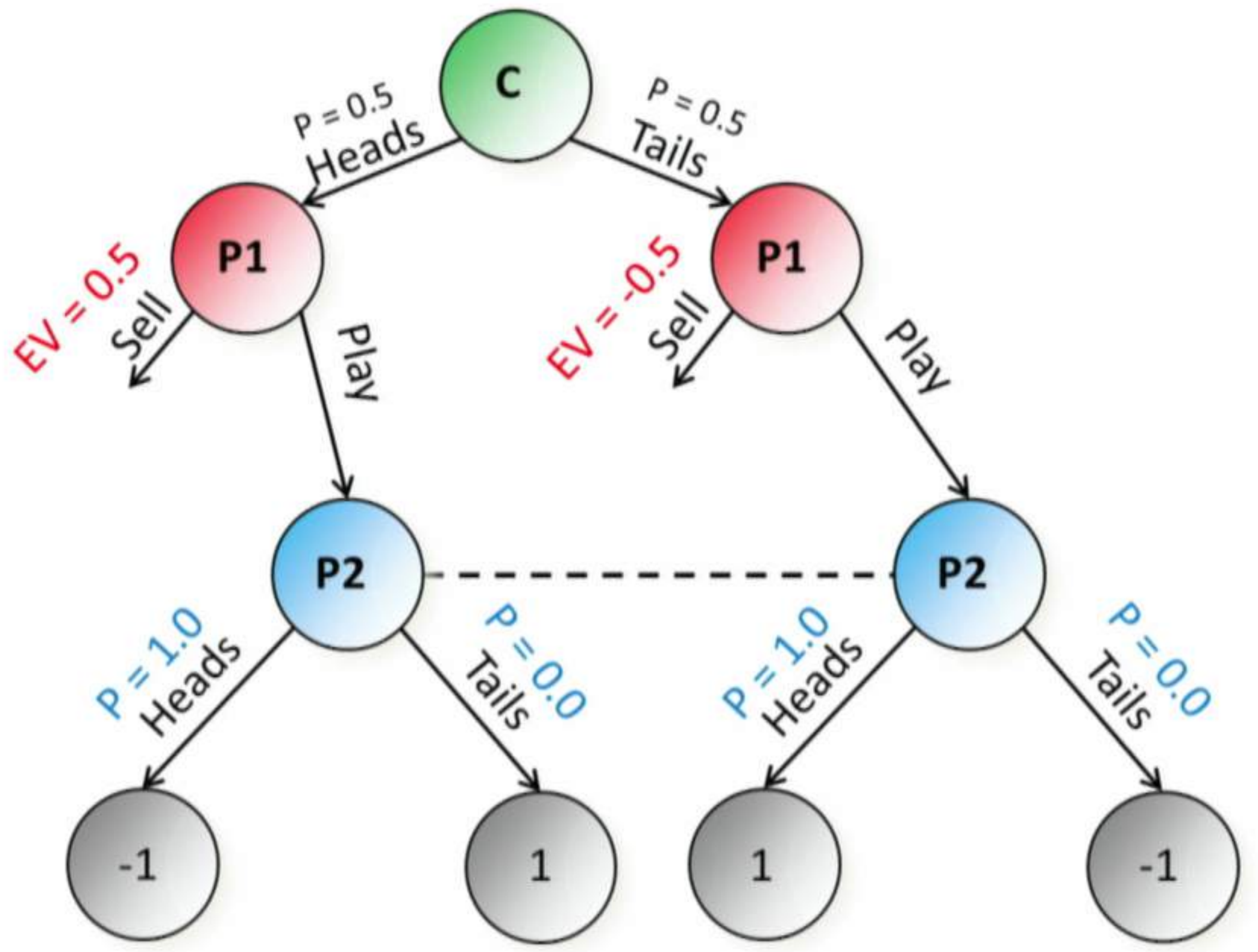
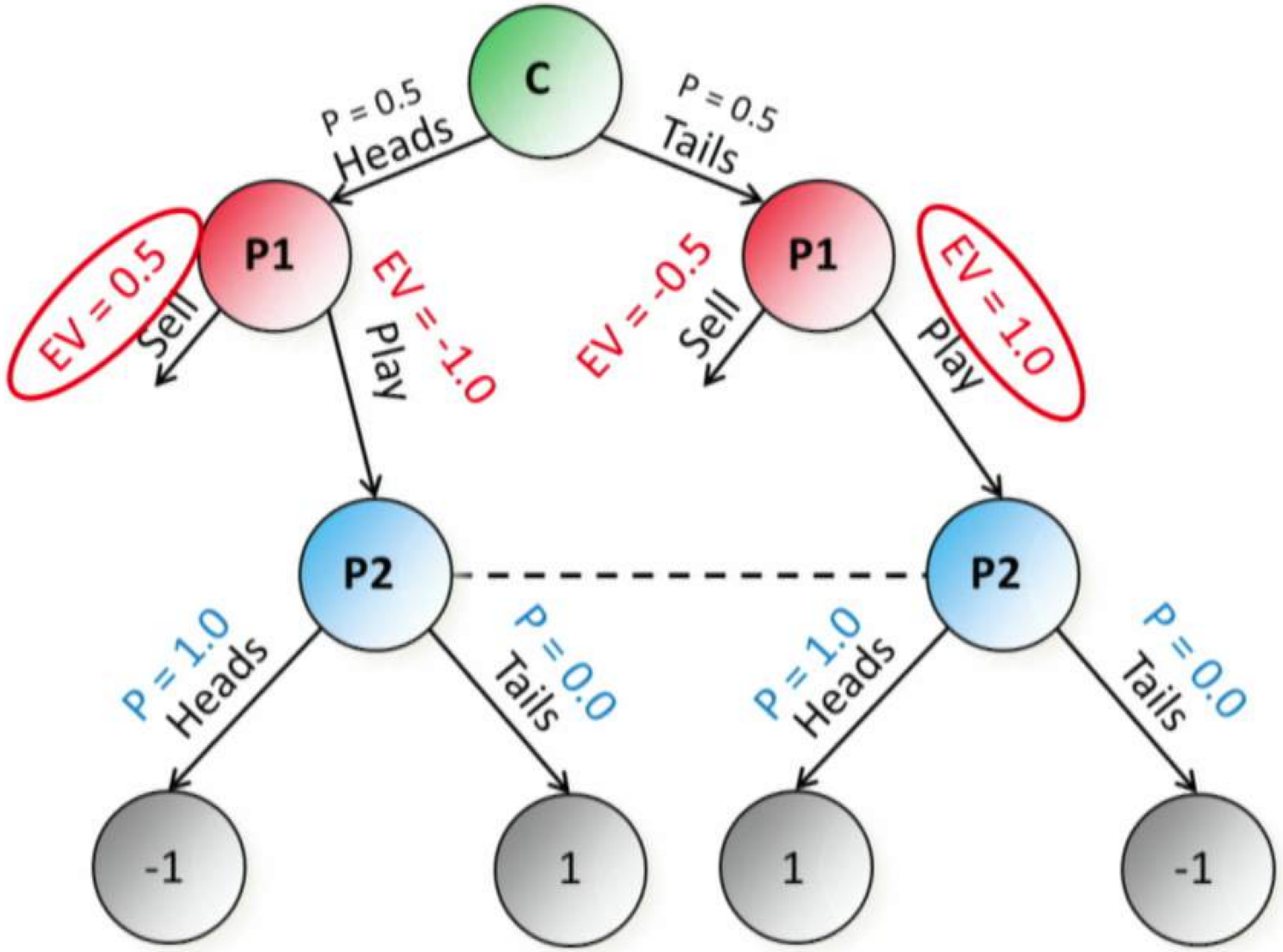


Example game: Coin Toss

Heads EV = 0.5
Tails EV = 1.0
Average = 0.75



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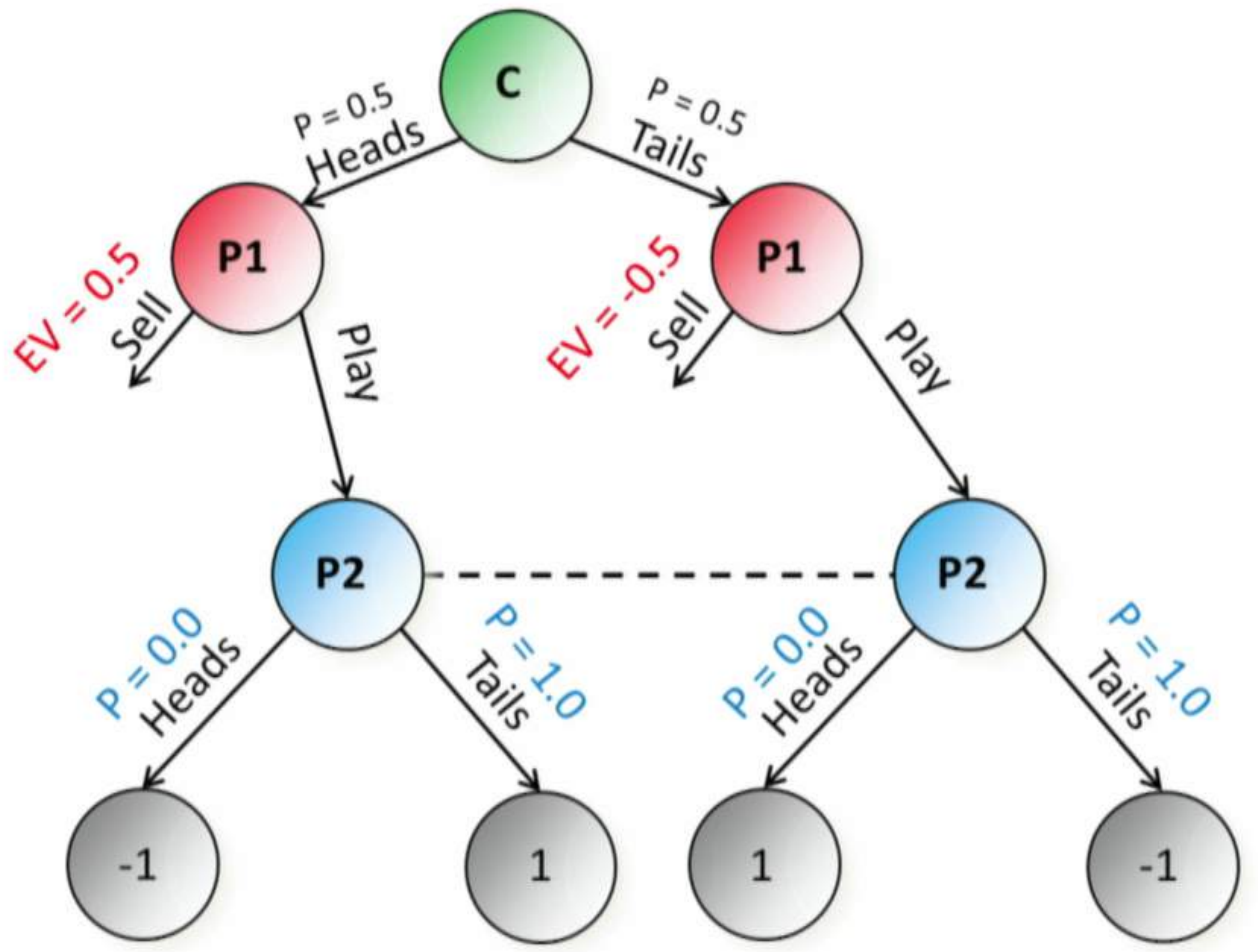

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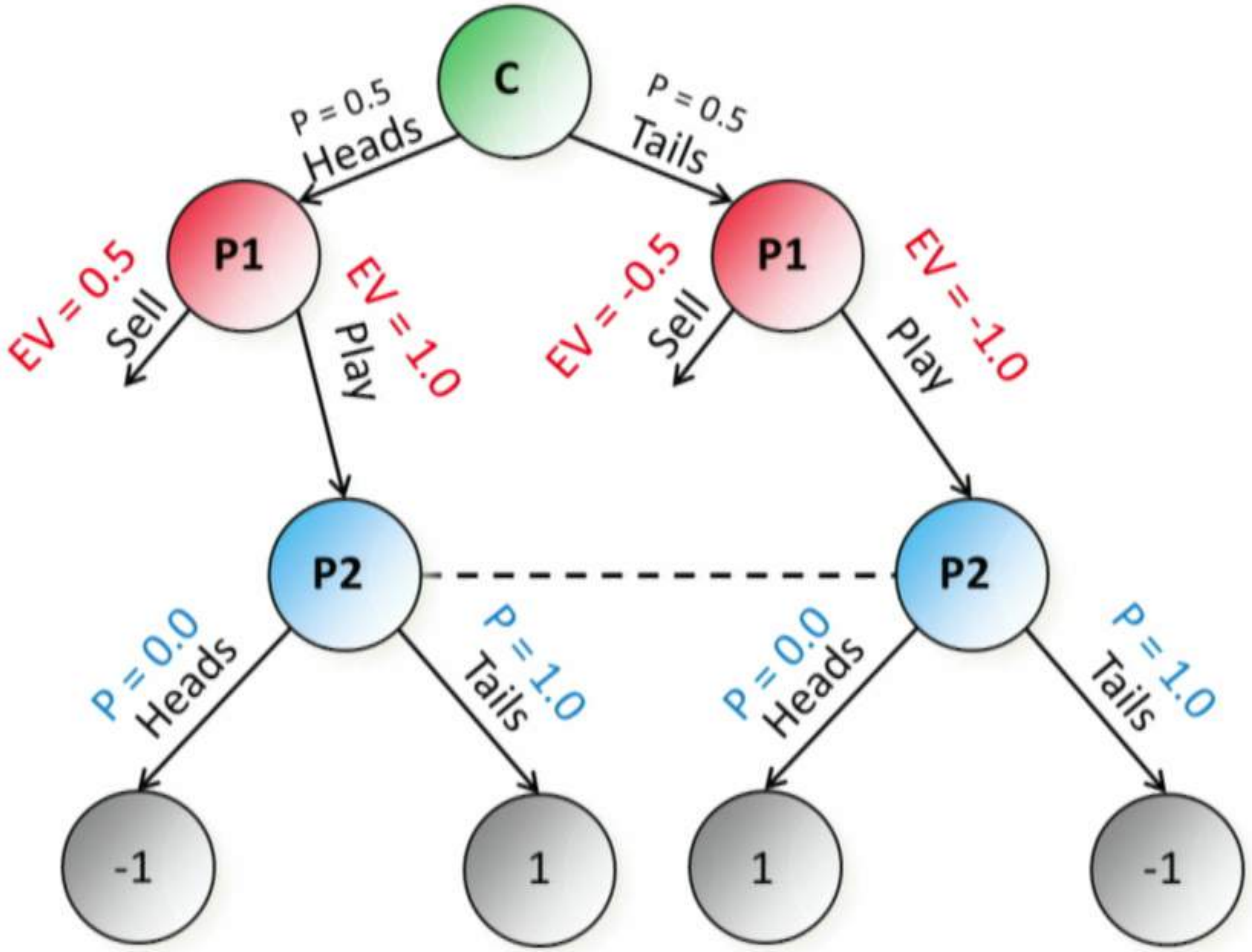
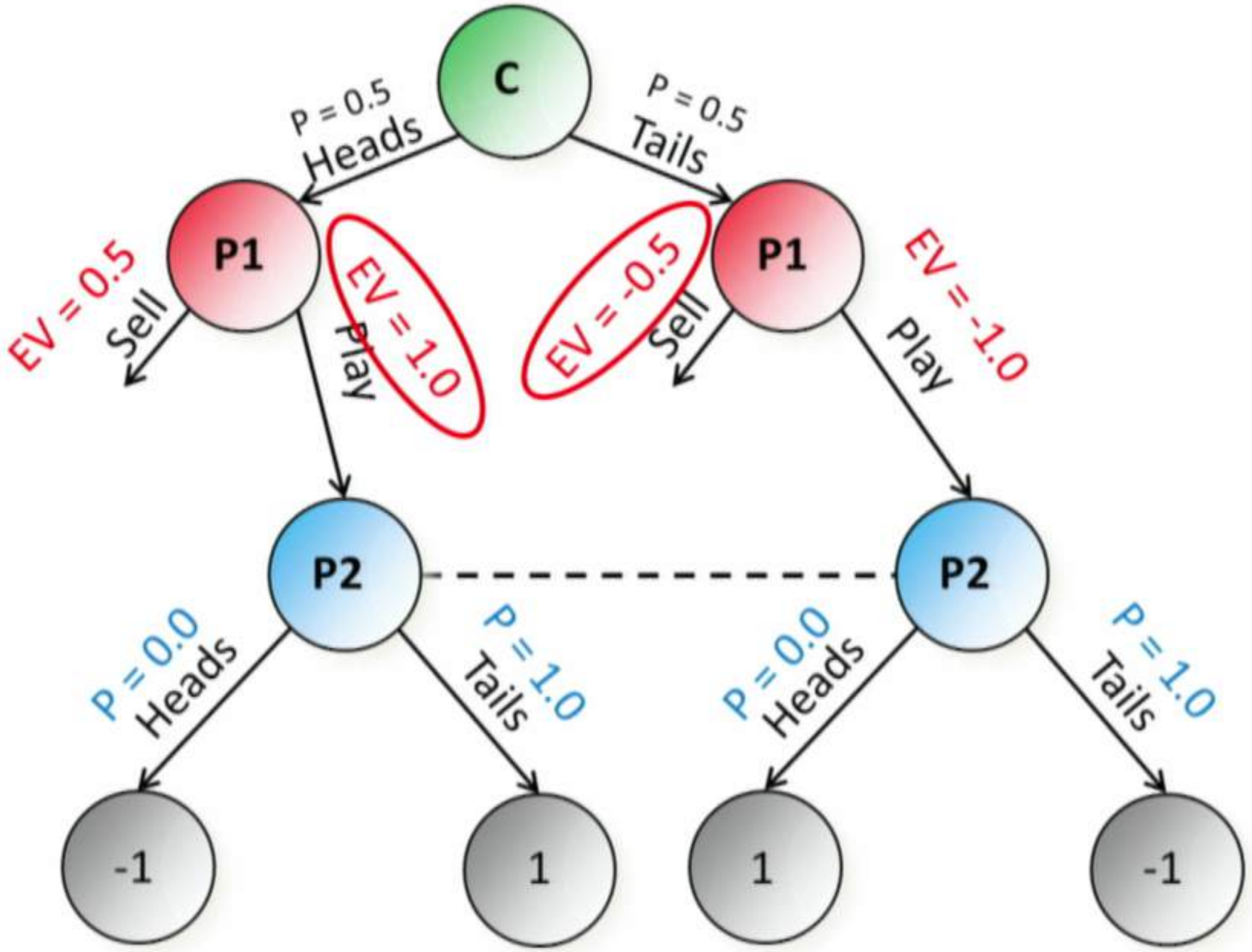


Example game: Coin Toss

Heads EV = 1.0
Tails EV = -0.5
Average = 0.25



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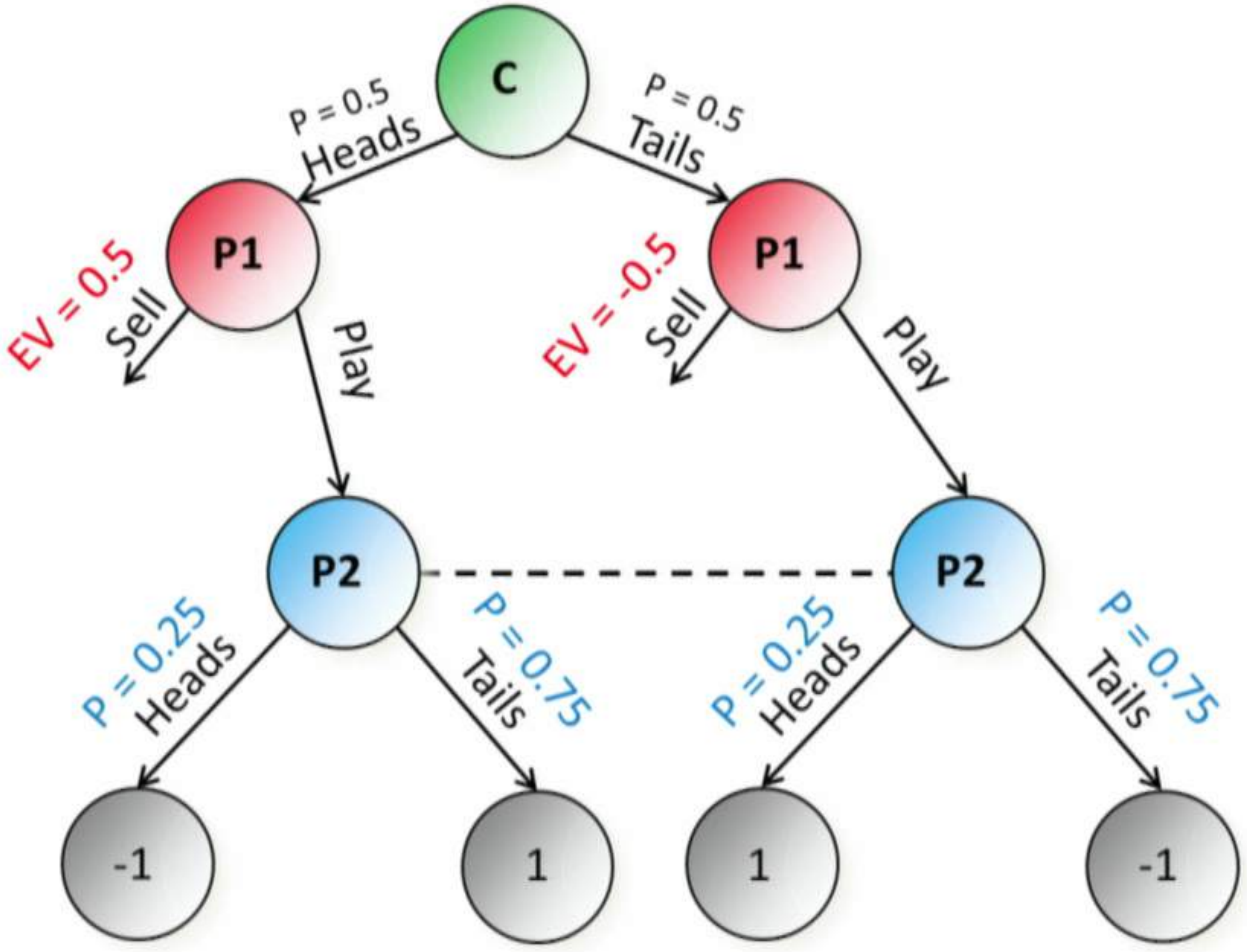

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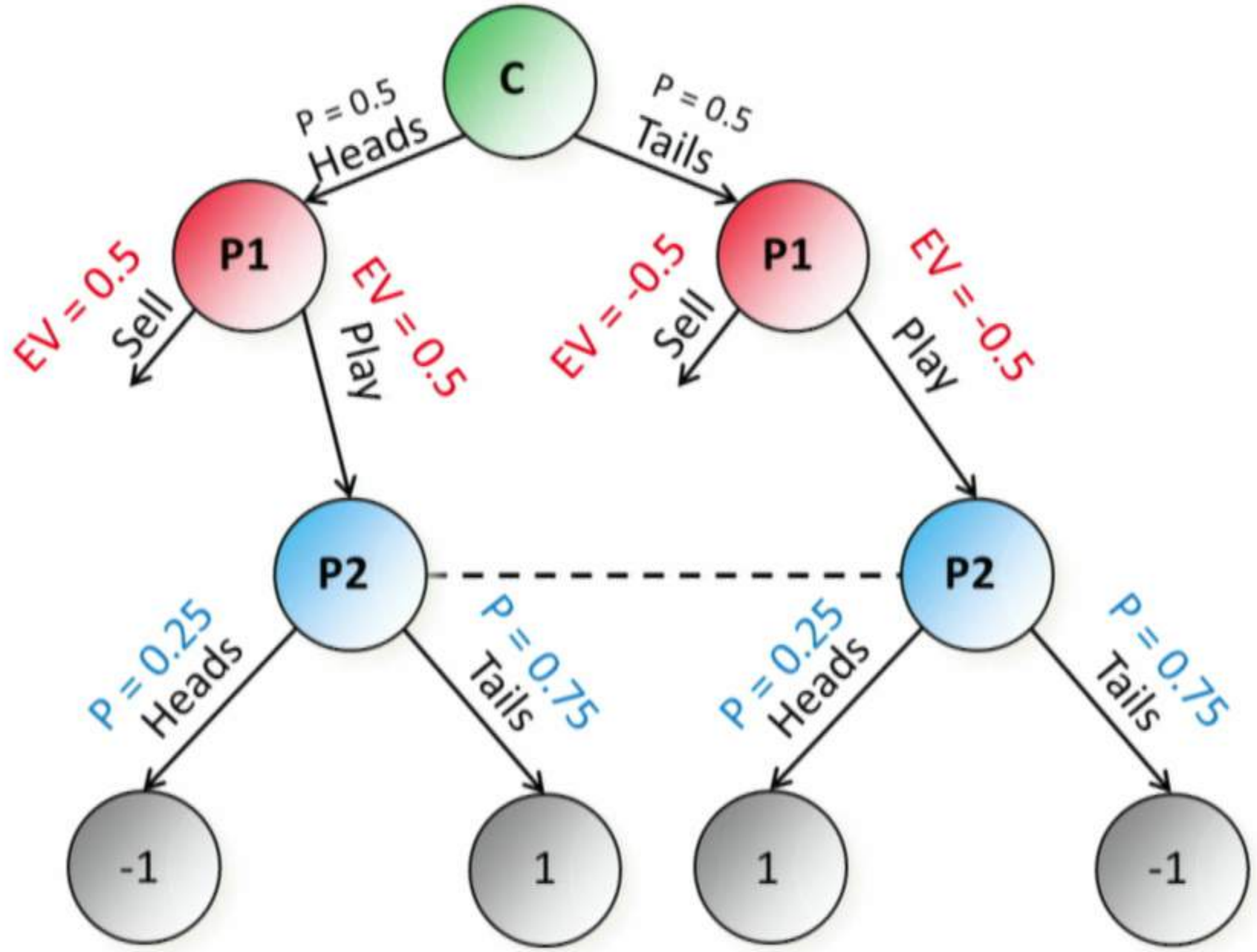

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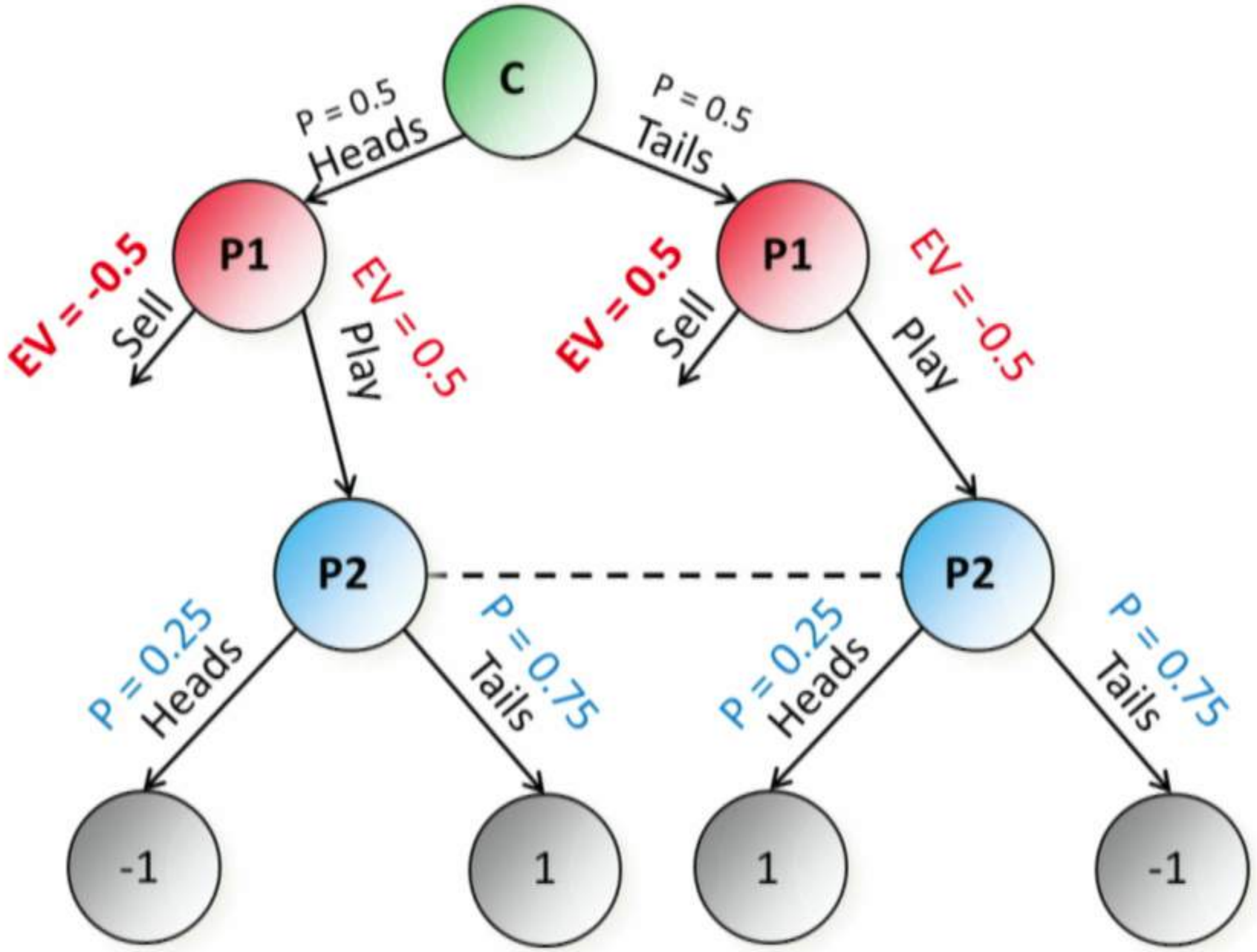
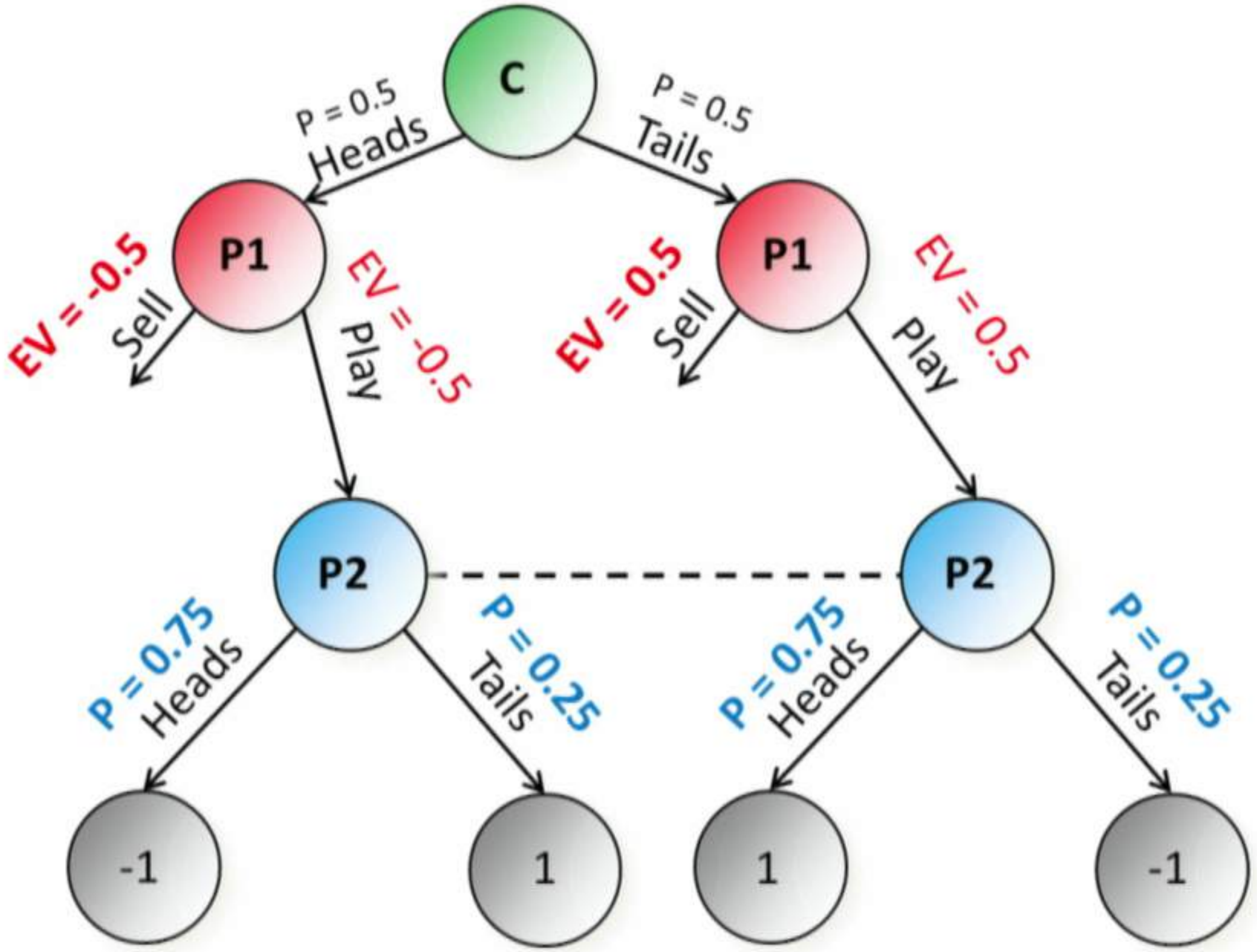


Example game: Coin Toss

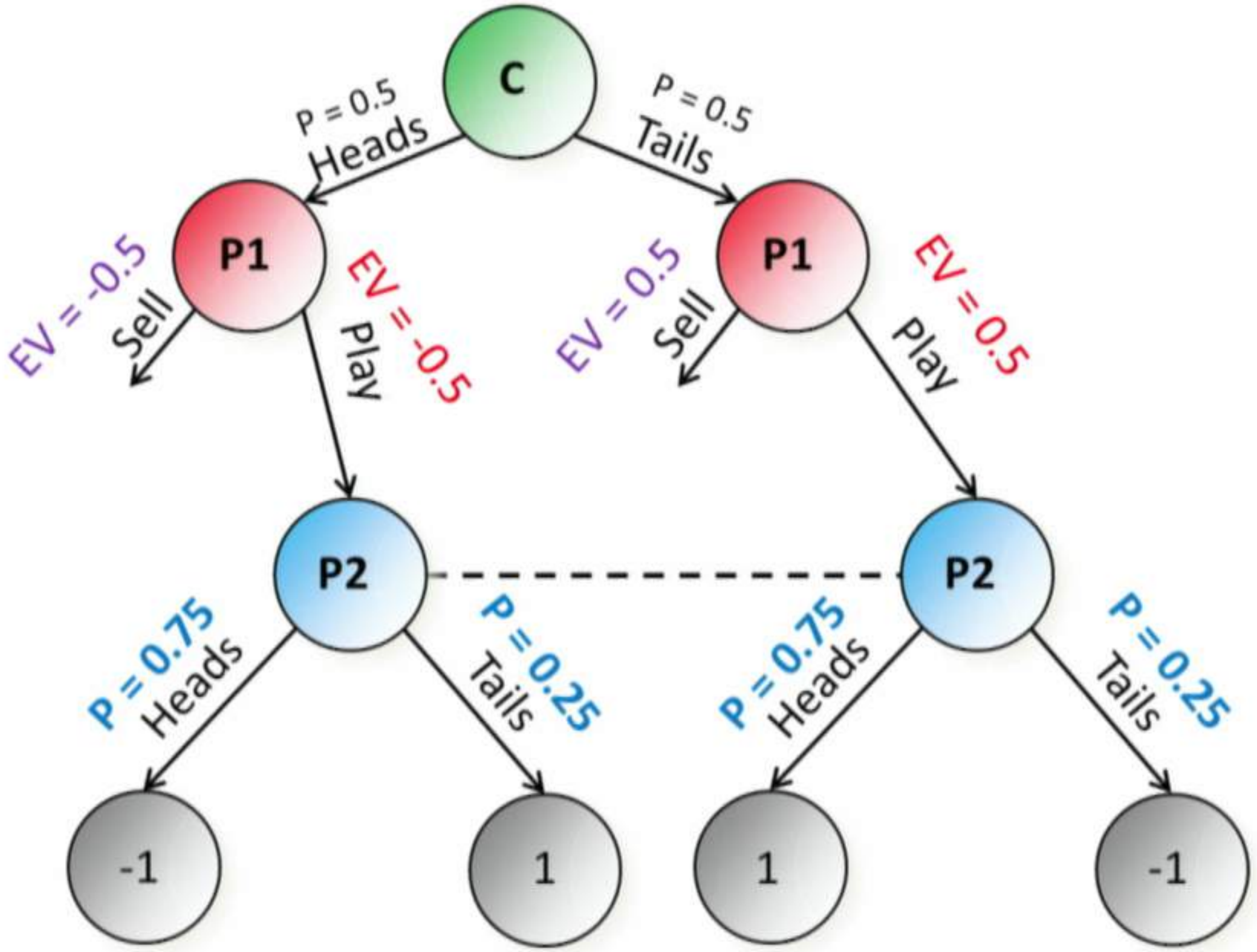
Determining the optimal strategy in the “Play” subgame requires knowledge of the **values** the opponent could receive in other subgames



Example game: Coin Toss

[Burch et al AAAI-14, Moravcik et al AAAI-16, Brown & Sandholm NIPS-17]

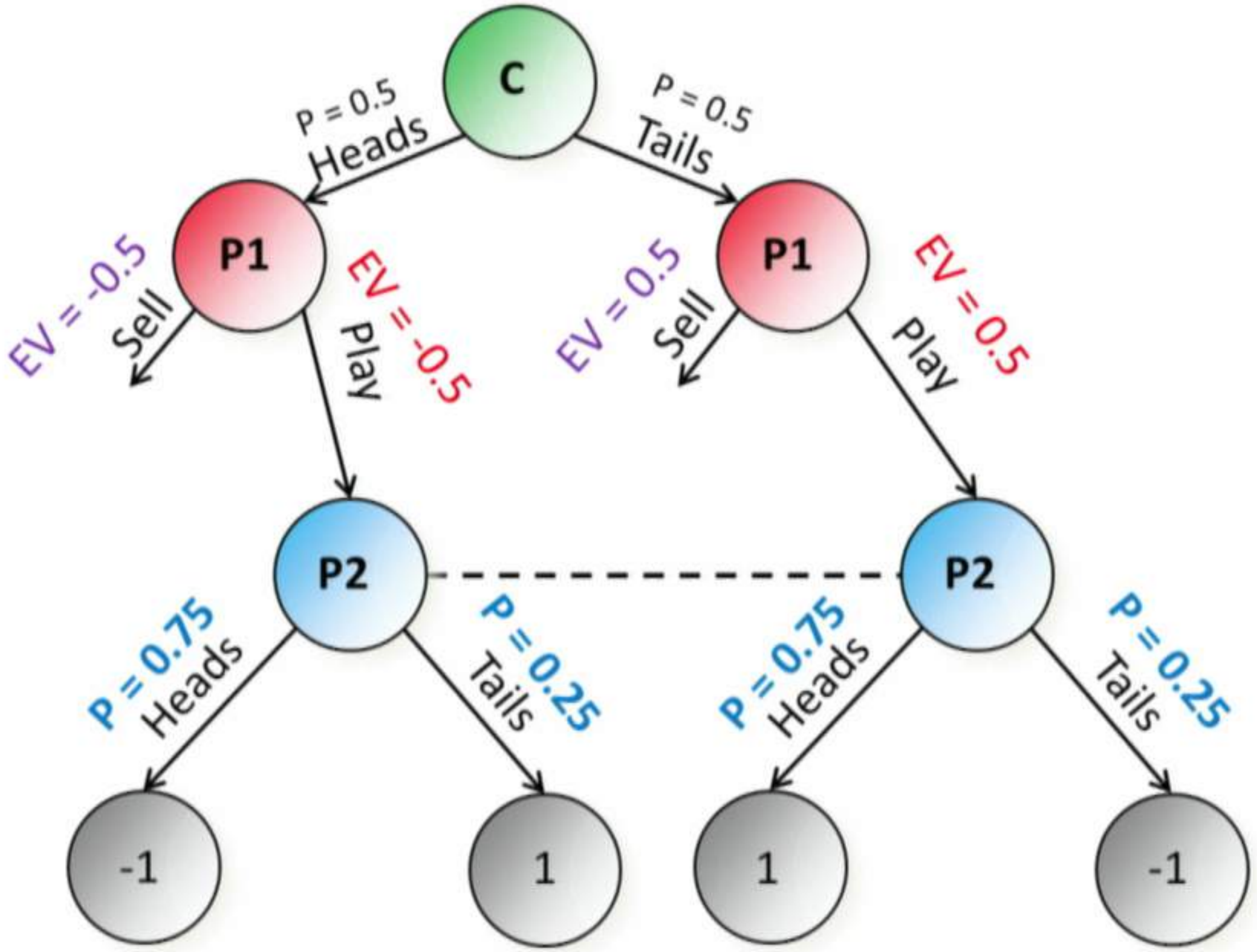
Idea: Estimate the **values** the opponent receives for actions in an equilibrium



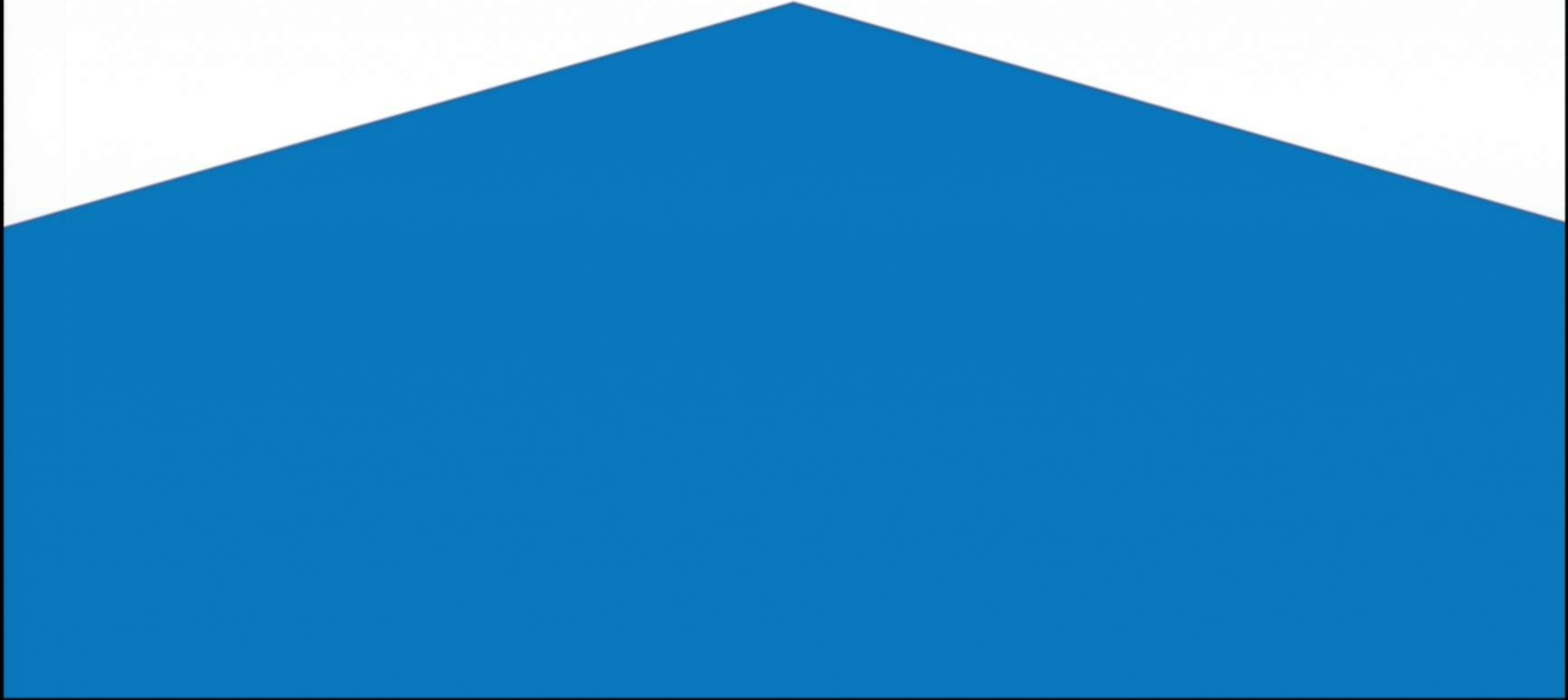
Example game: Coin Toss

[Brown & Sandholm Science-17]

Theorem: If estimates of opponent values are off by at most δ, then safe subgame solving has at most δ exploitability.



Nested subgame solving



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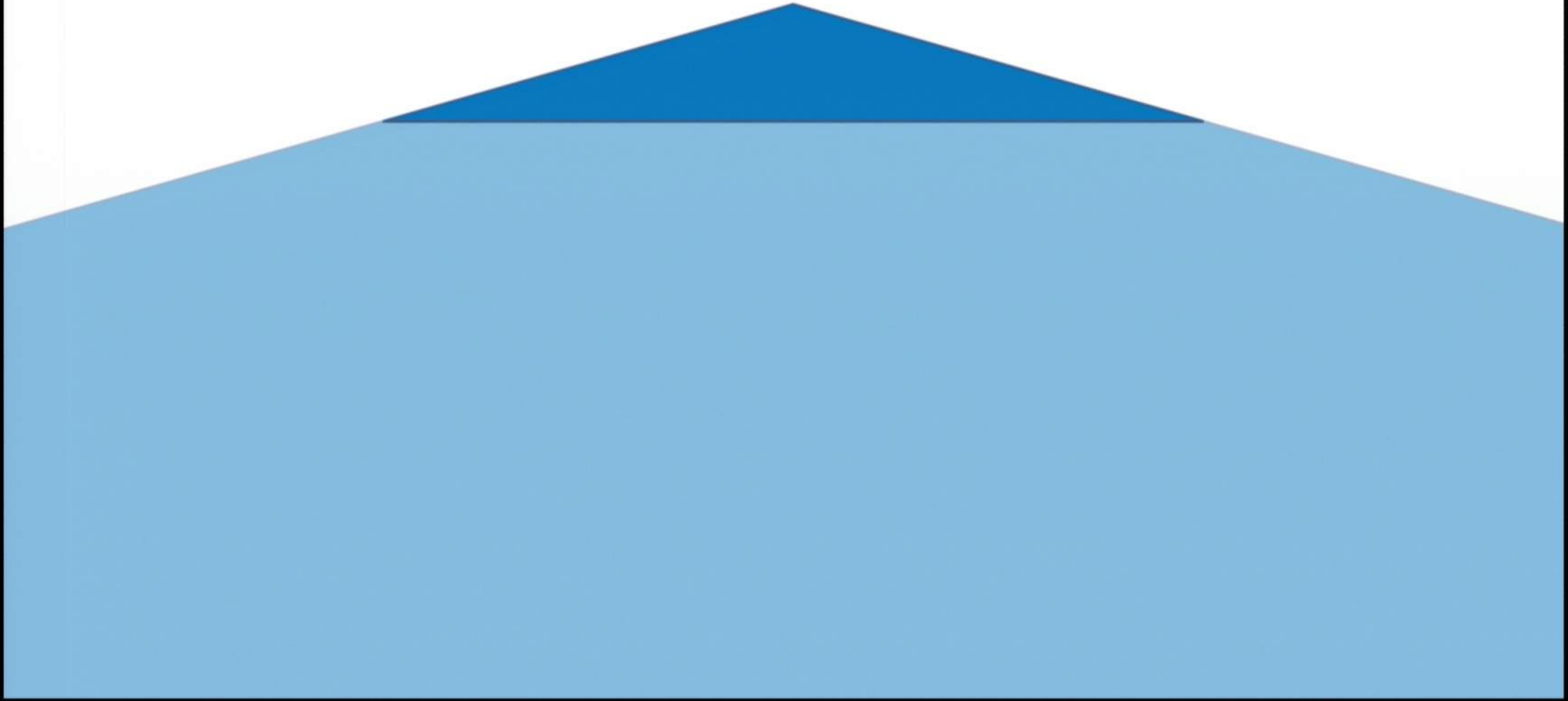

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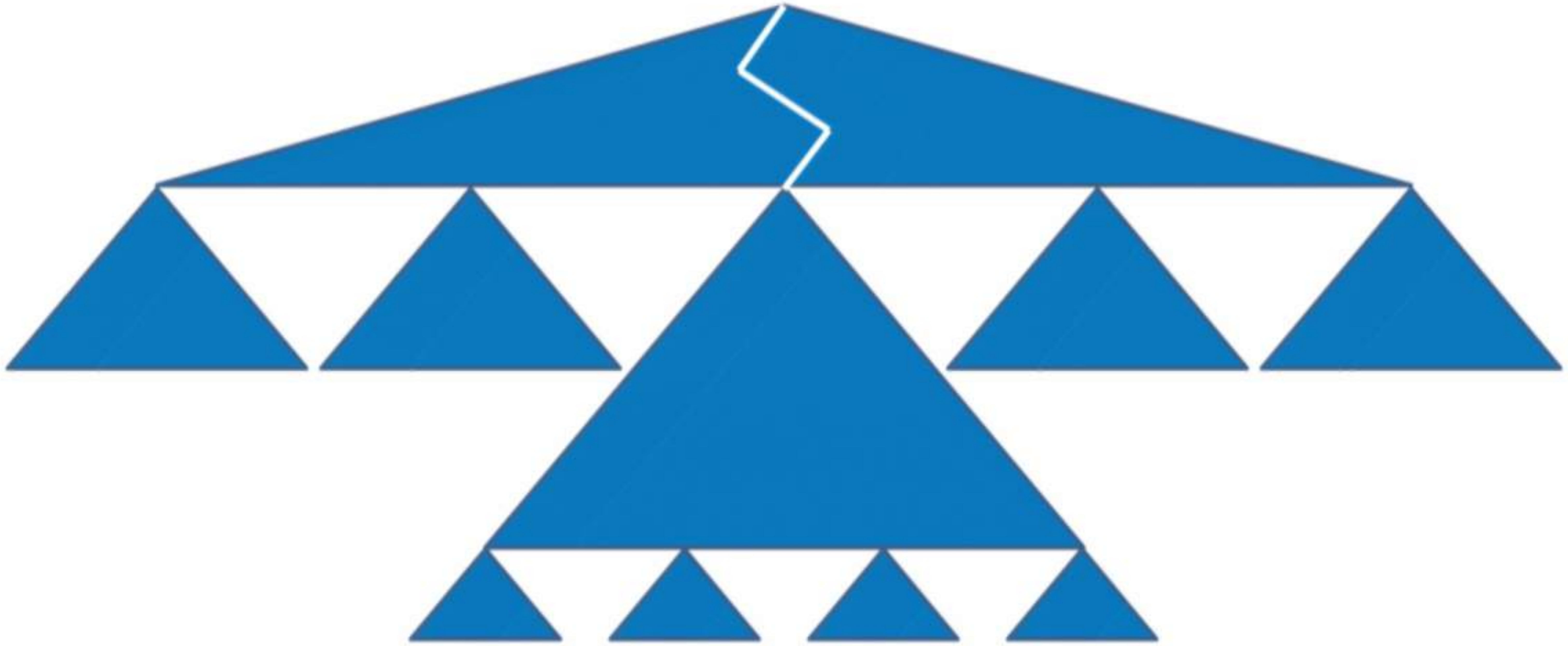

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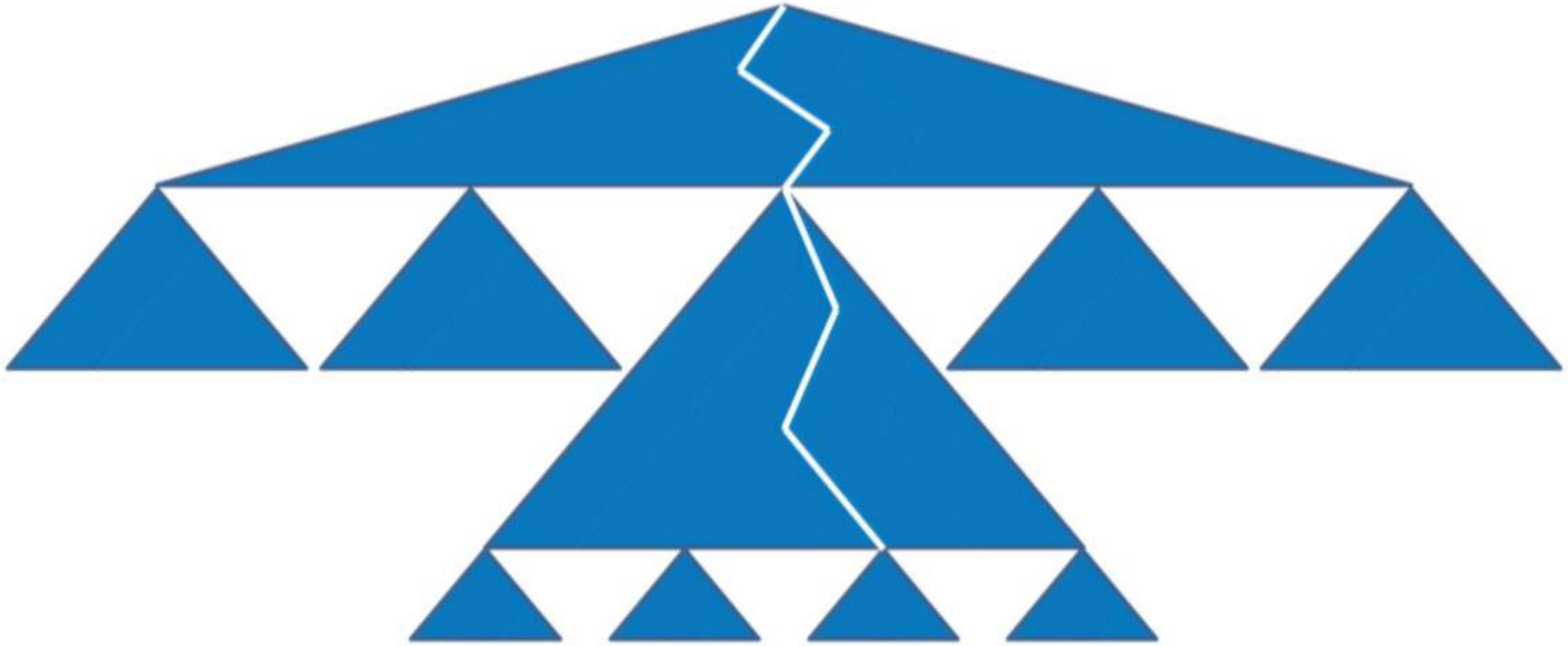
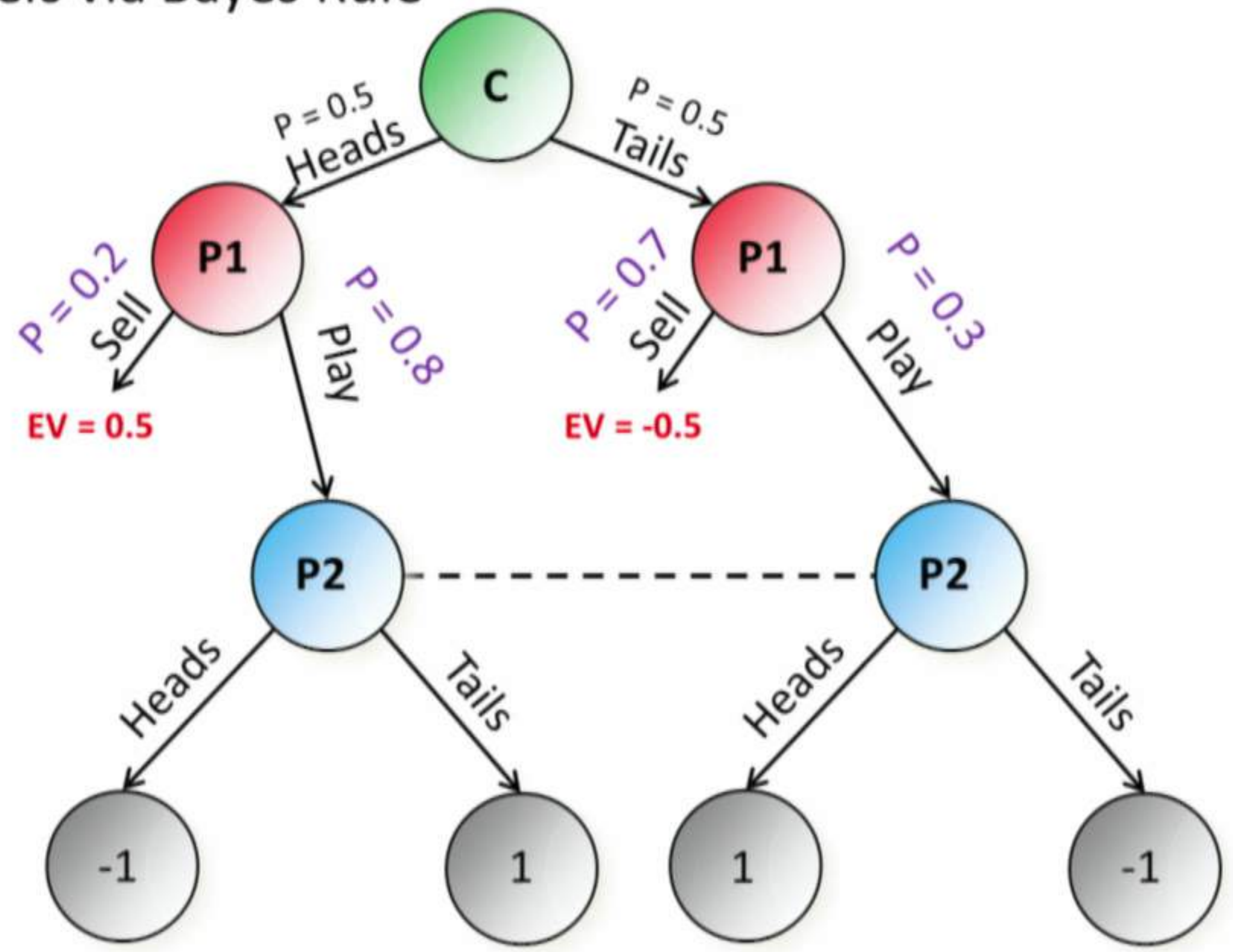


Nested subgame solving

Unsafe Subgame Solving

[Ganzfried & Sandholm AAMAS 2015]

- Estimate the opponent's strategy
- This gives a belief distribution over states
- Update beliefs via Bayes Rule



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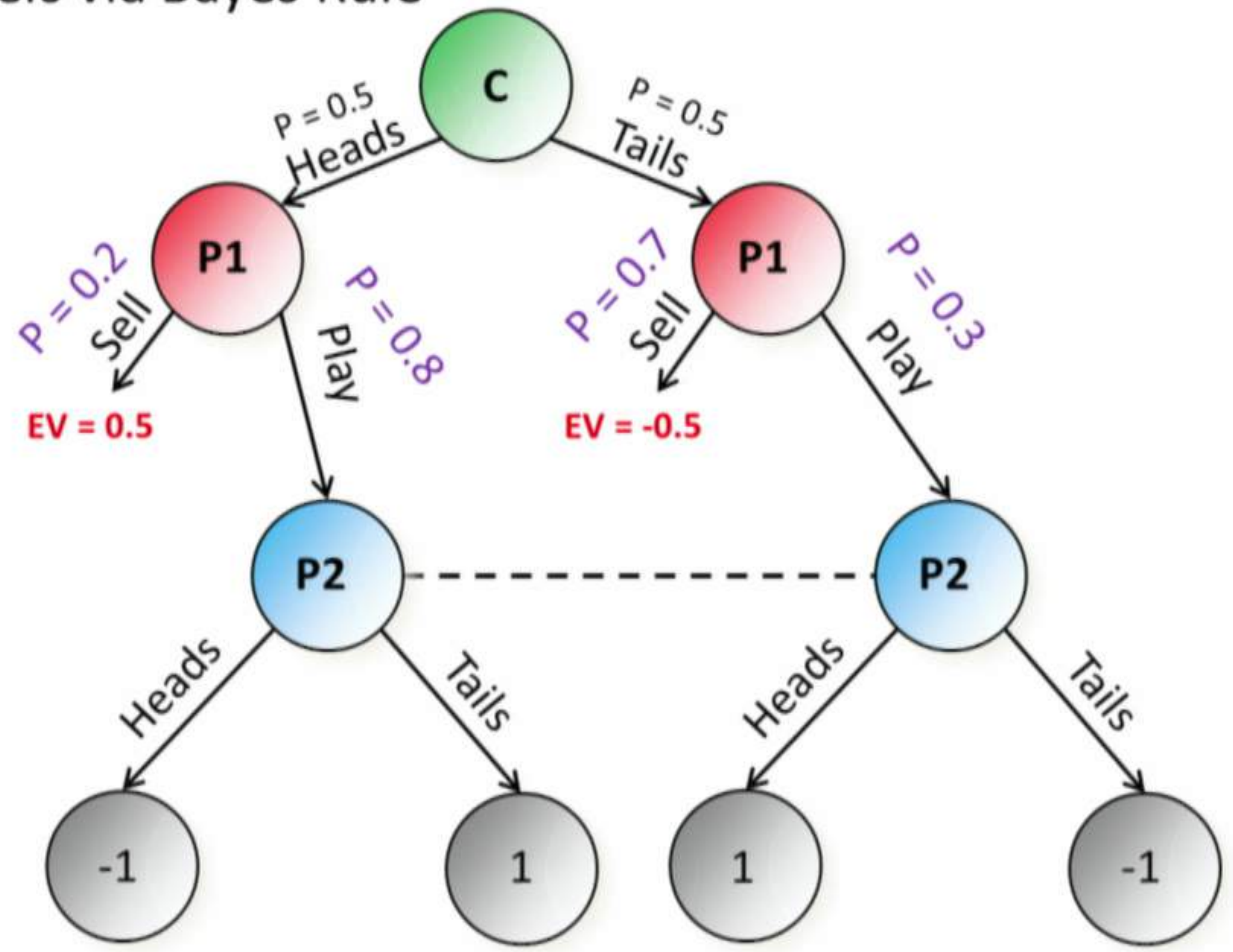

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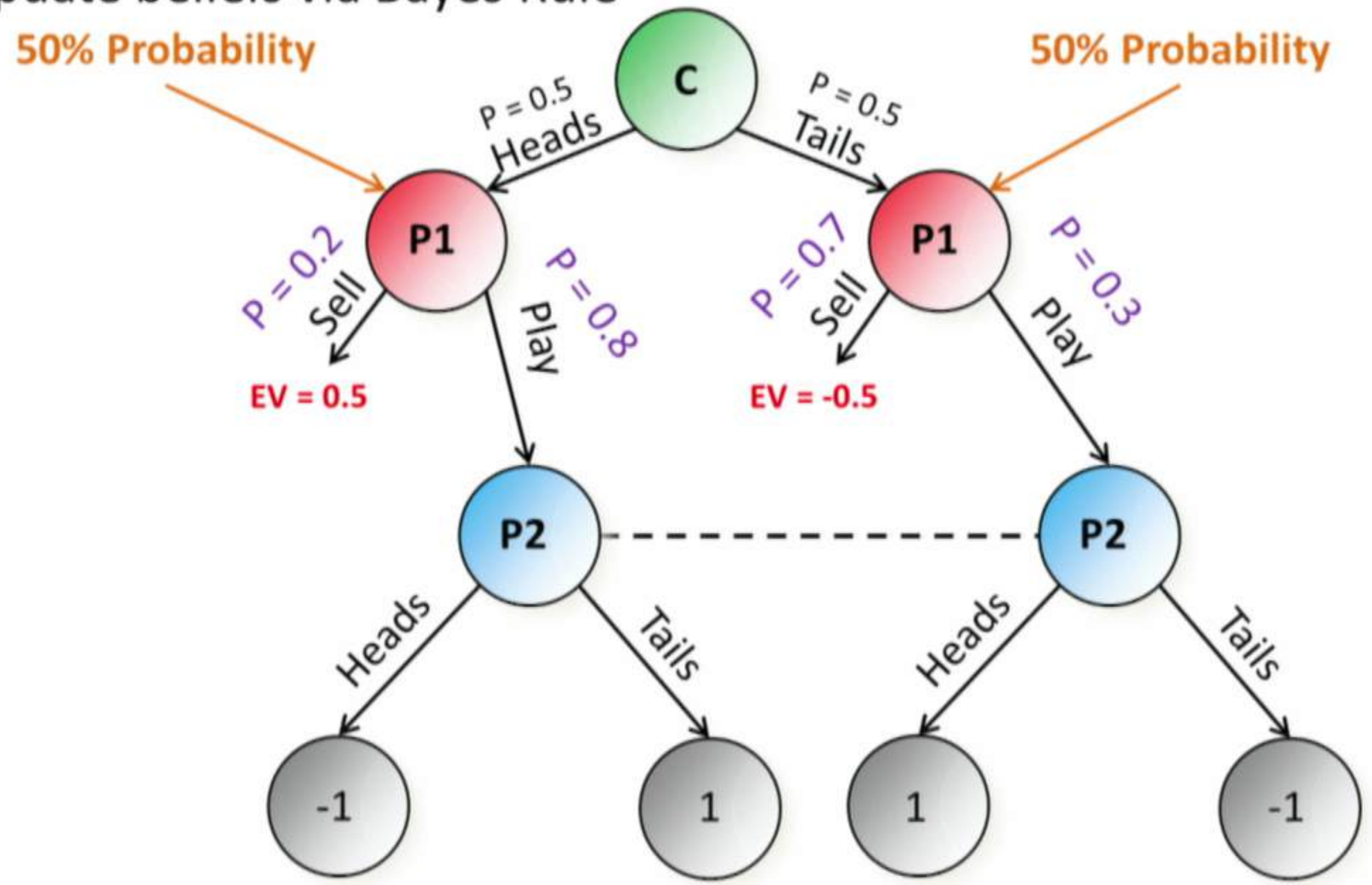

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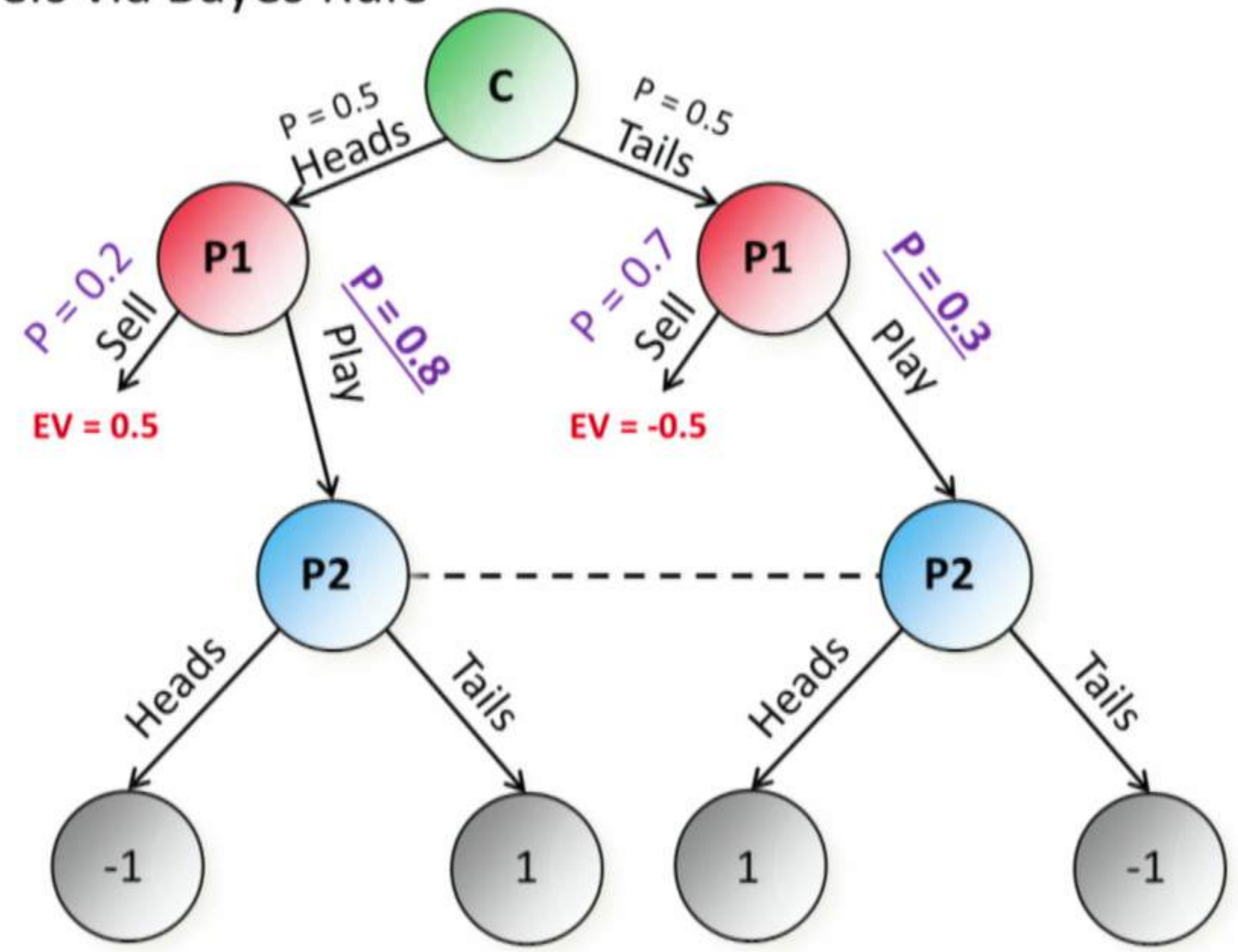

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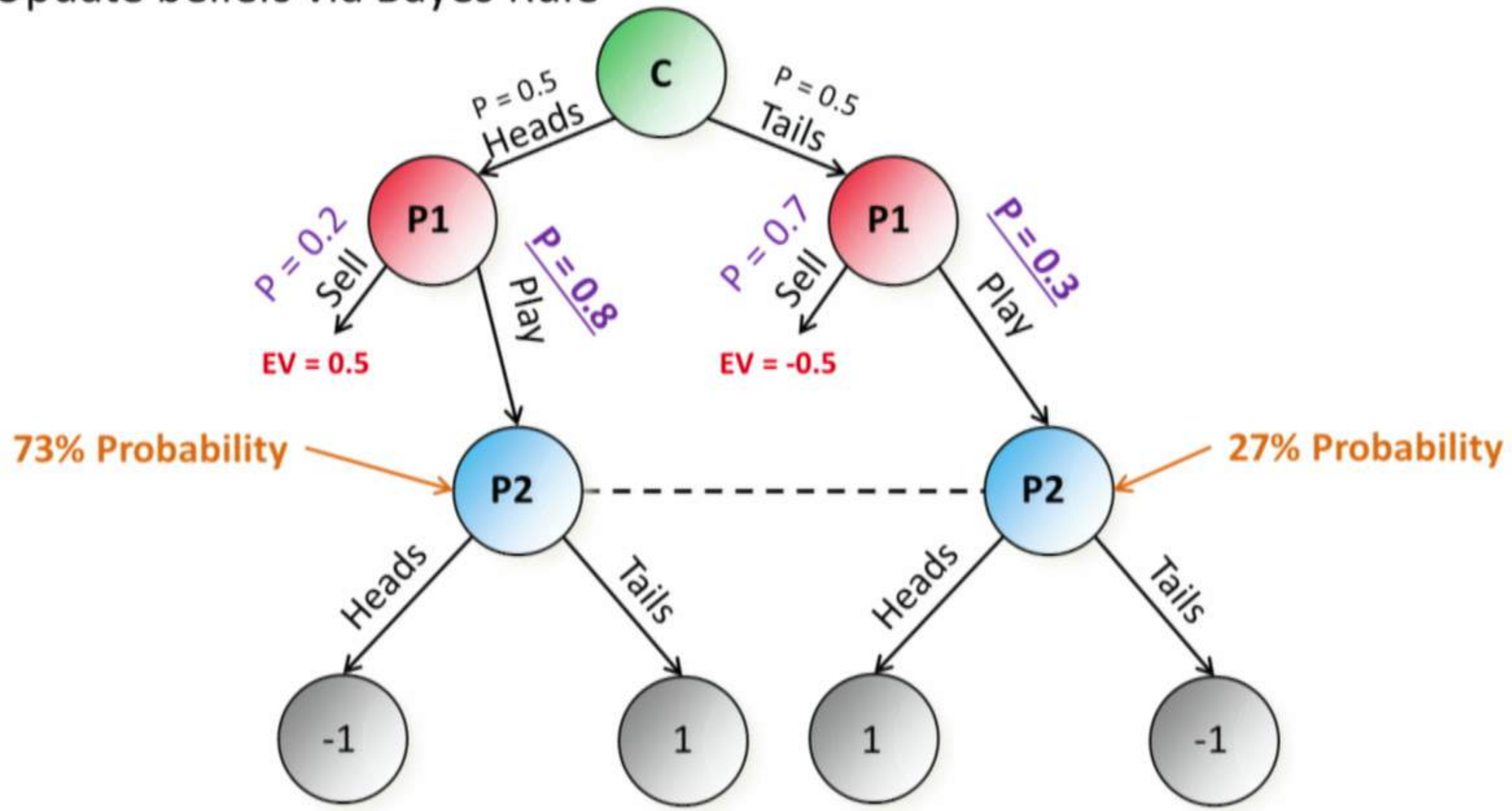

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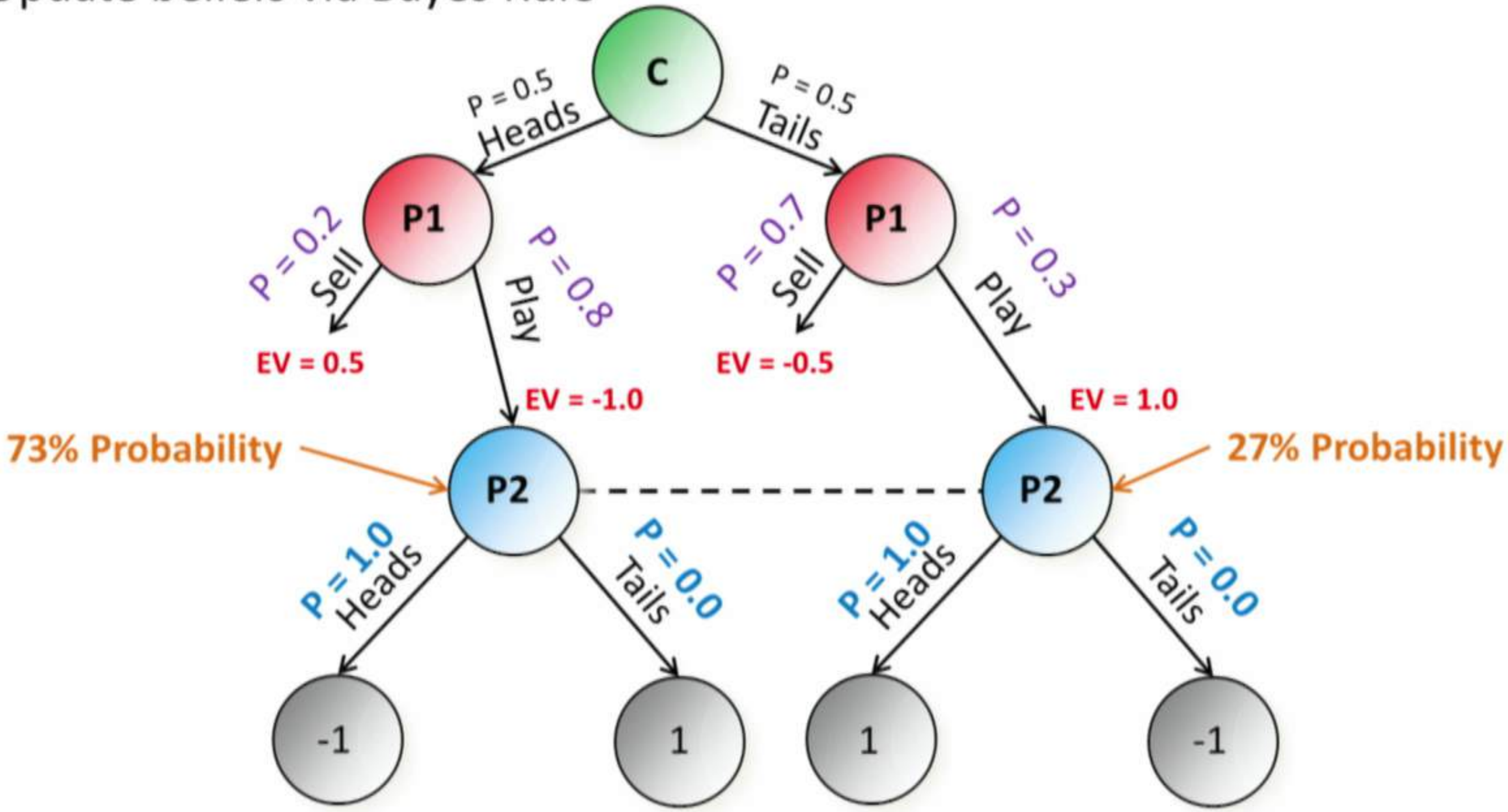
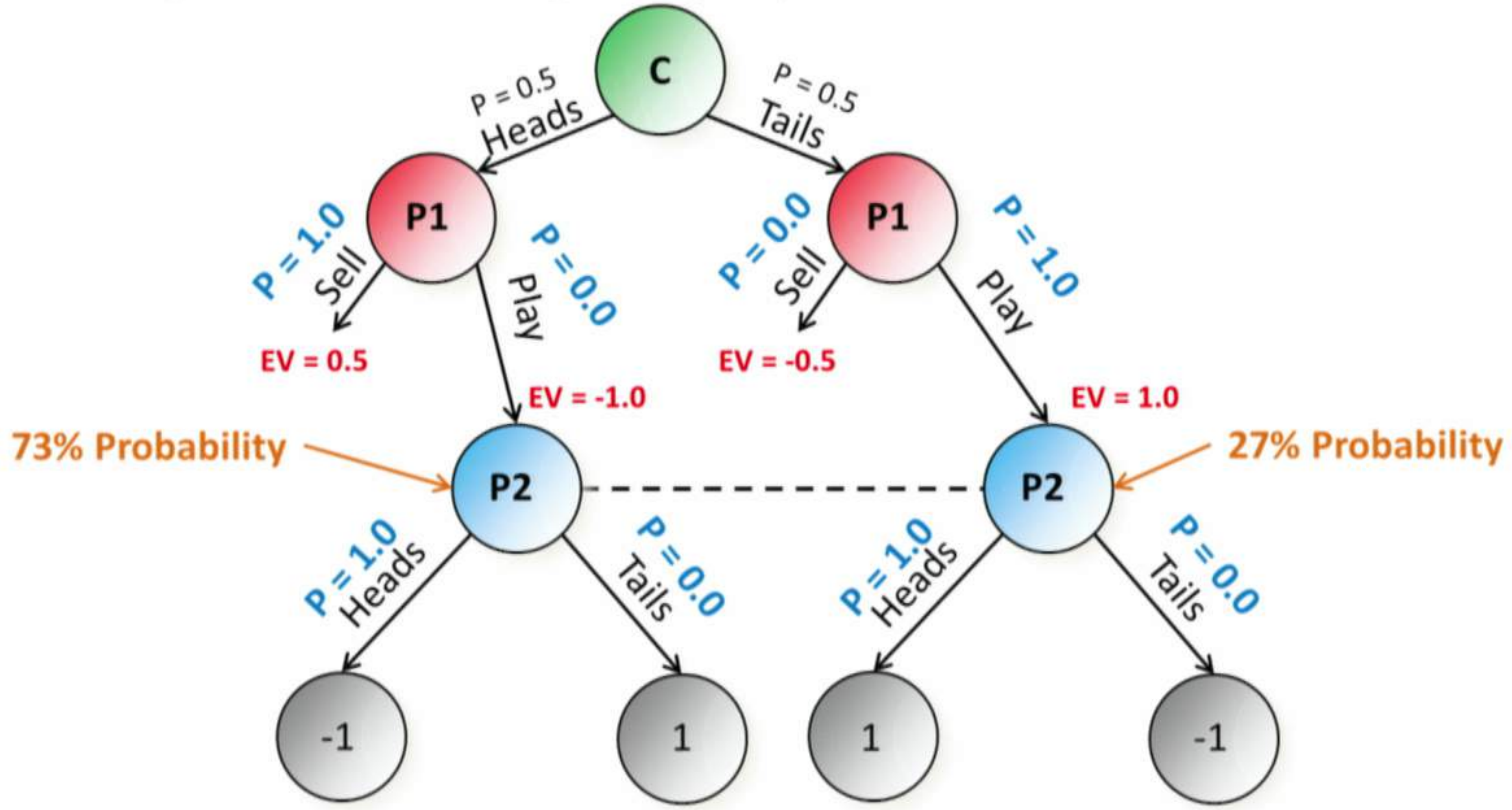


Unsafe Subgame Solving

[Ganzfried & Sandholm AAMAS 2015]

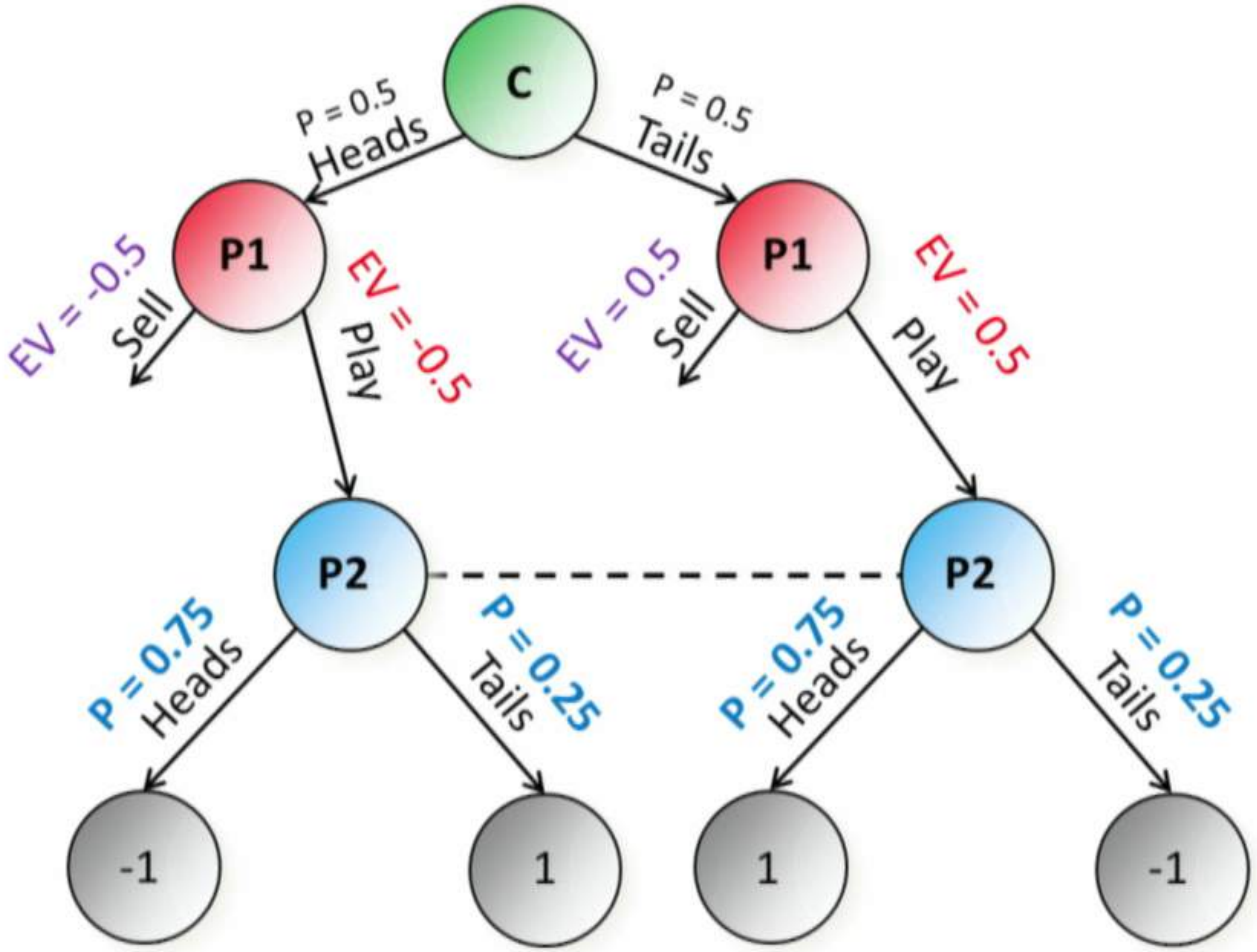
We must account for the opponent's ability to adapt!

But, in practice, unsafe solving works pretty well sometimes



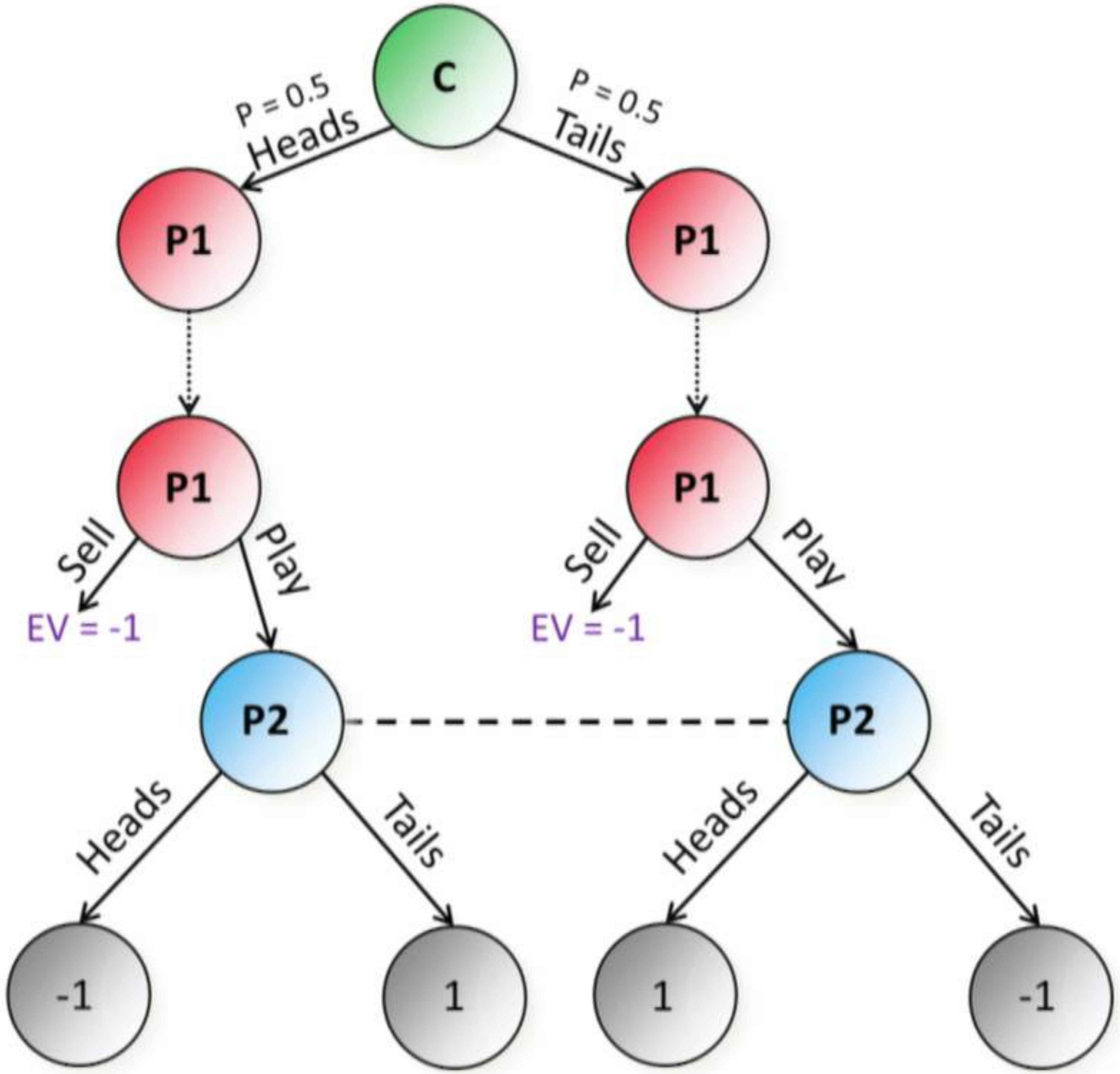
Safe Subgame Solving

[Burch et al AAAI-14, Moravcik et al AAAI-16, Brown & Sandholm NIPS-17]



Reach subgame solving

[Brown & Sandholm NIPS-17]



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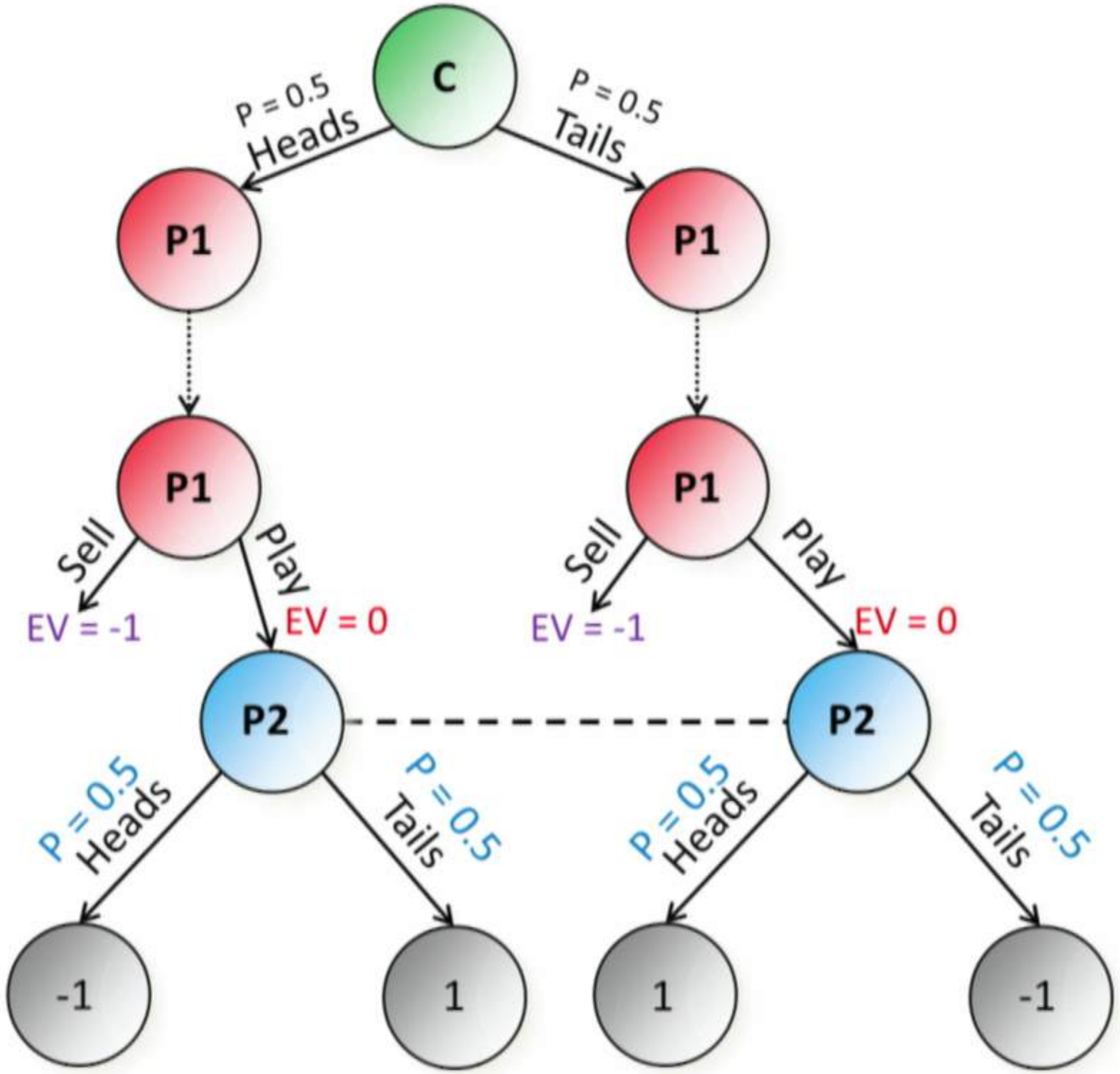

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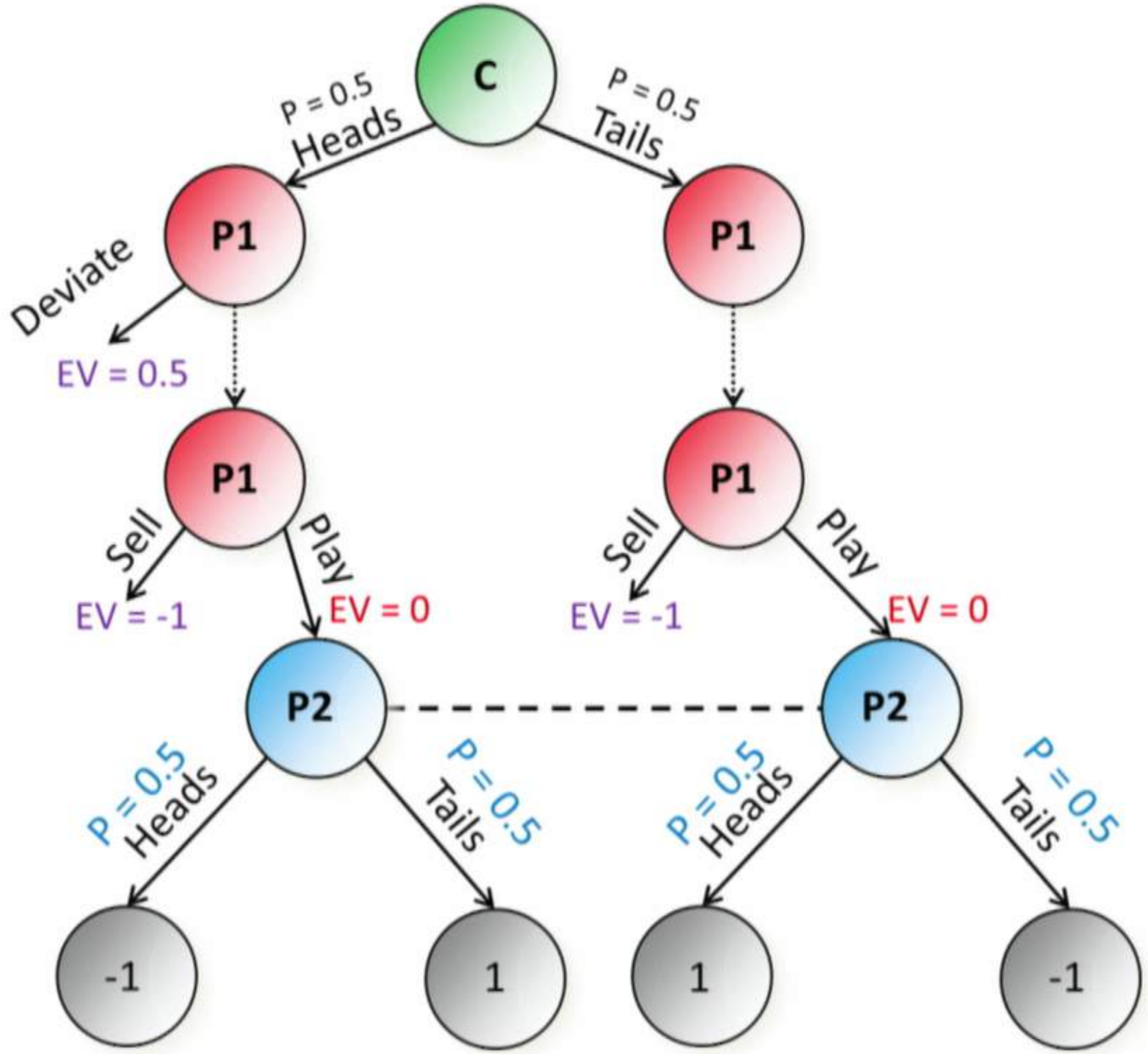

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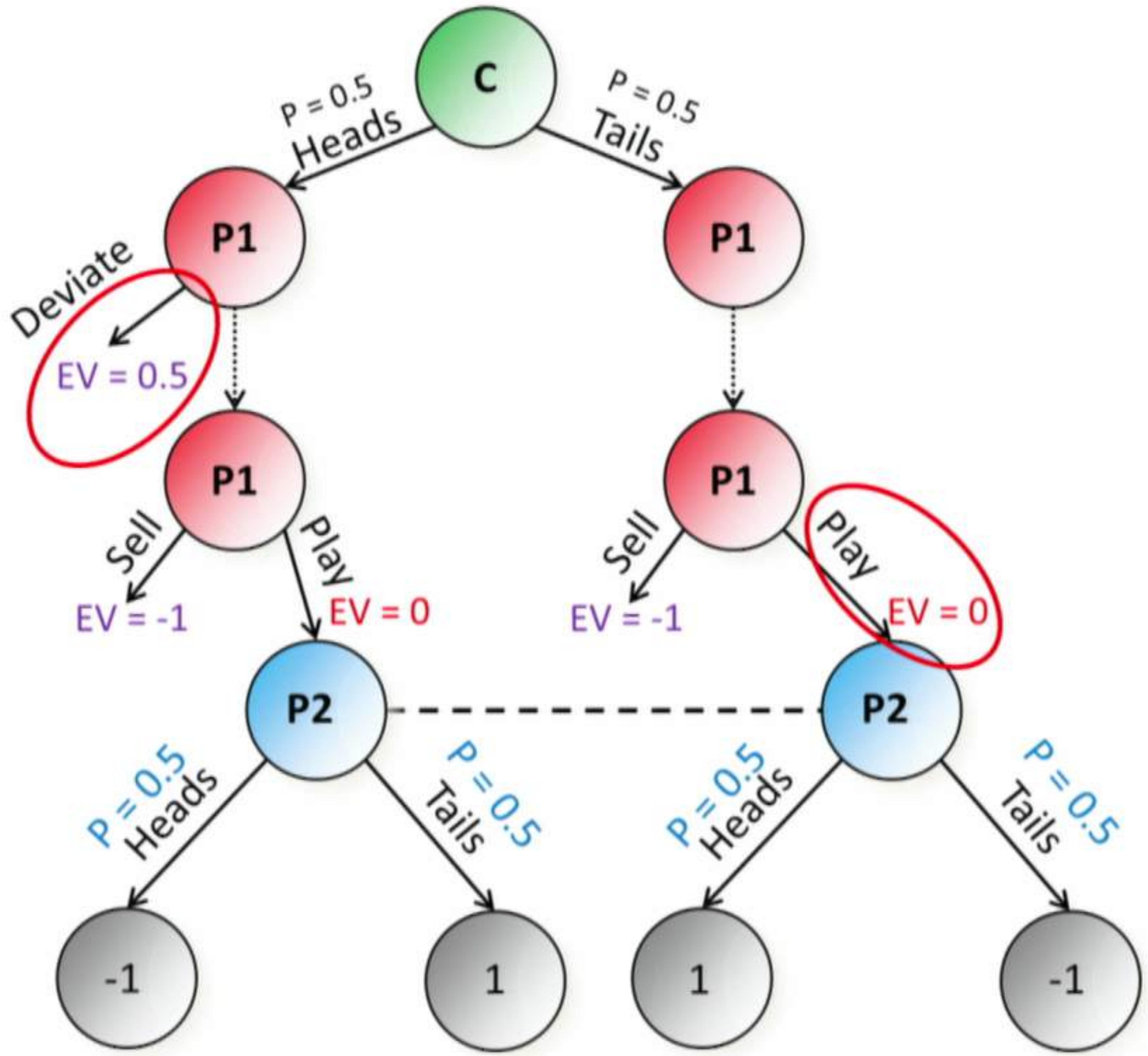

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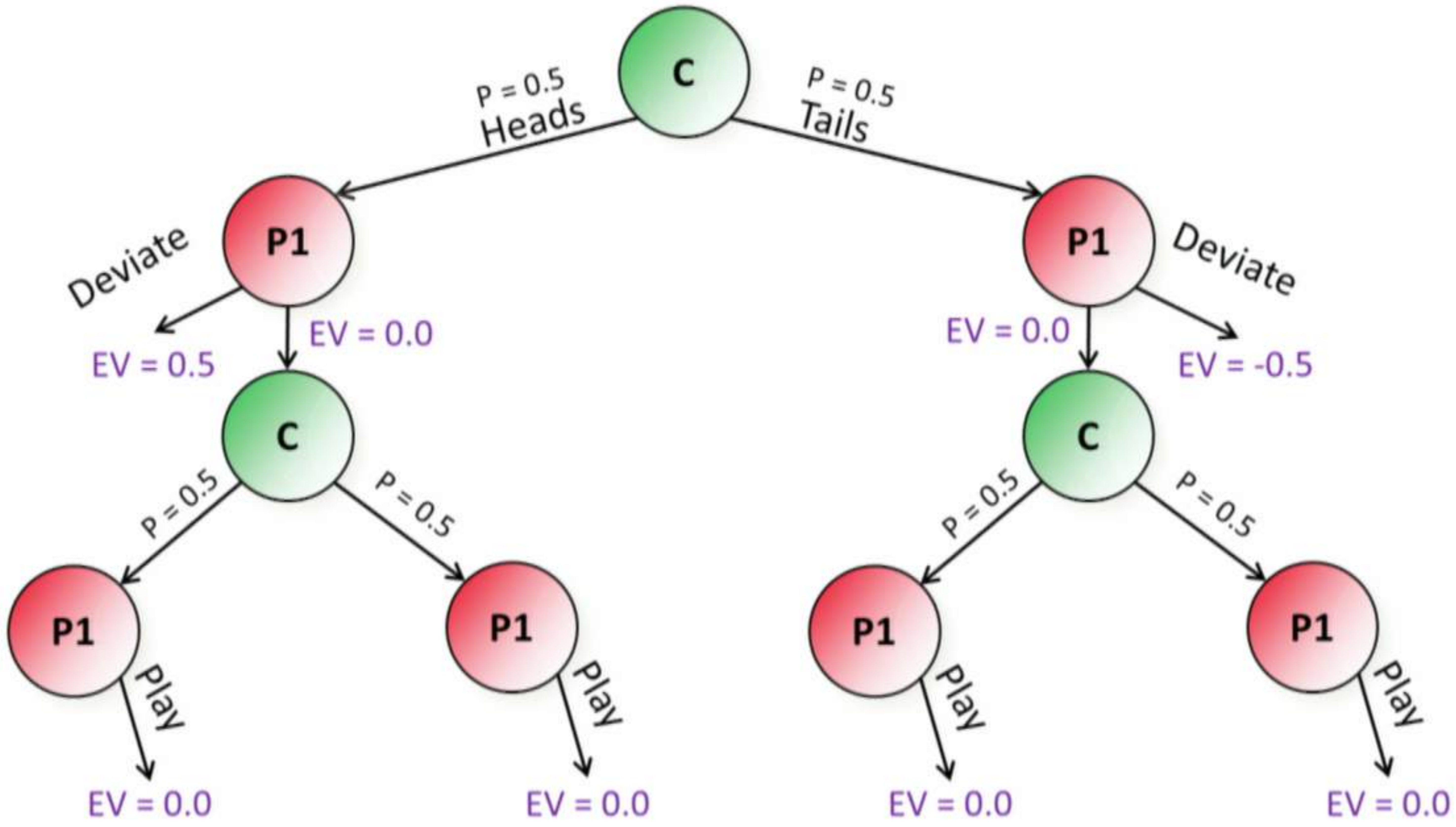

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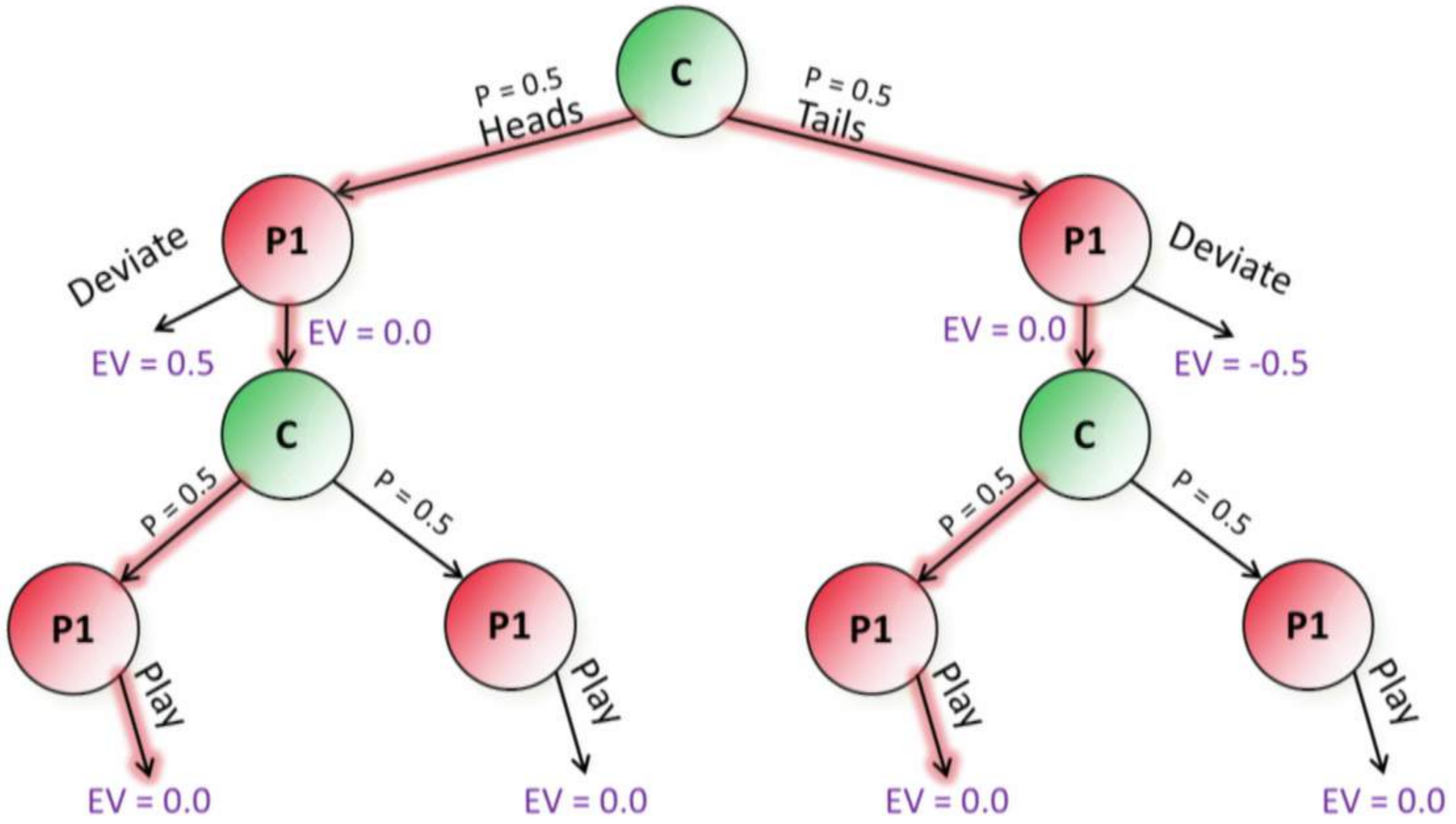

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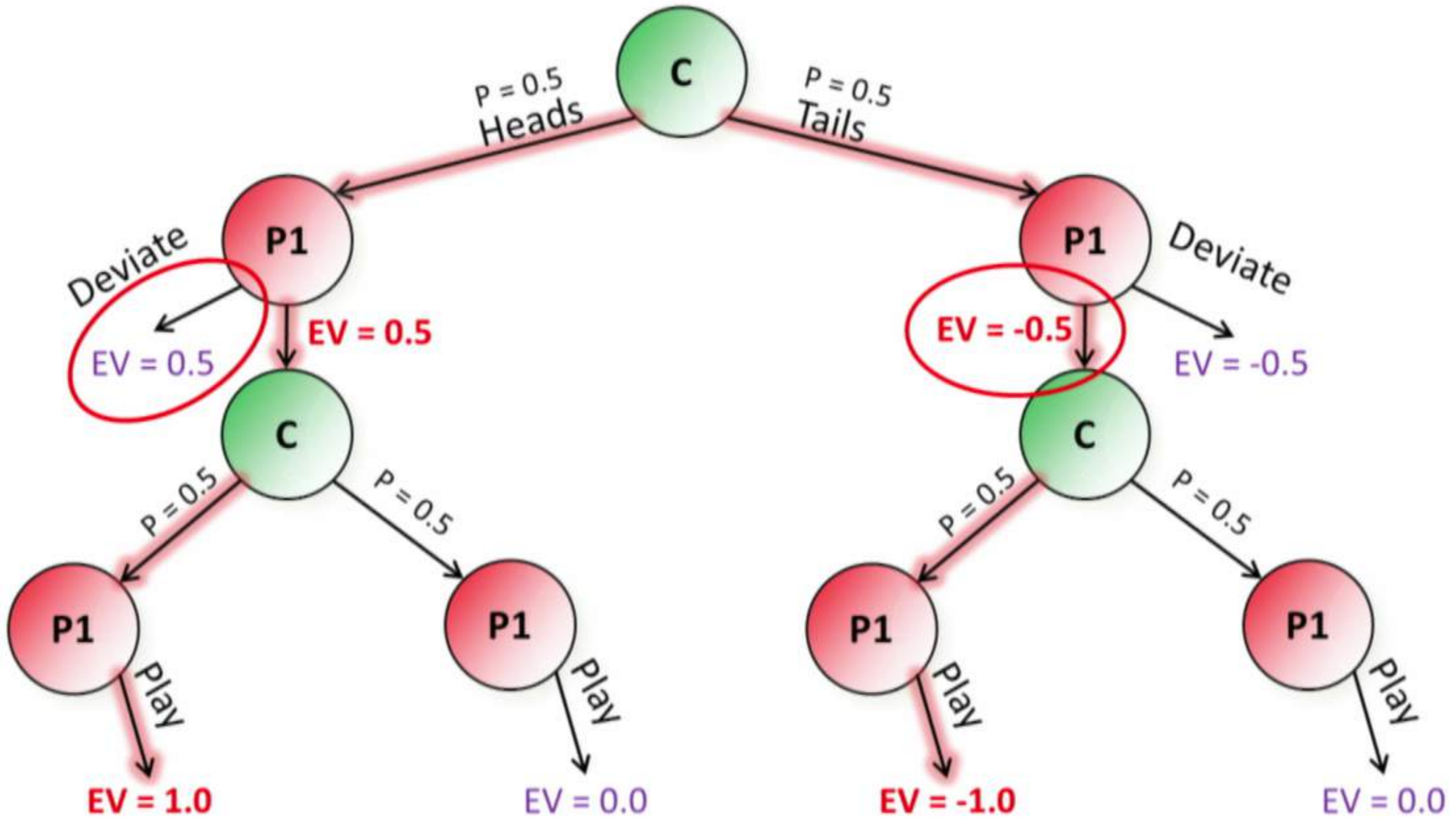

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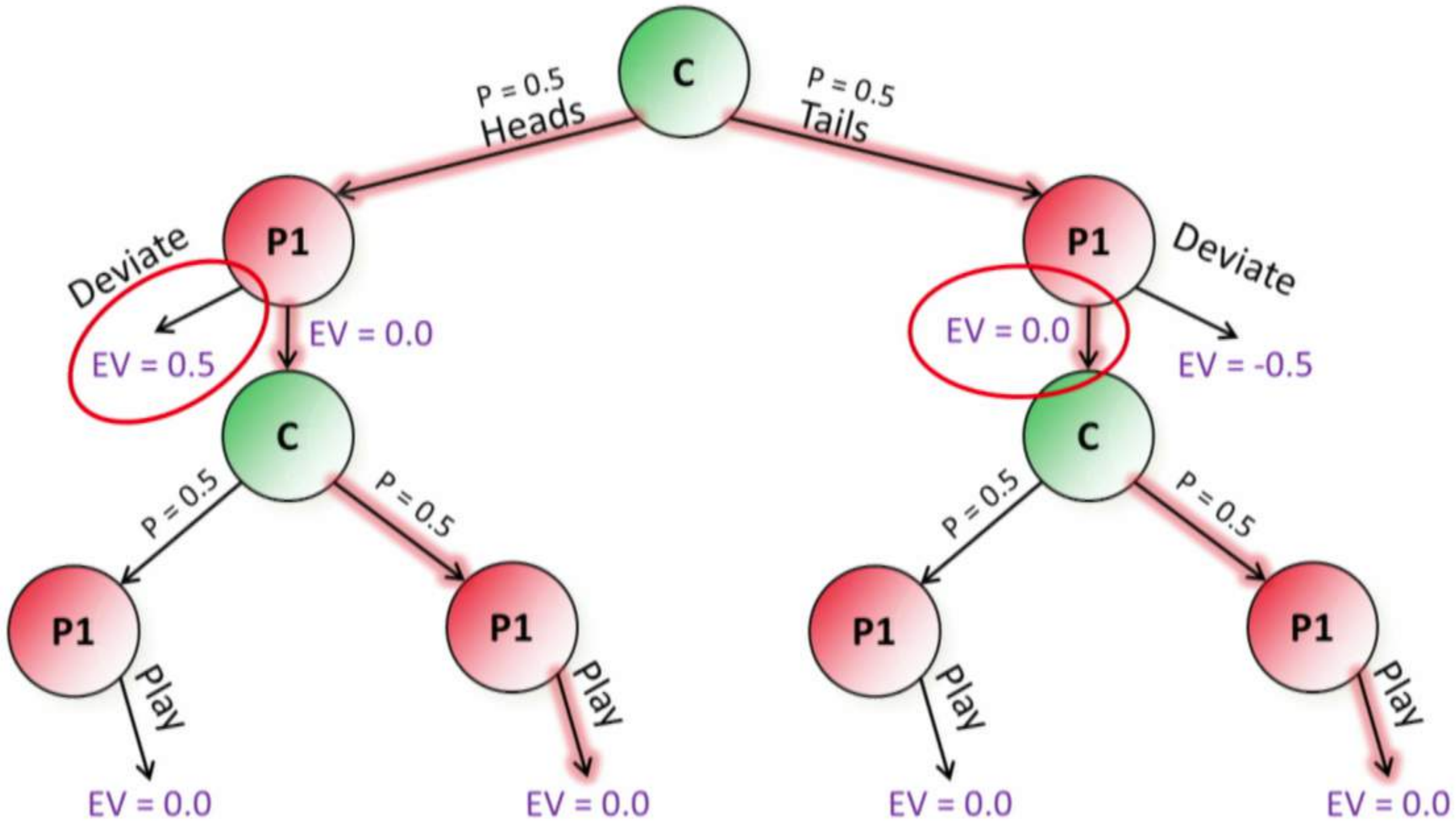

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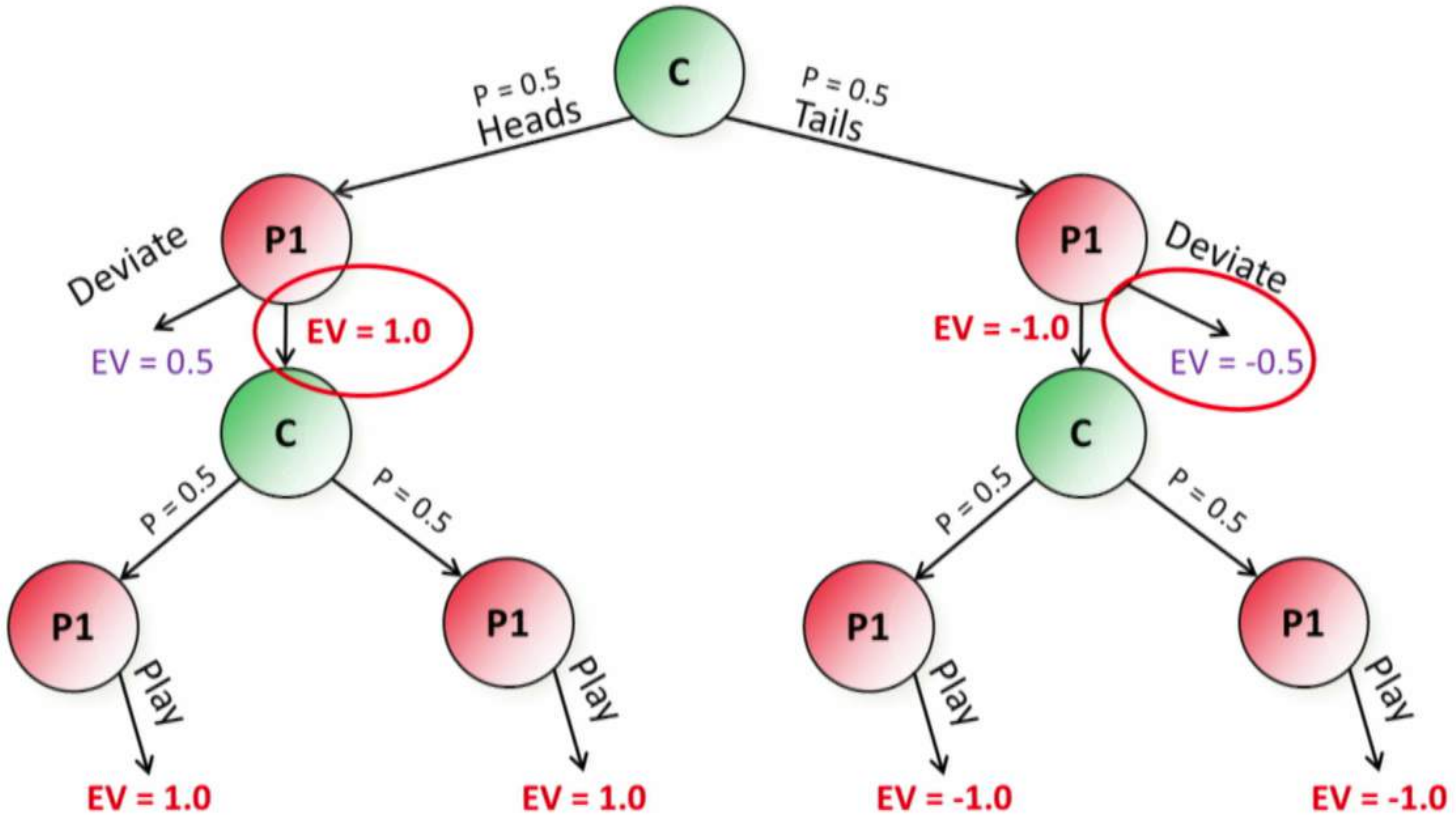

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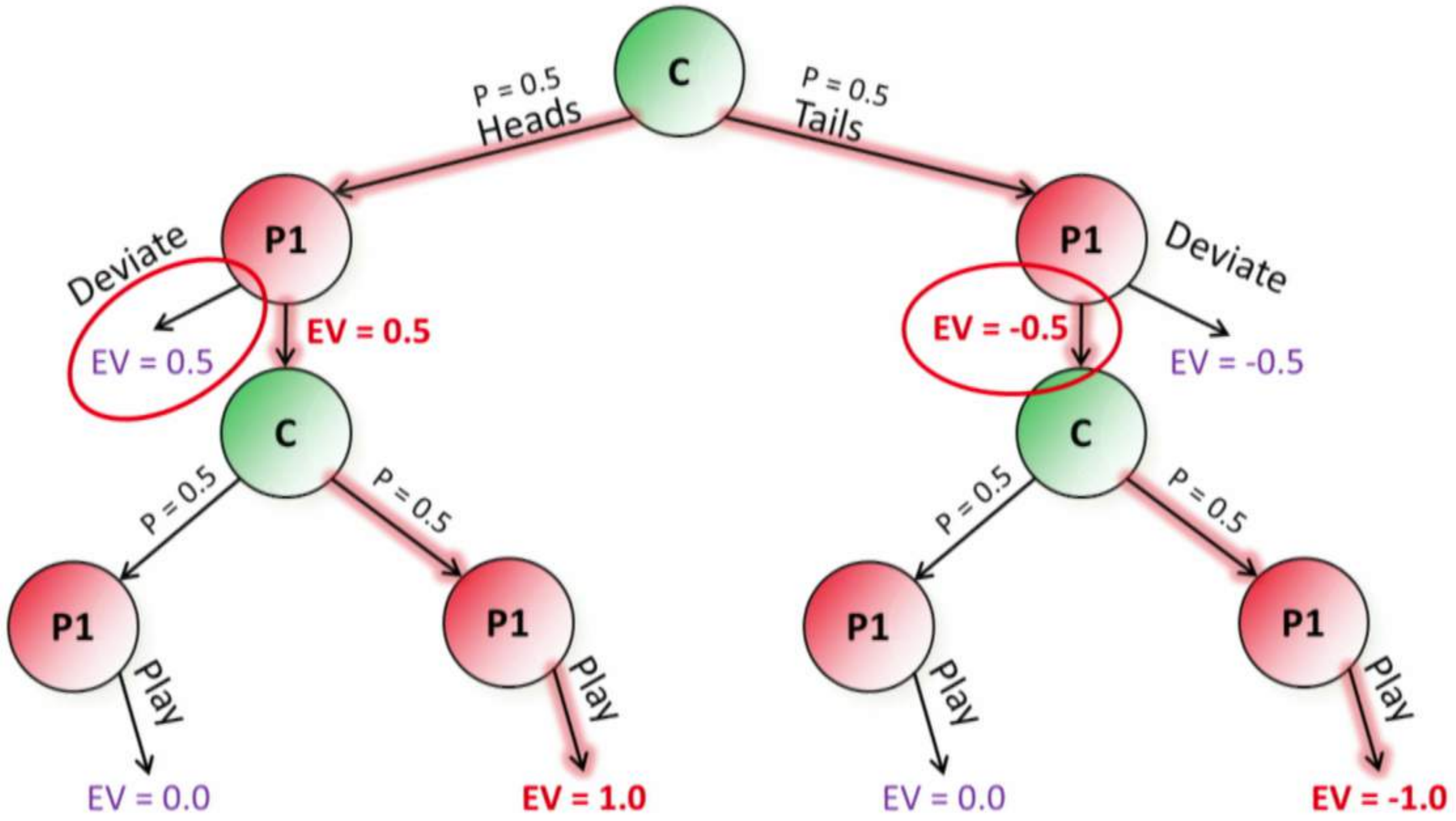

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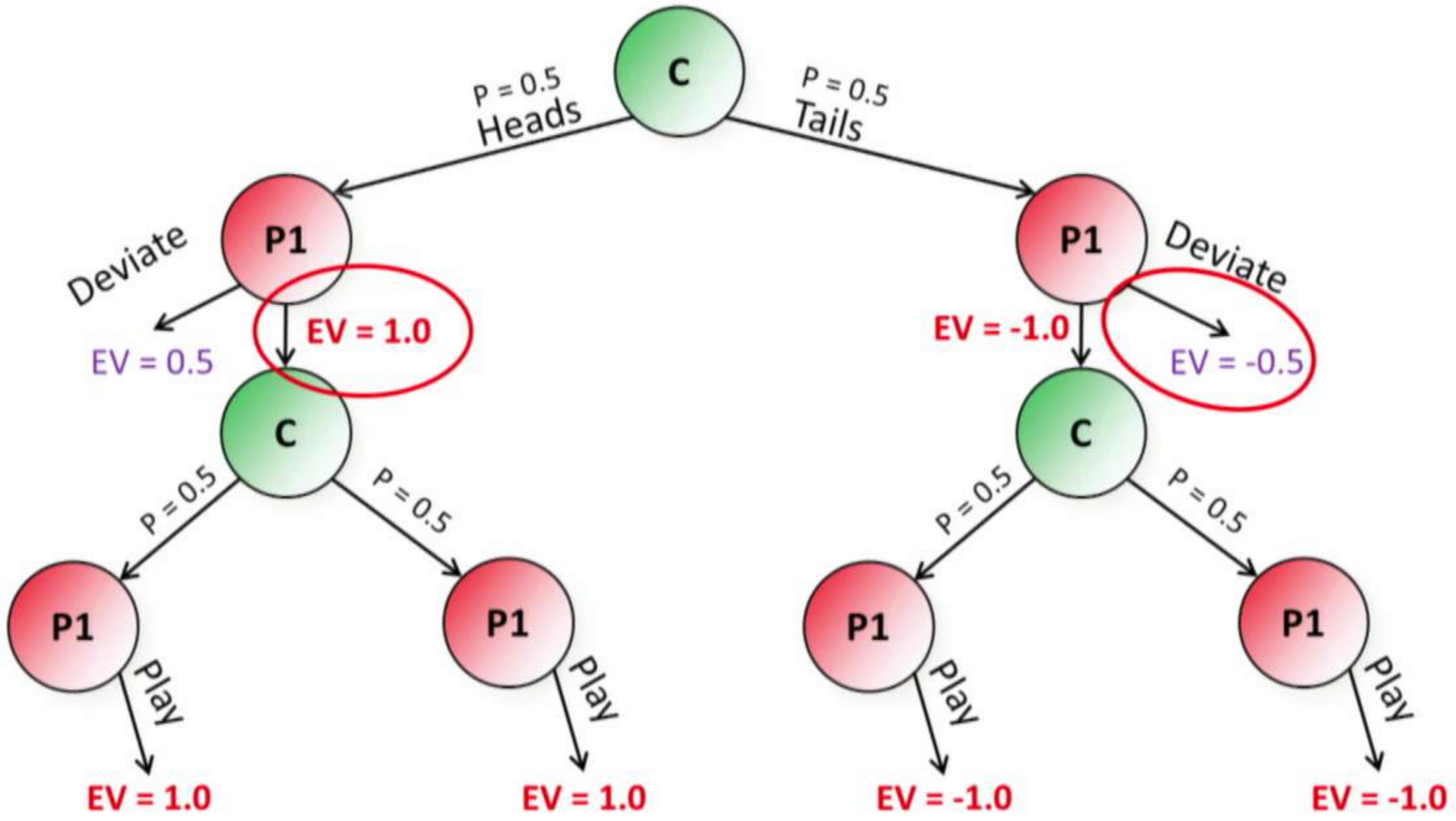

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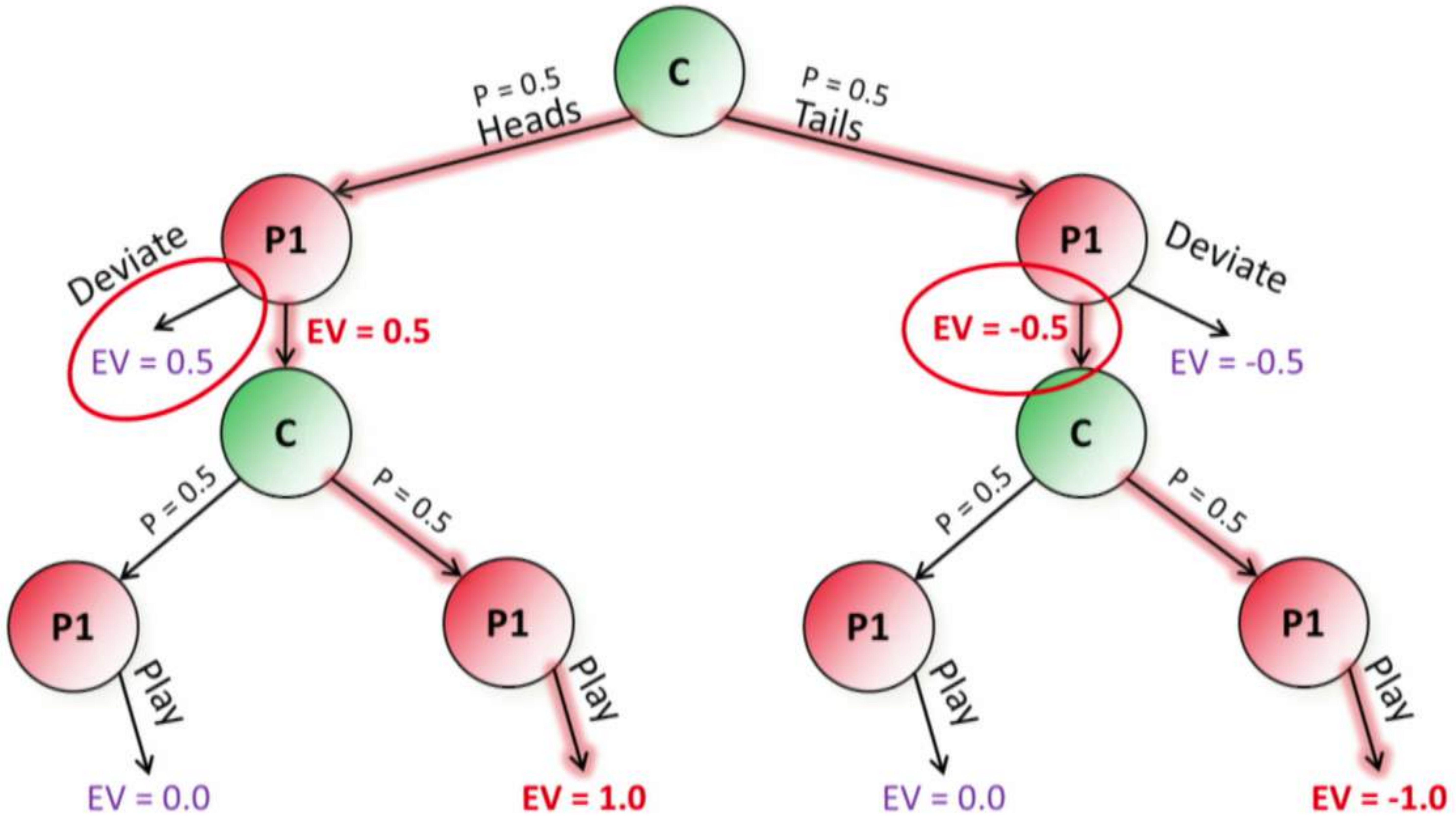

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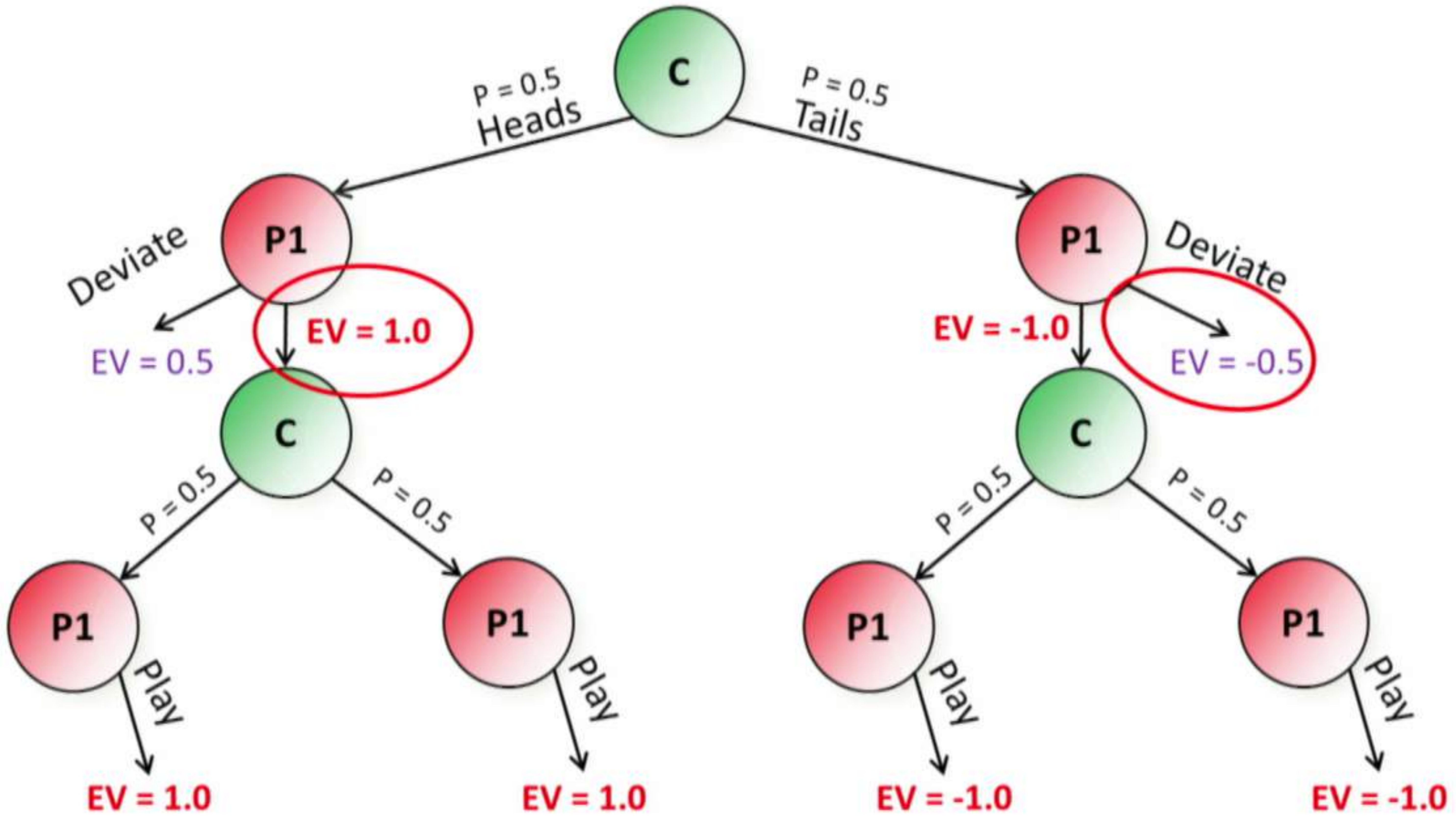

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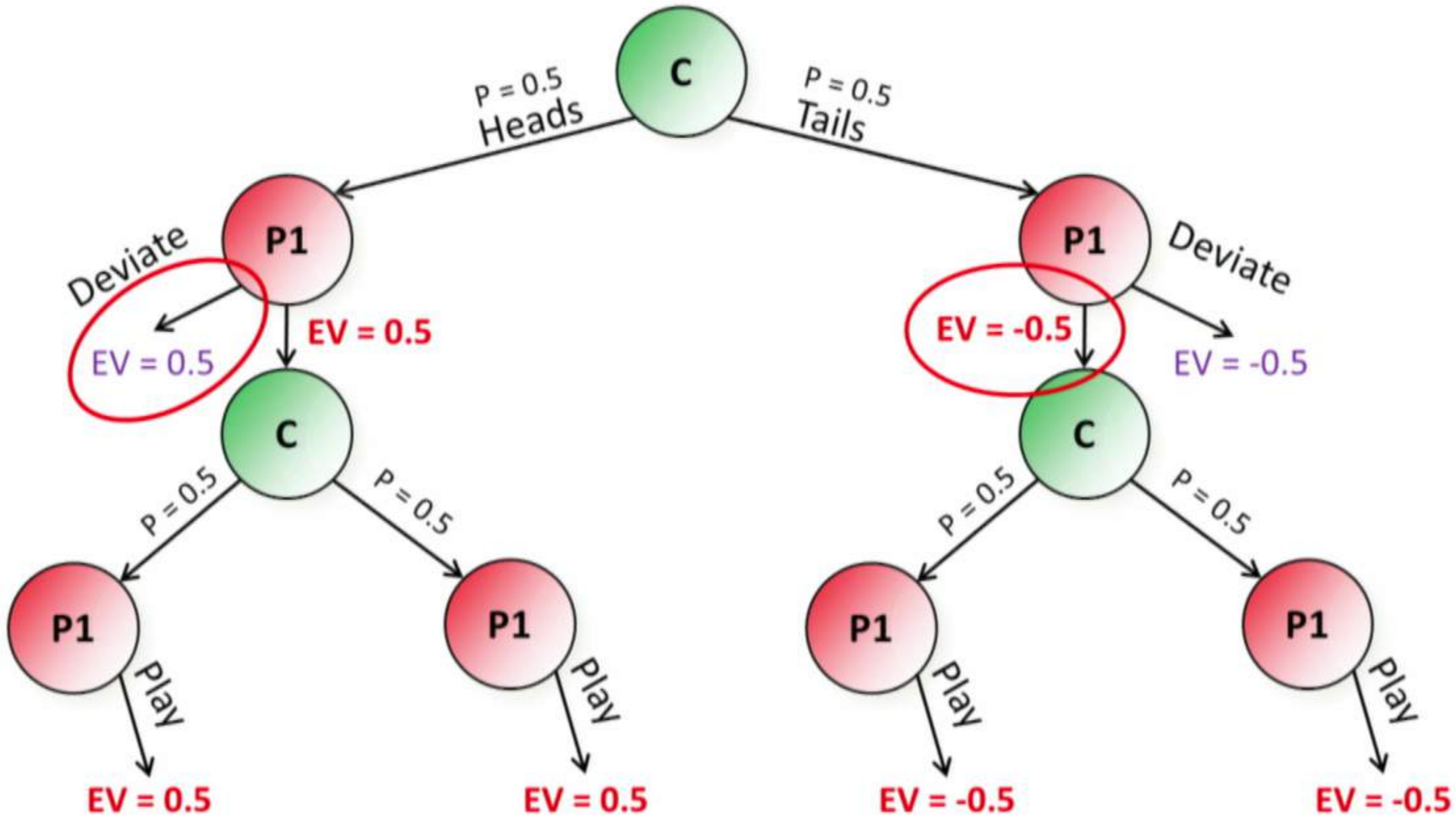

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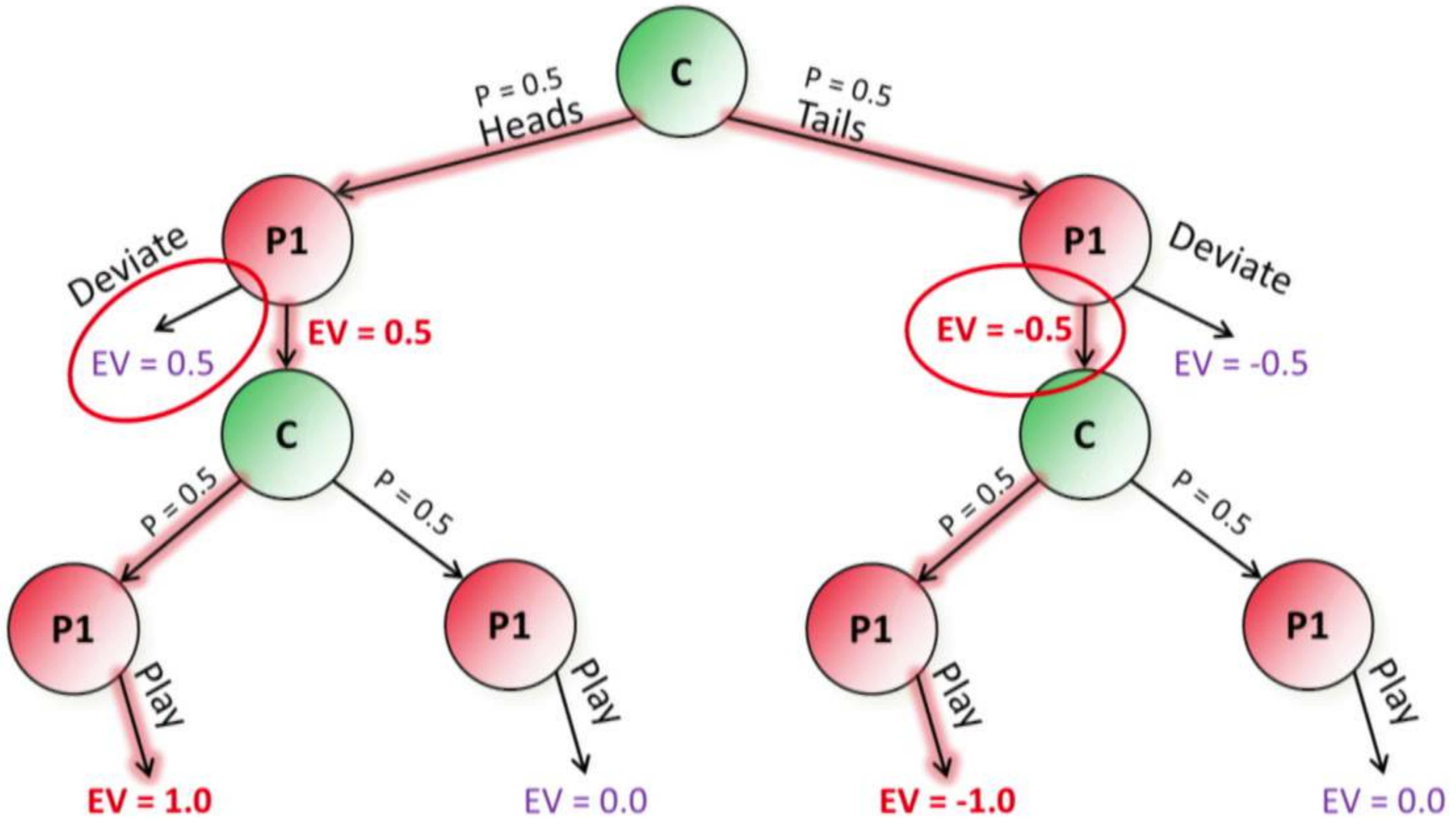
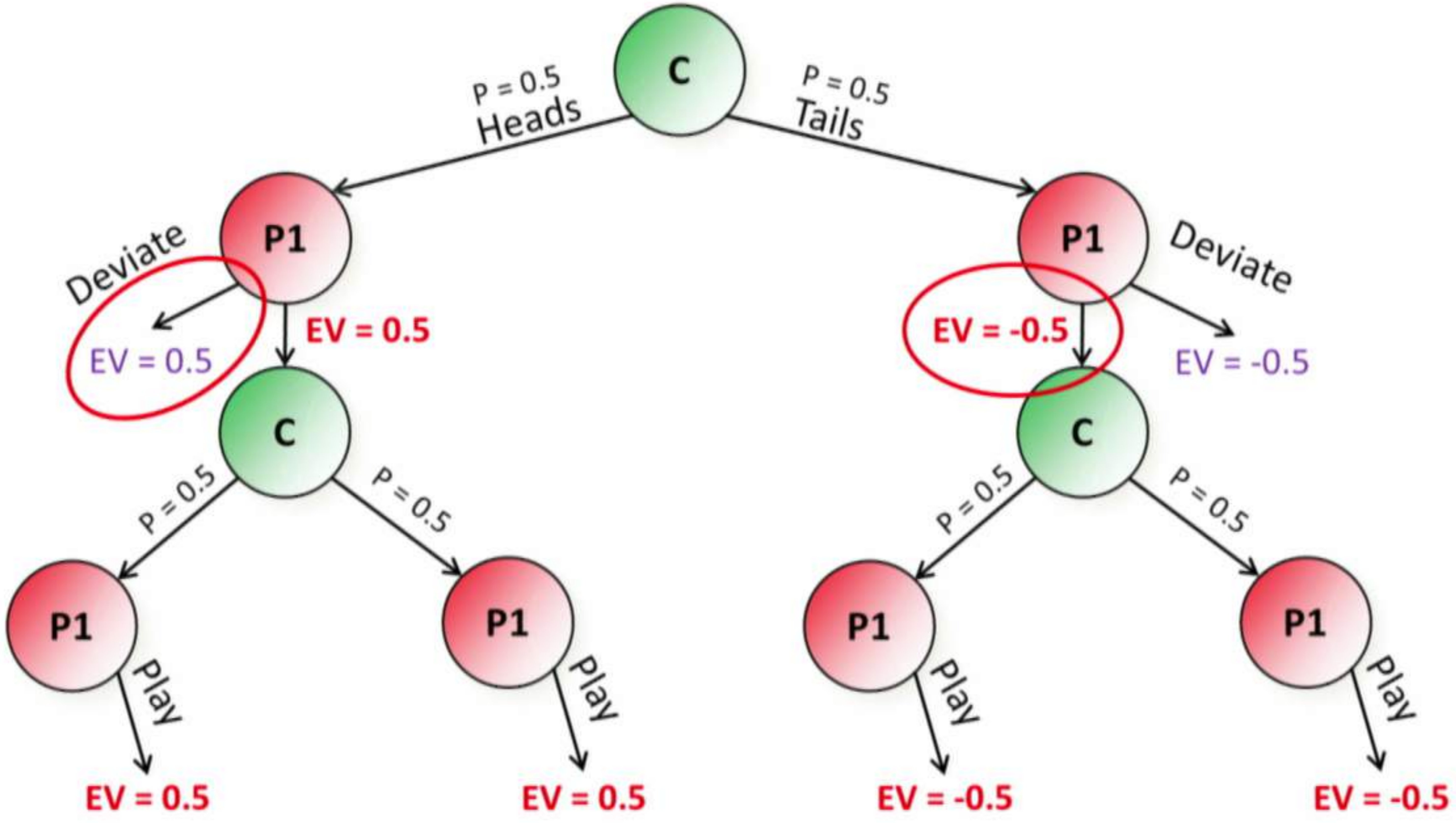


Reach subgame solving

[Brown & Sandholm NIPS-17]

- For off-path actions, we must consider how applying subgame solving **would have changed** their EVs if those actions were chosen instead
- Solution: split "slack" among actions by actions' probabilities



Theorem: Reach subgame solving will not increase a safe technique's exploitability, and lower it if there is slack.

Experiments on medium-sized games

- Our best reach subgame solving technique has **3x** less exploitability than the best prior safe subgame-solving technique

- Nested solving is **12x** less exploitable than techniques that do not use real-time reasoning

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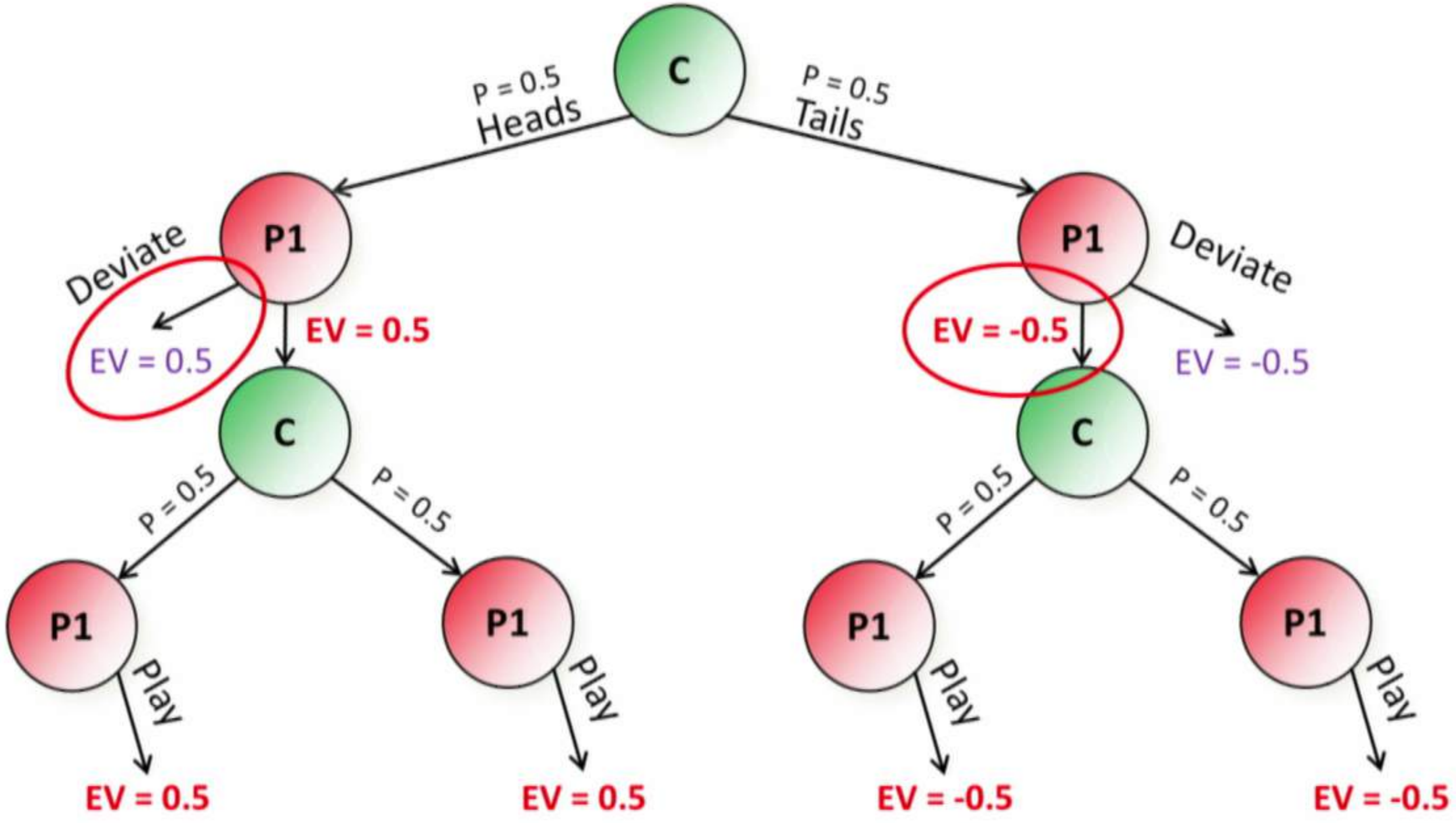

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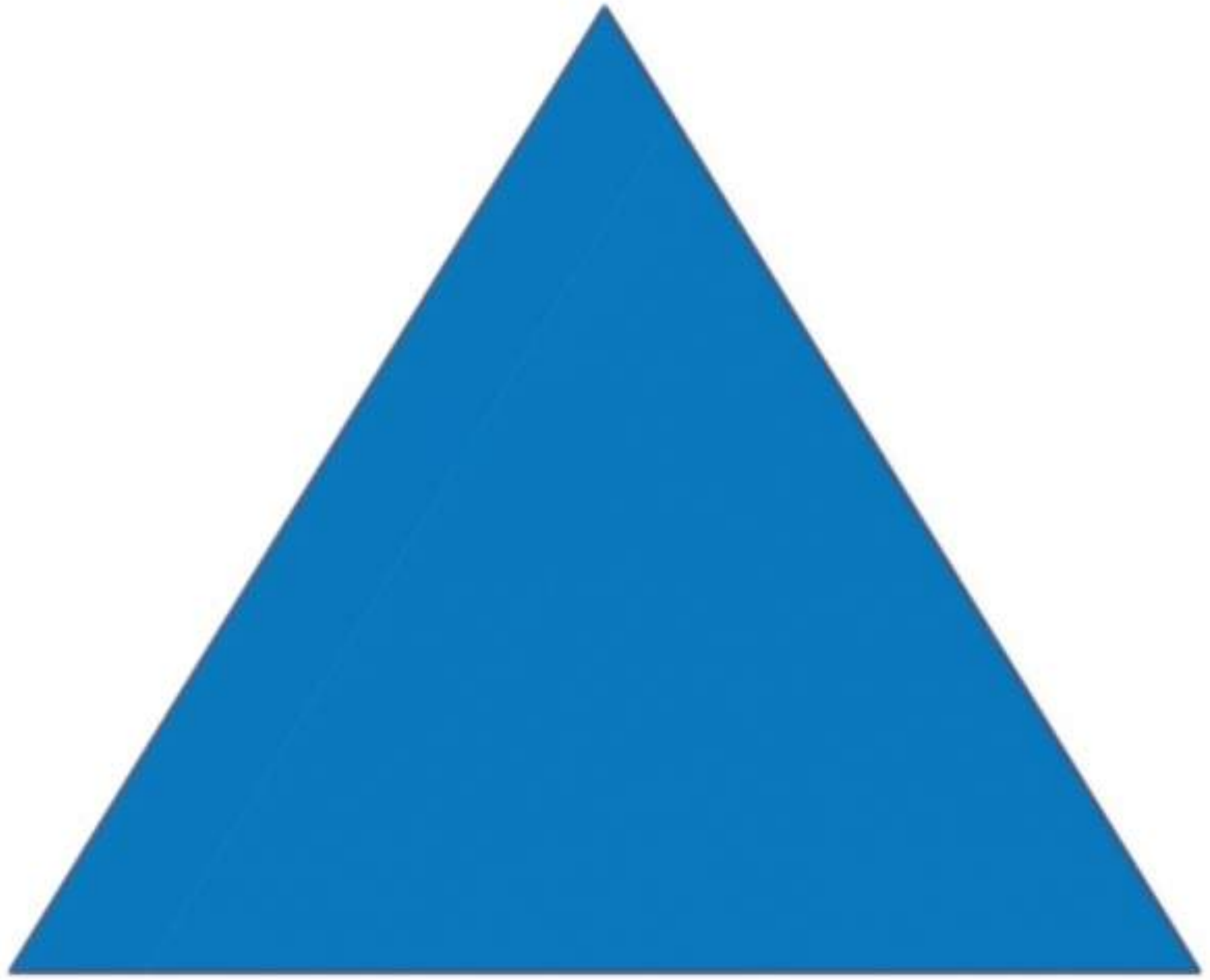
1) Because an optimal strategy for a subgame cannot be determined from that subgame alone

2) Because states don't have well-defined values

Perfect-Information Games and Single-Agent Settings



Perfect-Information Games and Single-Agent Settings

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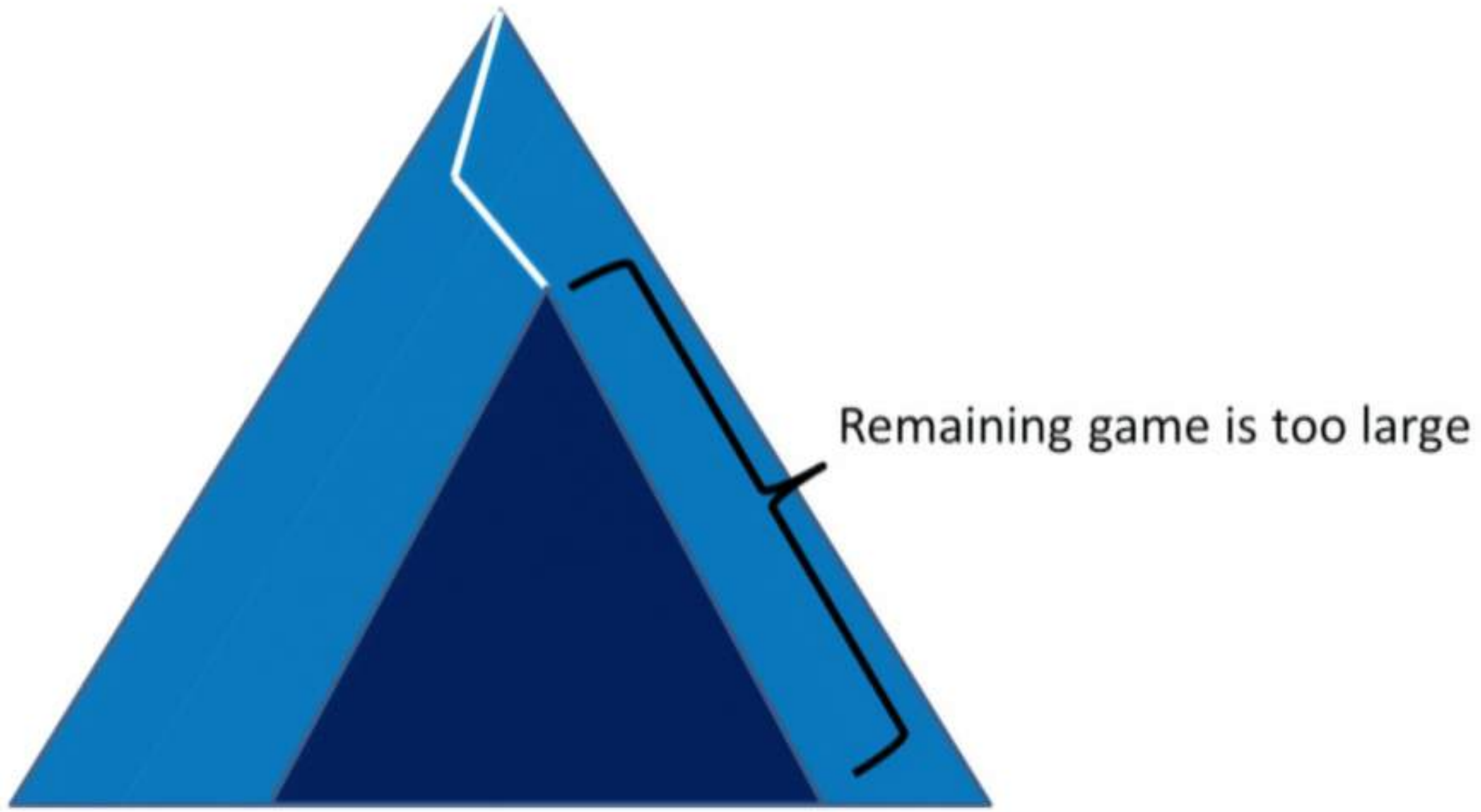

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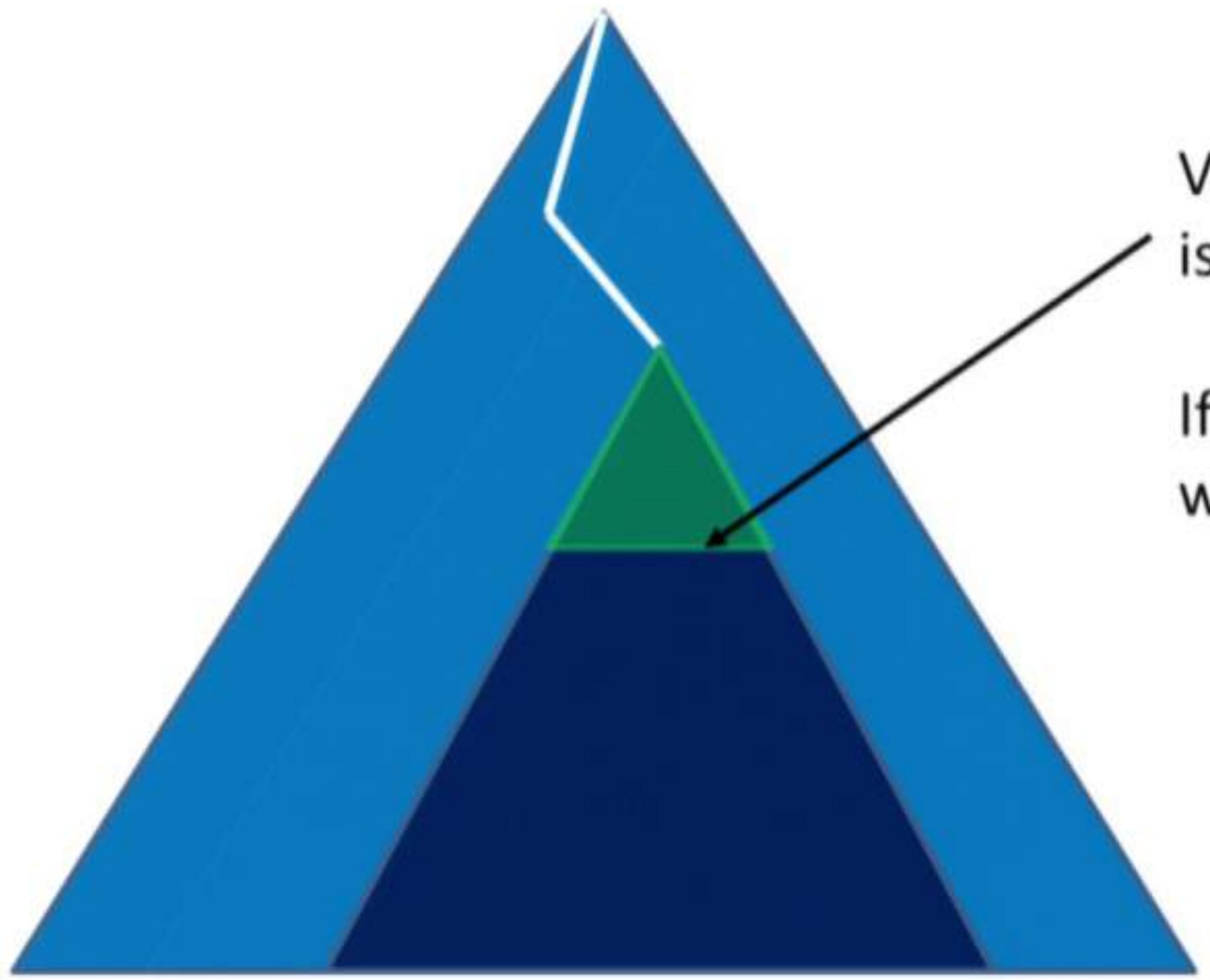
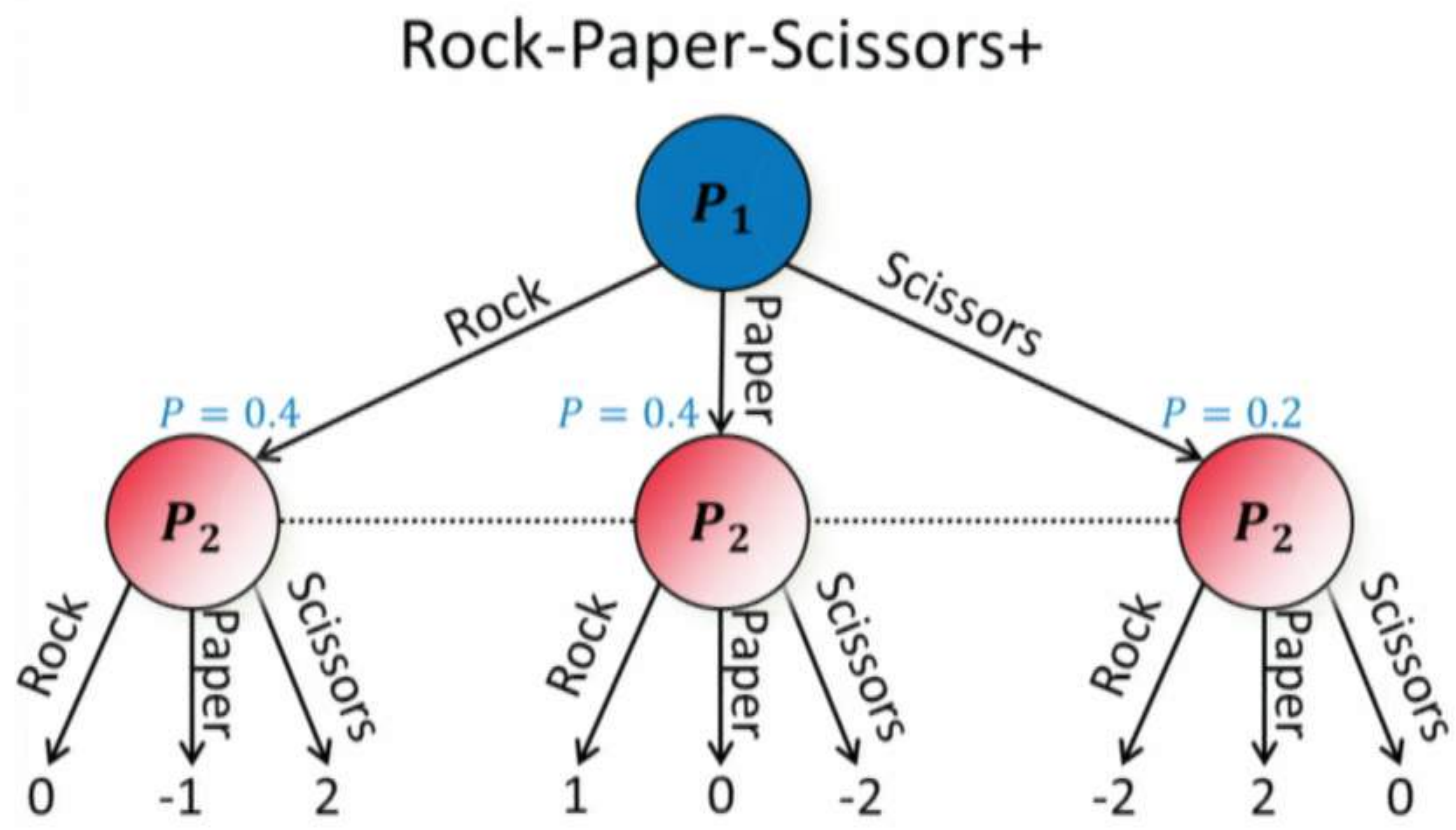
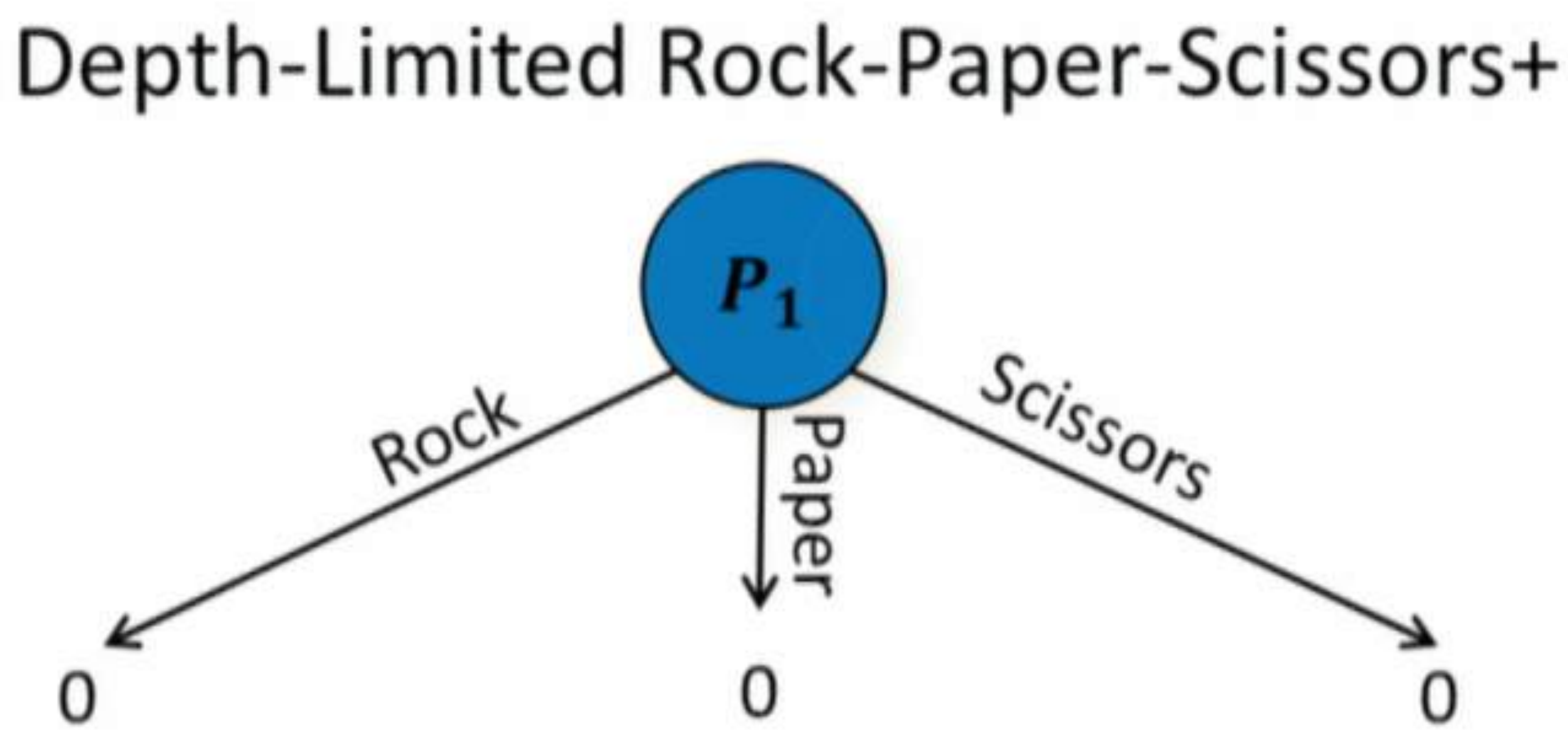


Value substituted at leaf node s
is estimate of equilibrium value $\hat{v}(s)$

If estimate is perfect, local search
will find optimal policy (the equilibrium)

Depth-Limited Solving





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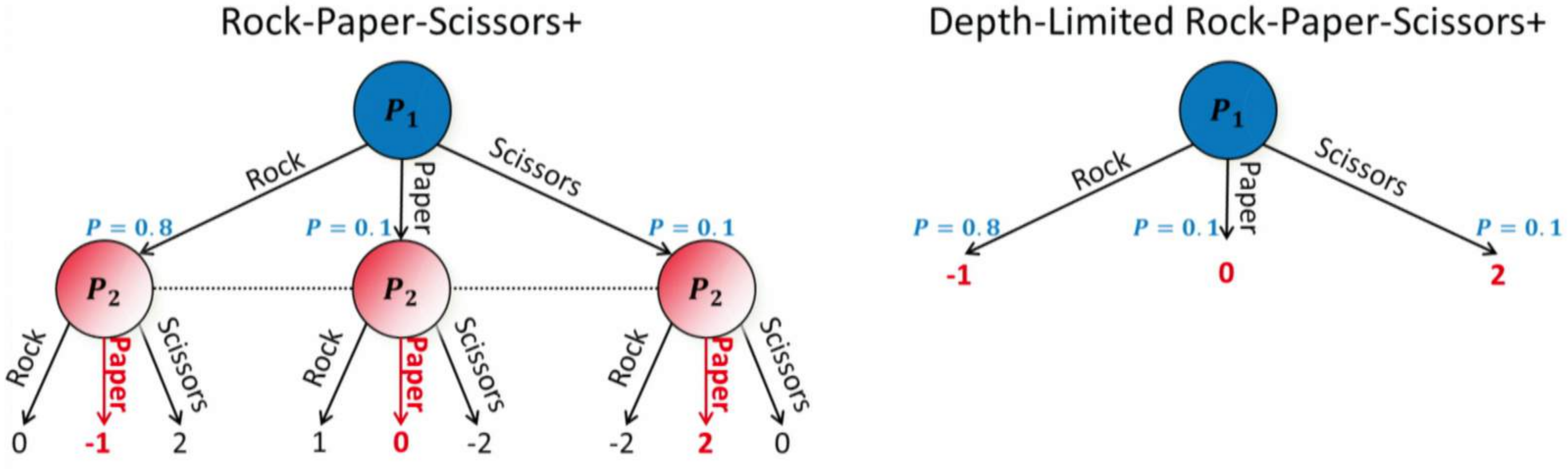
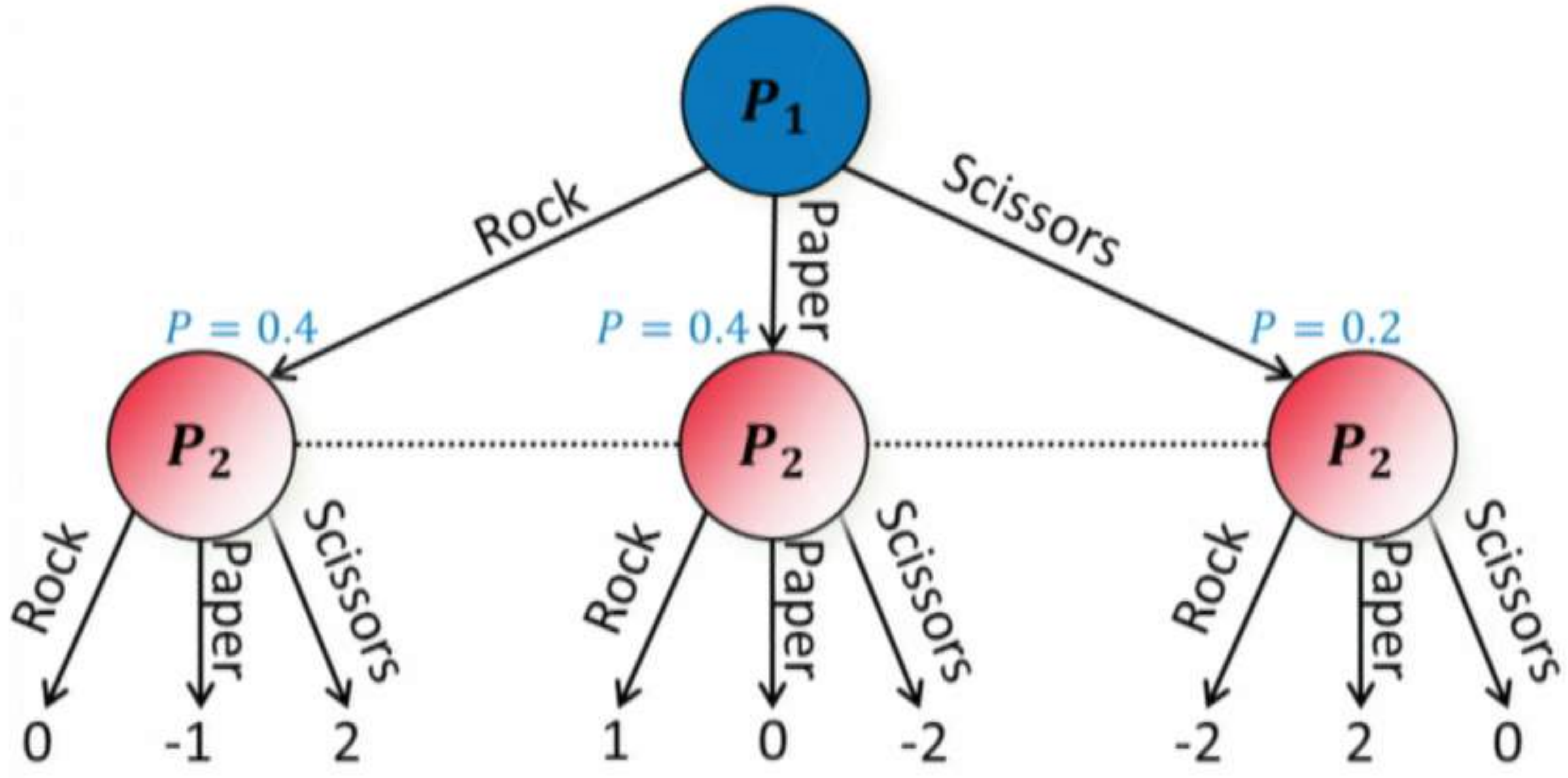
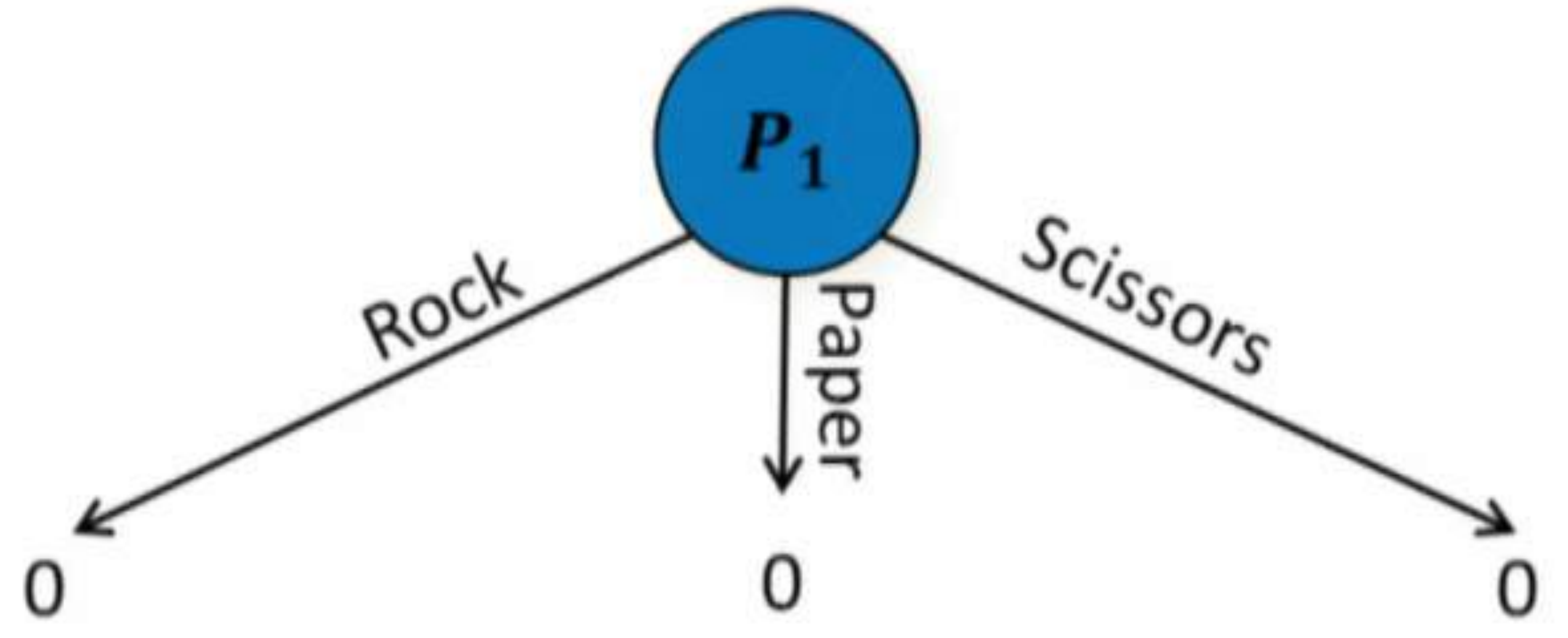


Depth-Limited Solving

Rock-Paper-Scissors+



Depth-Limited Rock-Paper-Scissors+



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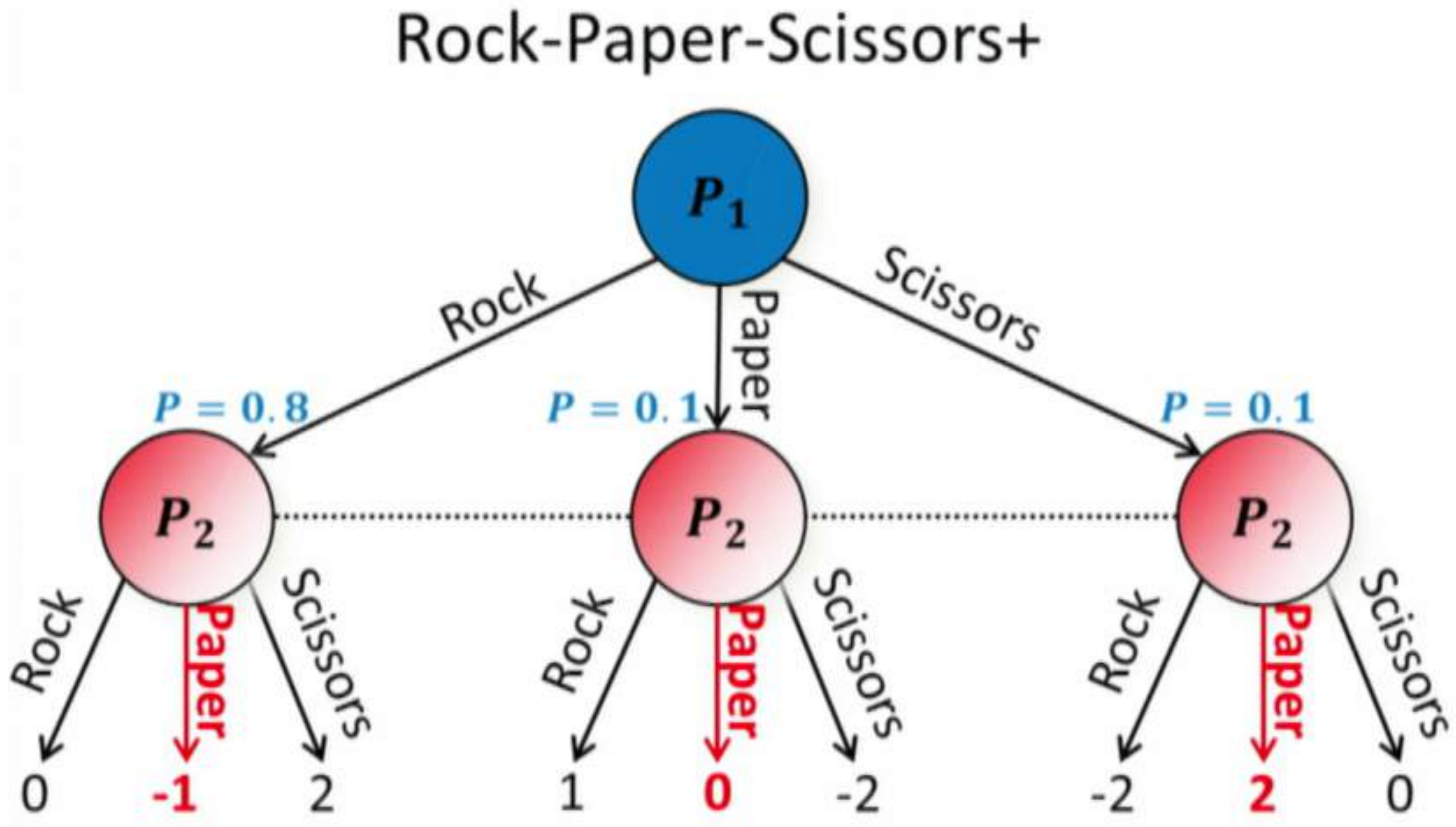

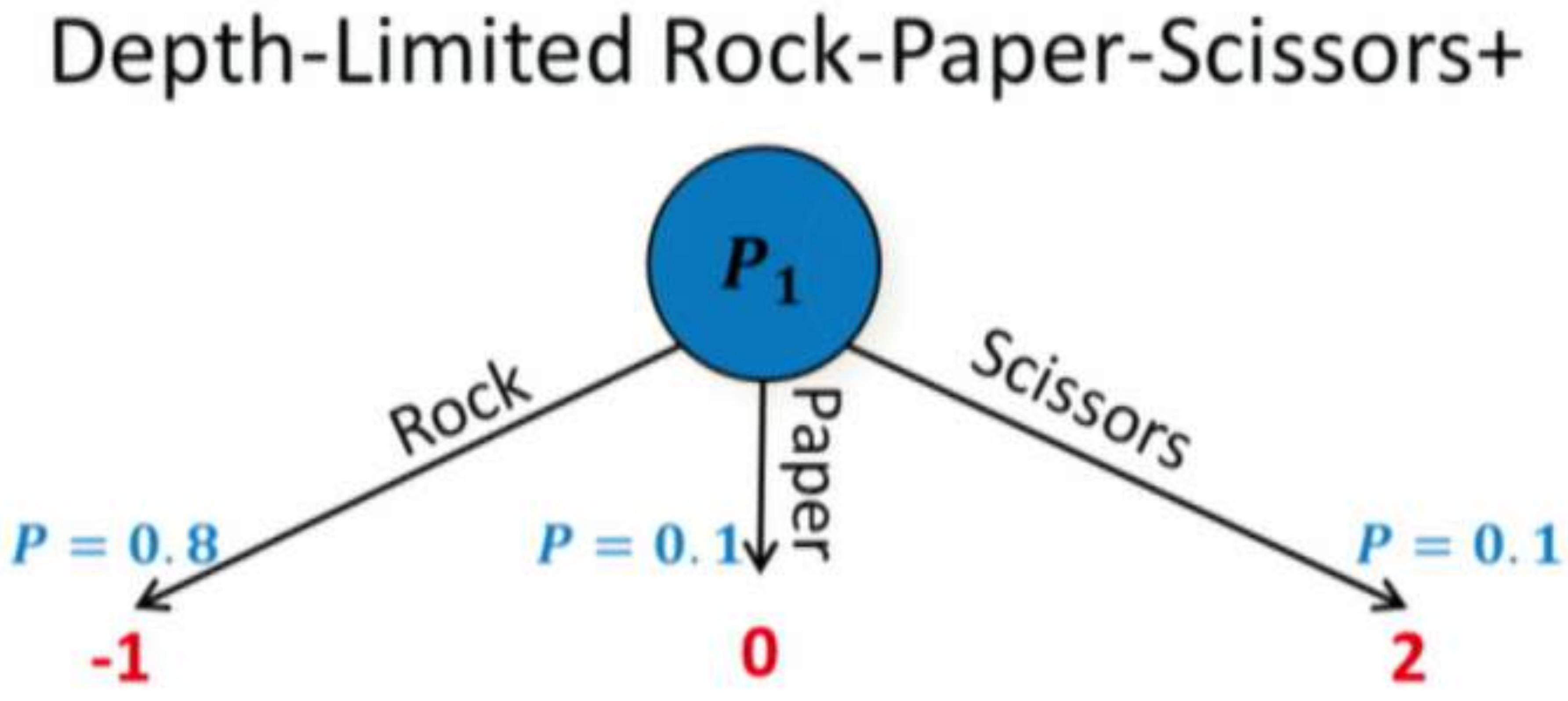

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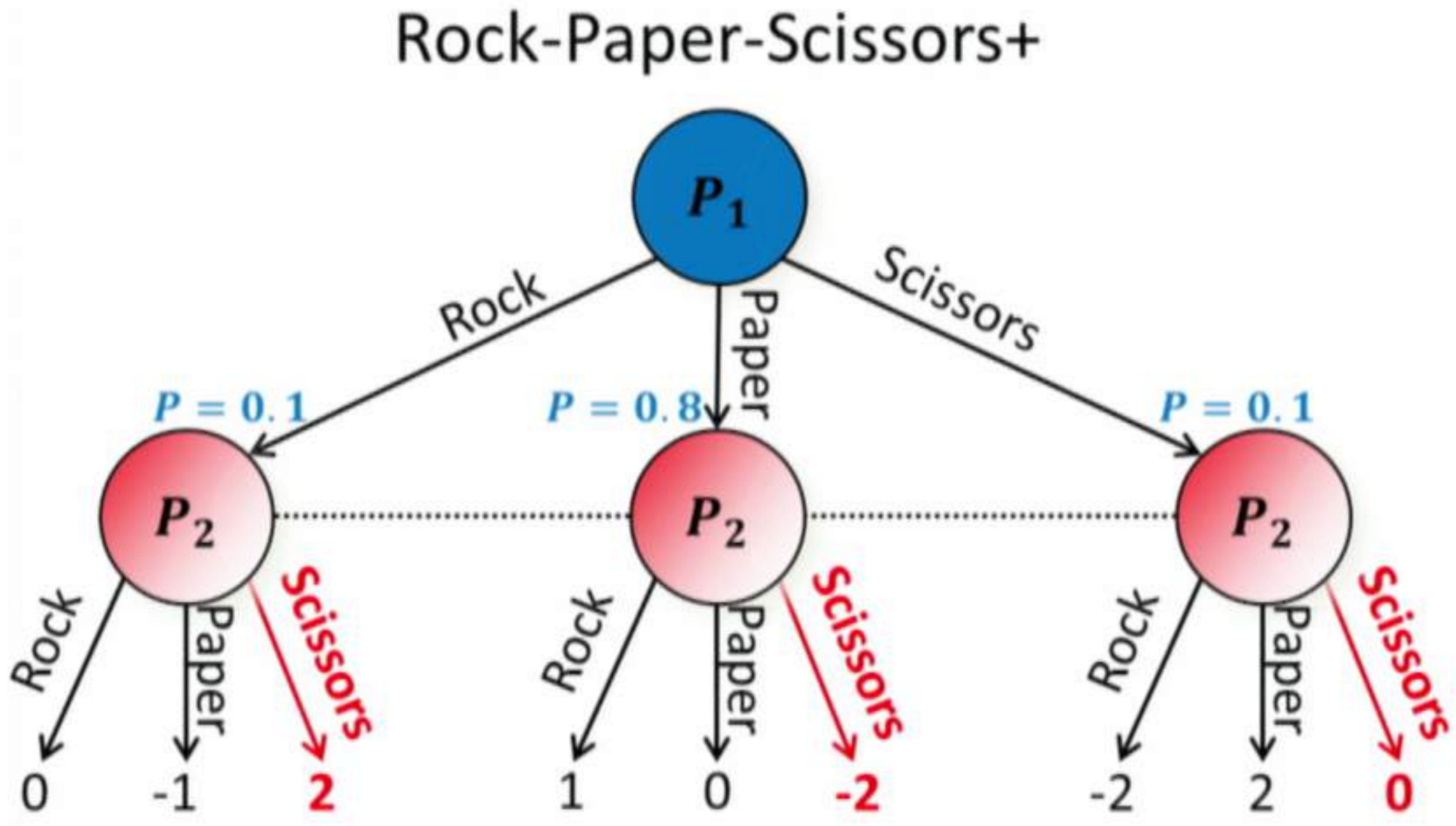
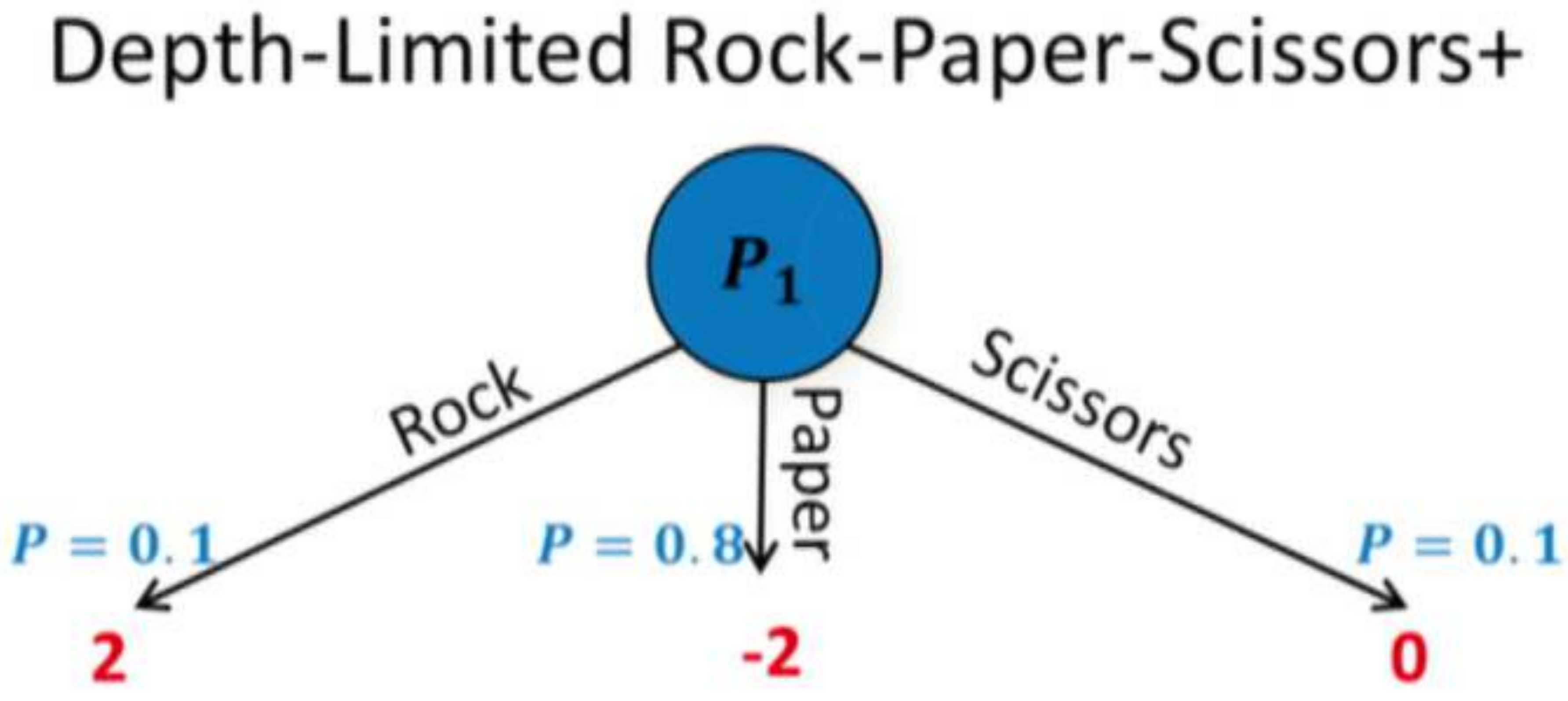
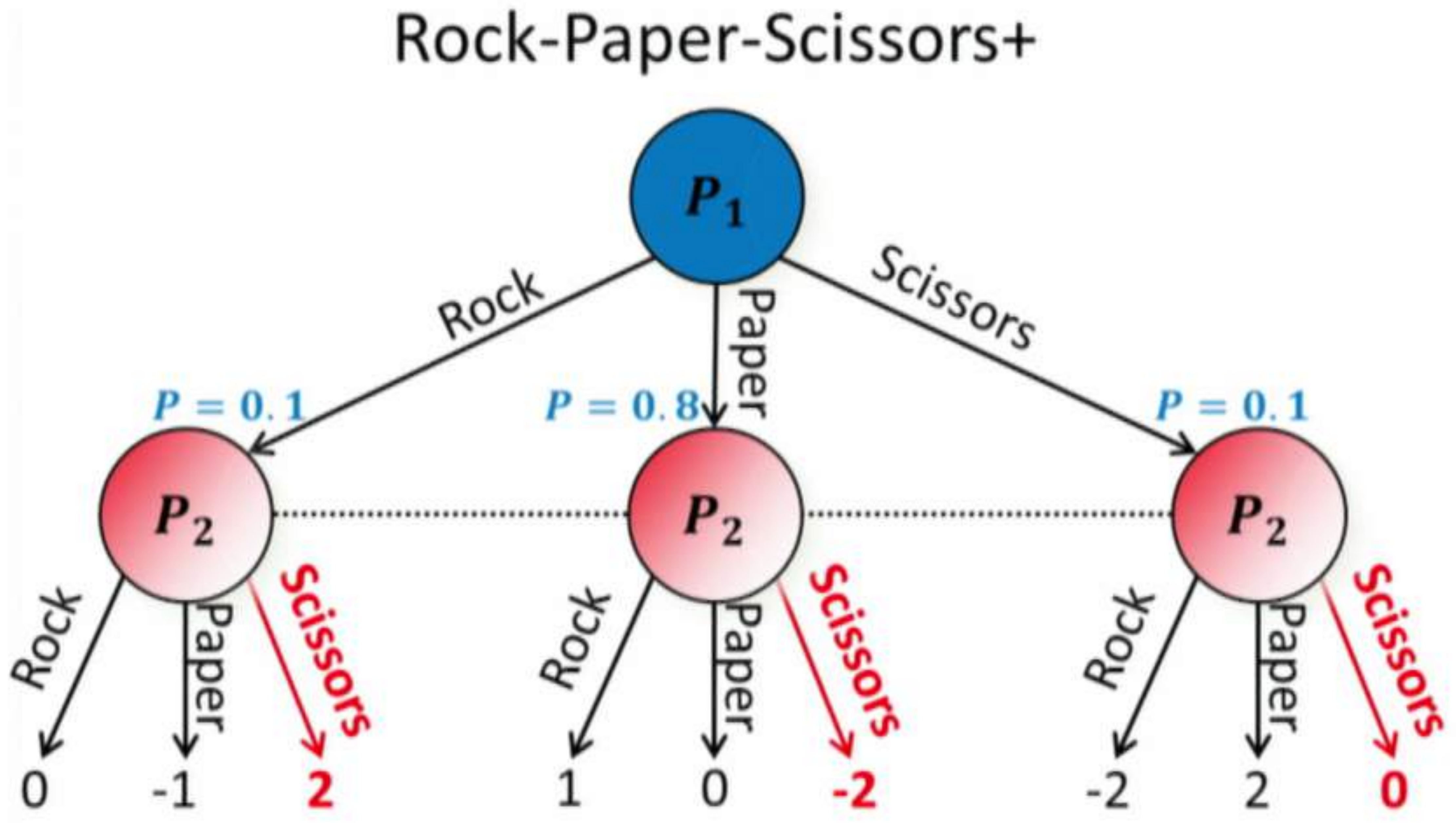
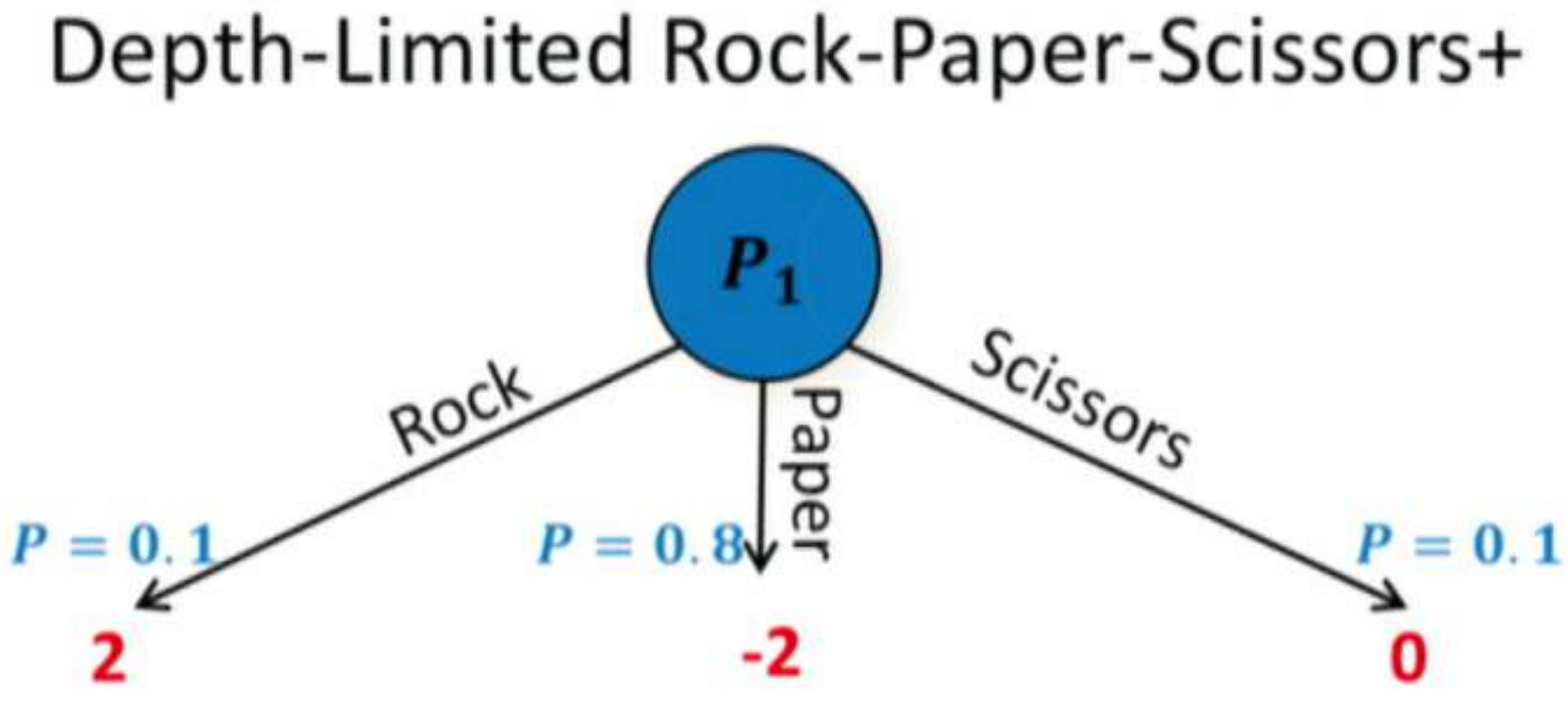




- Naïve approach: make state values a function of the state **and the entire policy** π
- Problem: extremely expensive

Depth-Limited Solving



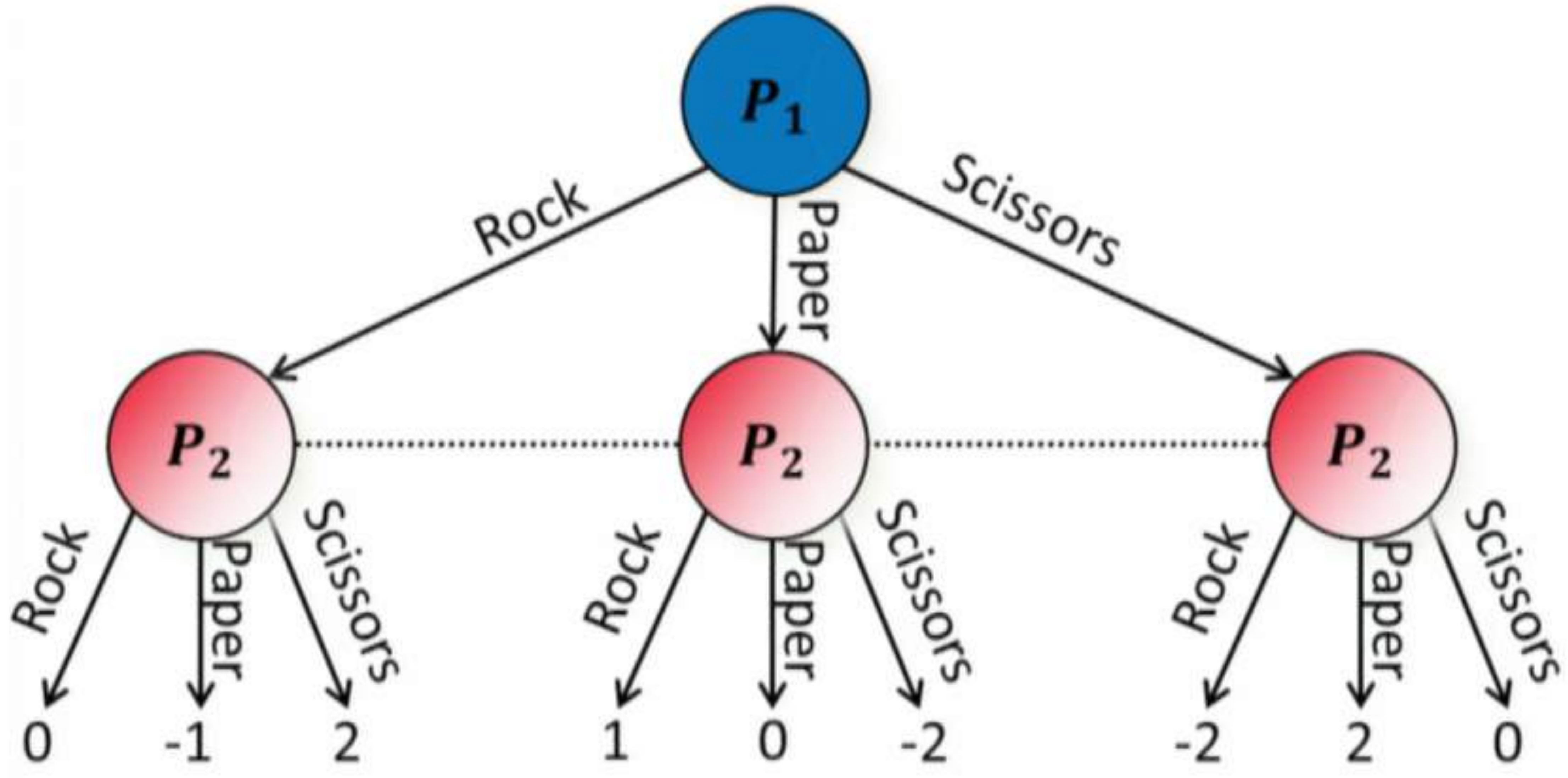


- *DeepStack* [Moravcik et al. Science-17] approach: condition state values on **believed state distribution** of each player
- Problem: still very expensive (DeepStack used 1.5 million core hours and could not beat prior top AIs)
- Problem: does not (currently) scale. In Texas Hold'em, input is $\sim$2,000 floats. In five-card draw, input is $\sim$5 billion floats. In Stratego, input is $> 10^{20}$.

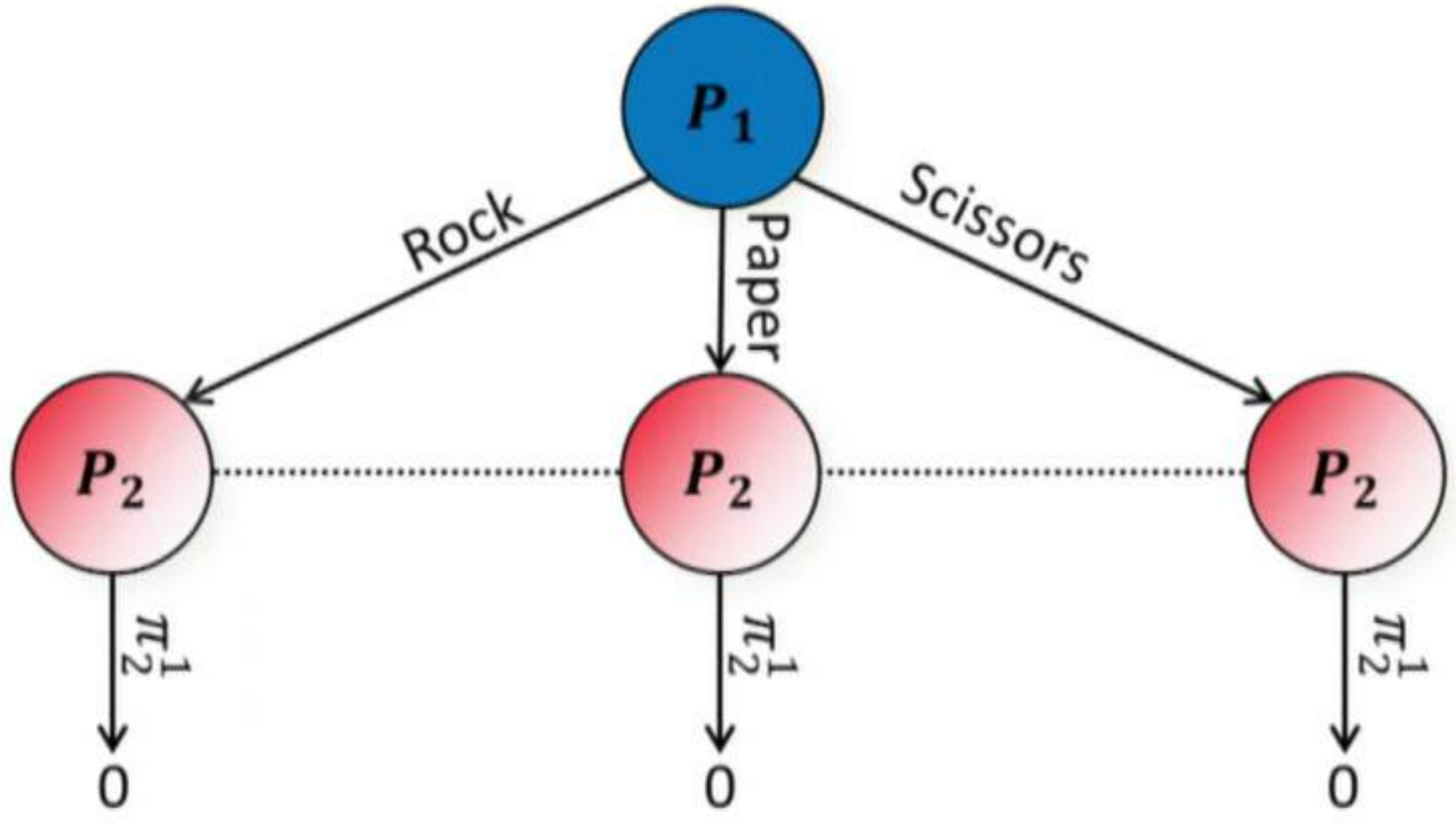
Depth-Limited Solving in *Modicum*

[Brown et al. NIPS-18]

Rock-Paper-Scissors+



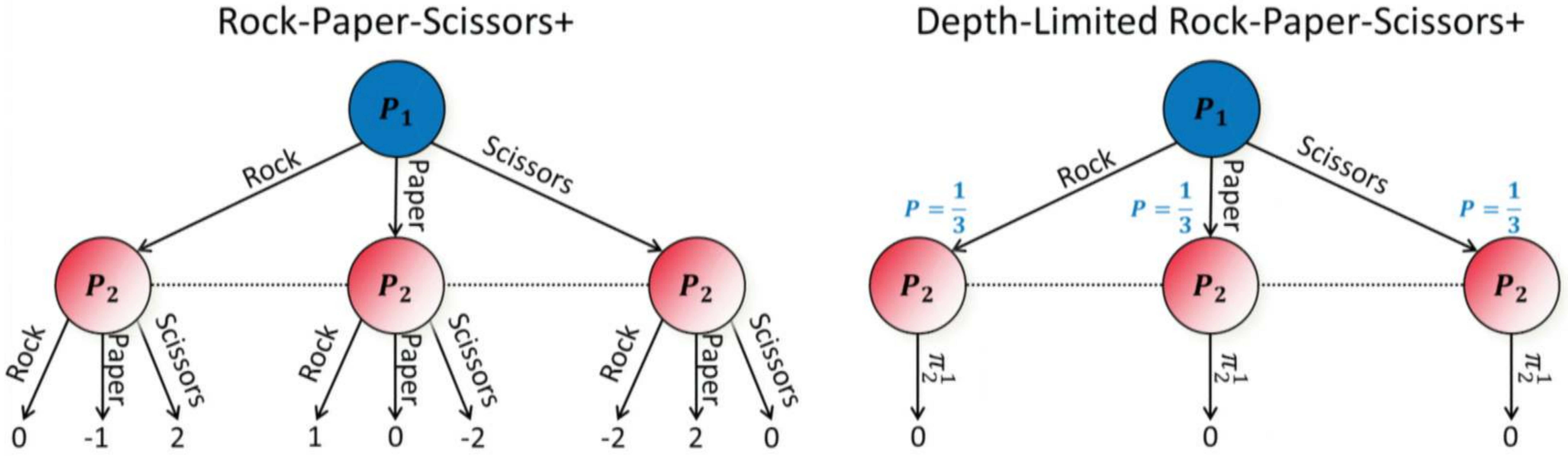
Depth-Limited Rock-Paper-Scissors+



- At leaf nodes, allow other player(s) one final action choosing between multiple policies for the remaining game
- Step 1: Solve subgame with current set of P_2 leaf-node policies
- Step 2: Calculate a P_2 best response
- Step 3: Add P_2 best response to set of leaf-node policies
- Repeat

Depth-Limited Solving in *Modicum*

[Brown et al. NIPS-18]



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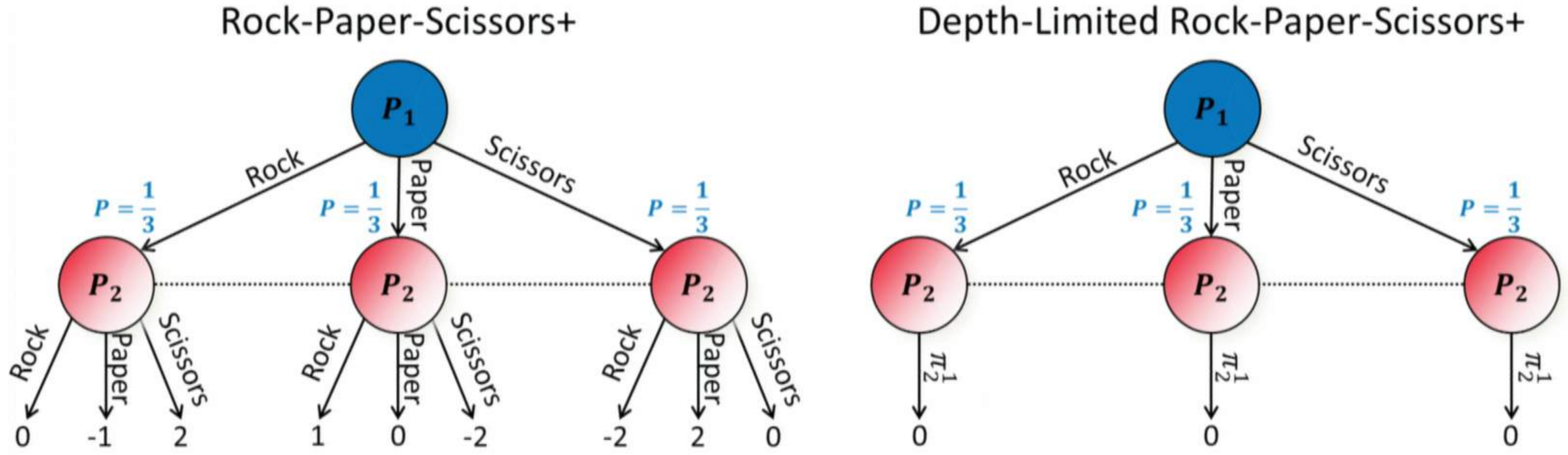
[Brown et al. NIPS-18]



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[Brown et al. NIPS-18]

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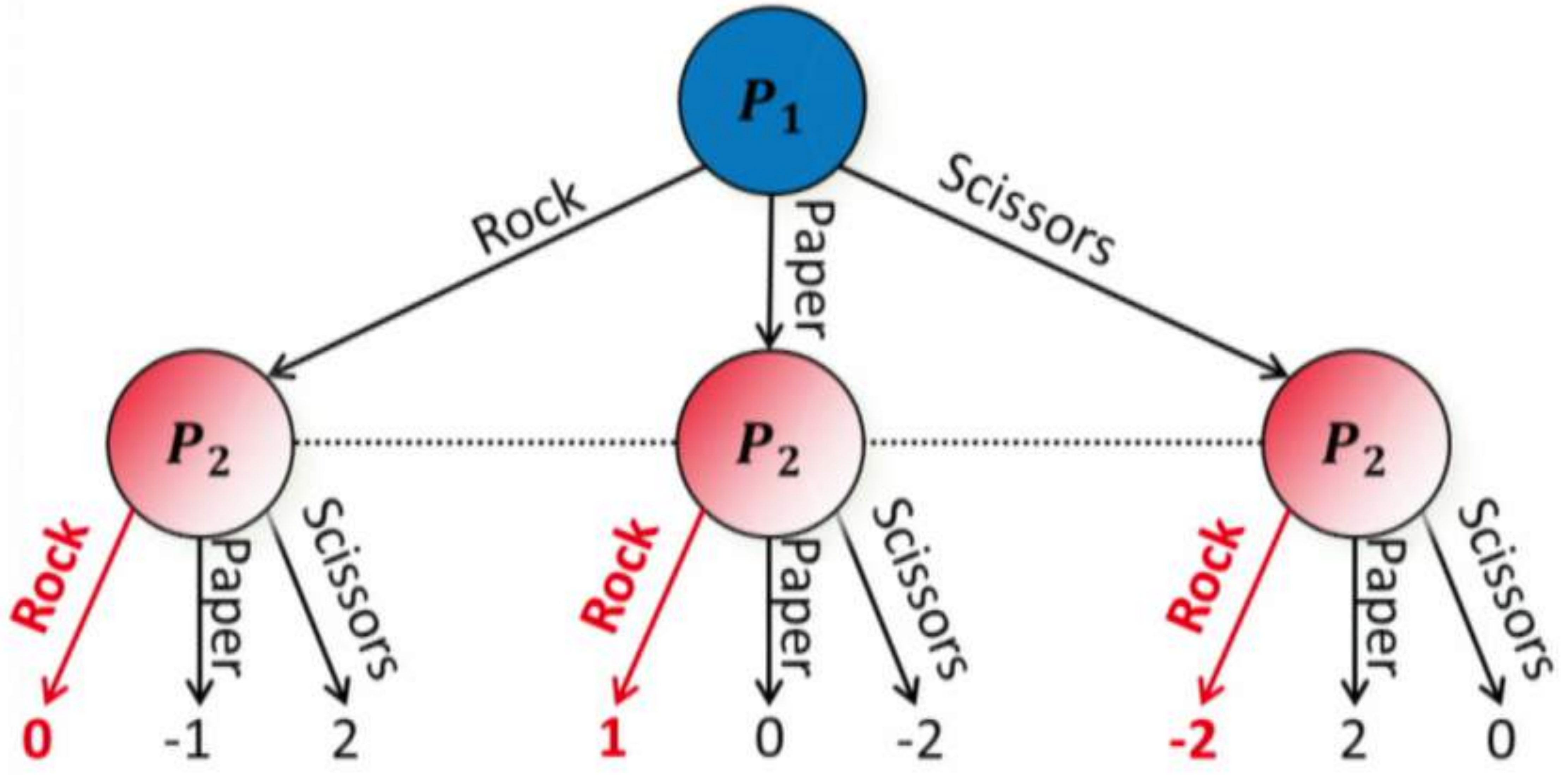

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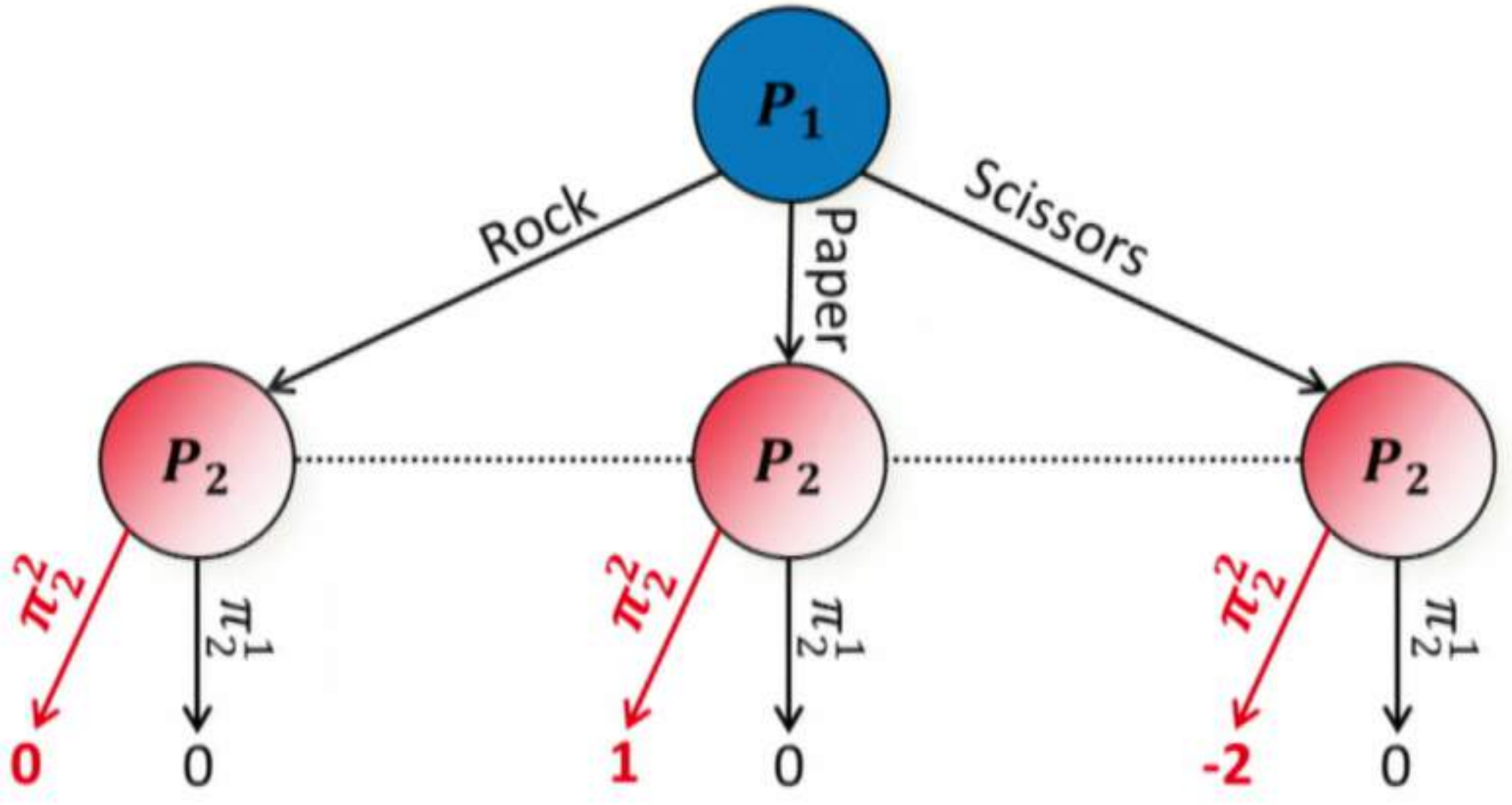

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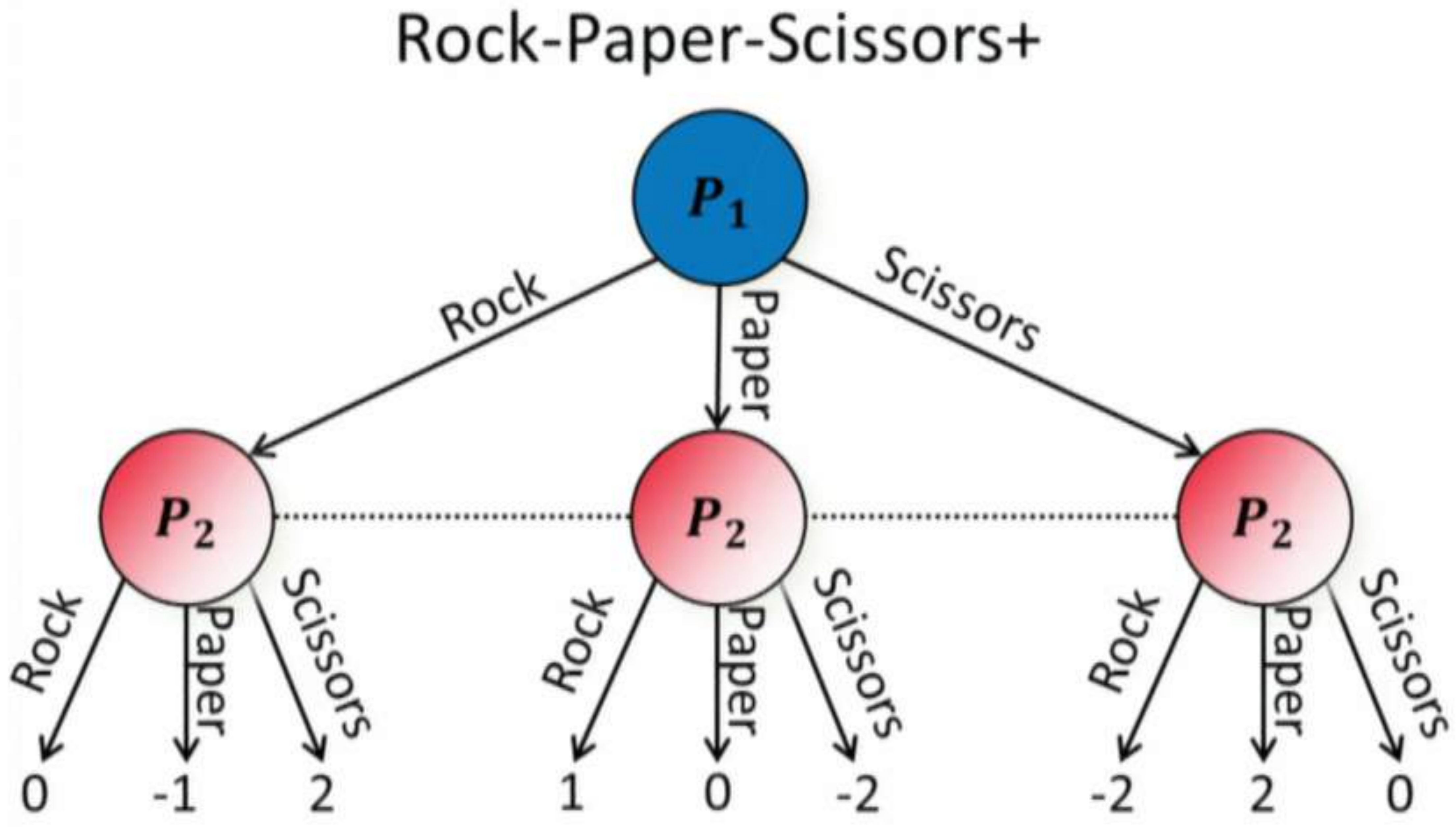
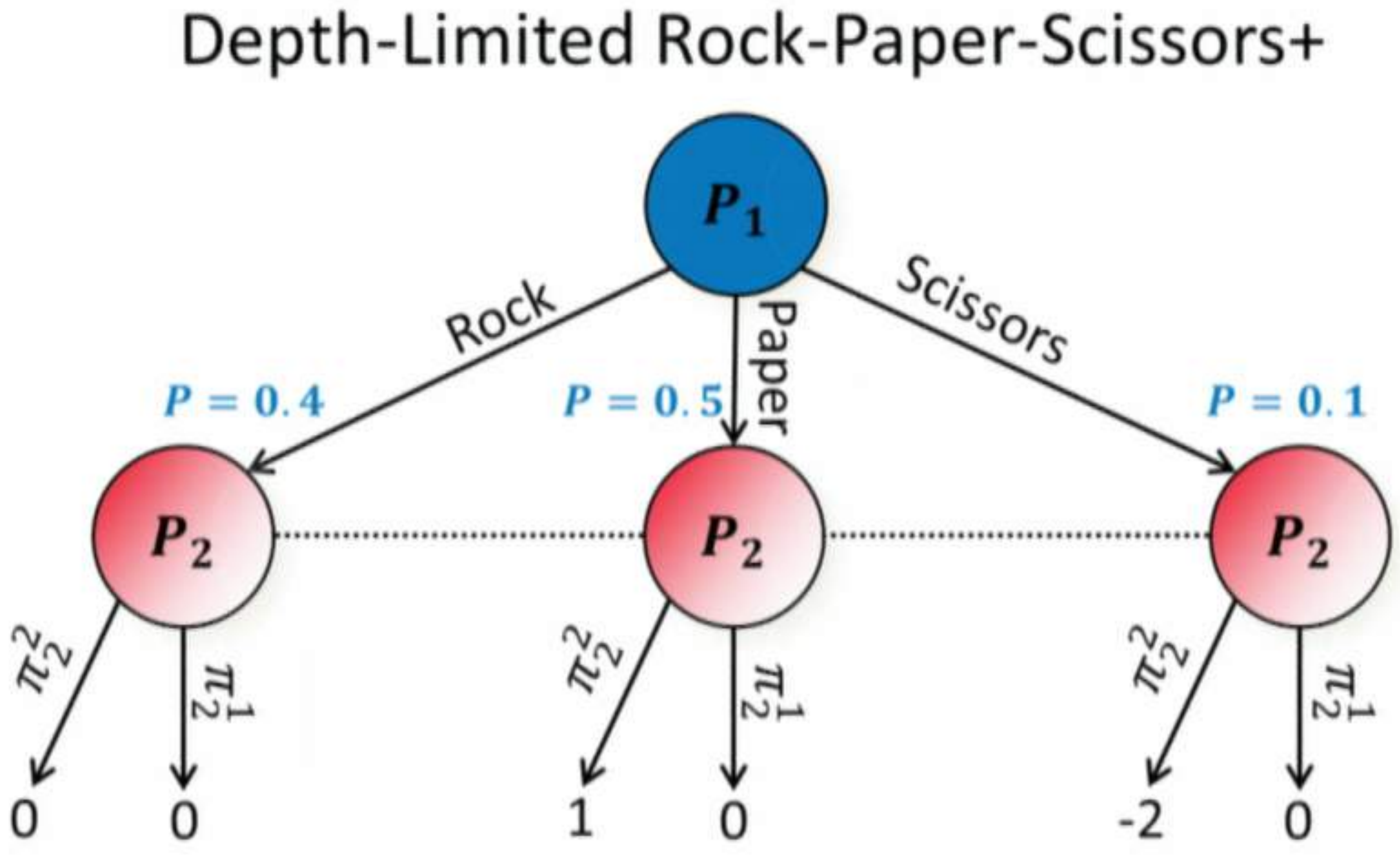
[Brown et al. NIPS-18]





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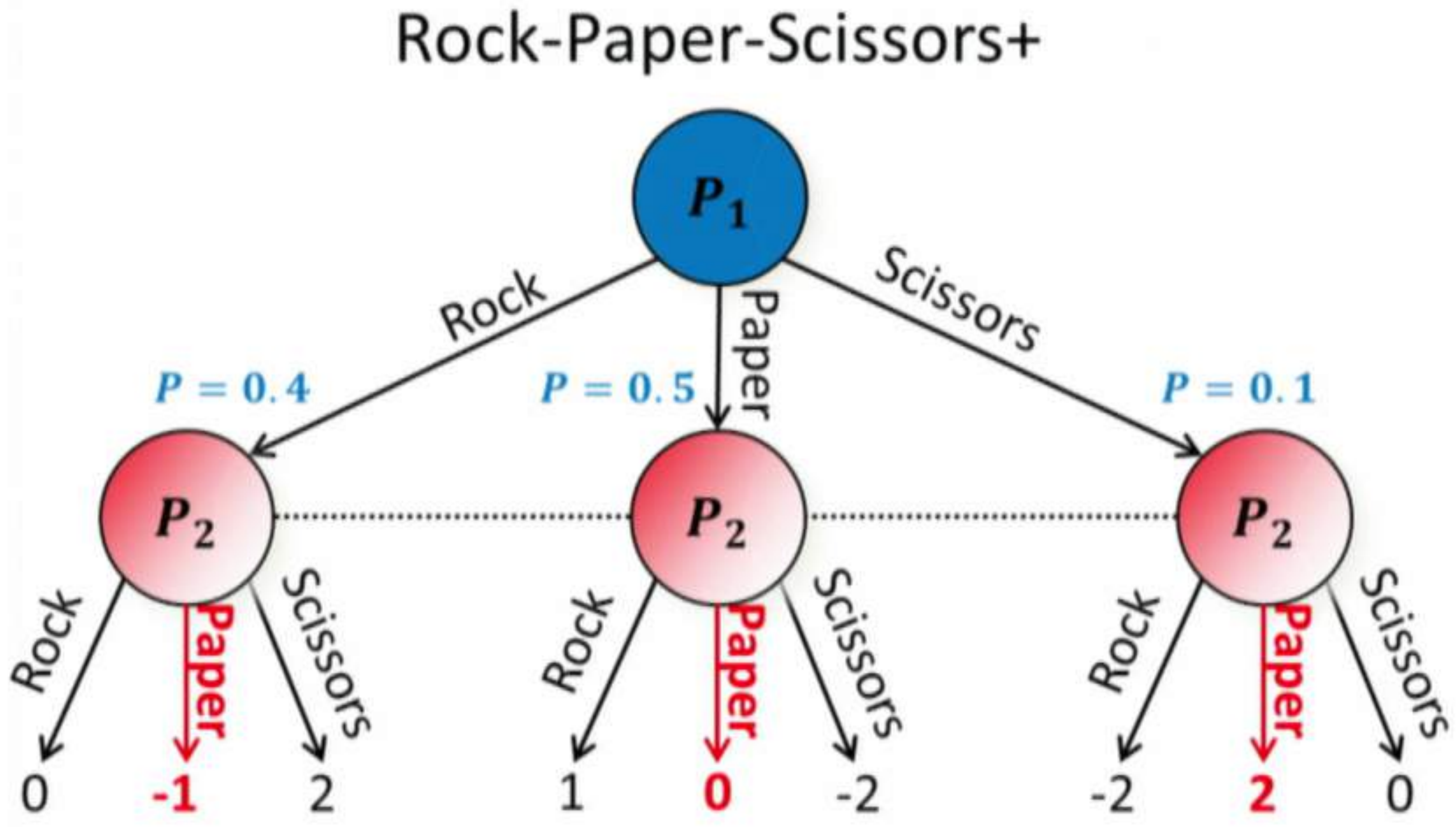
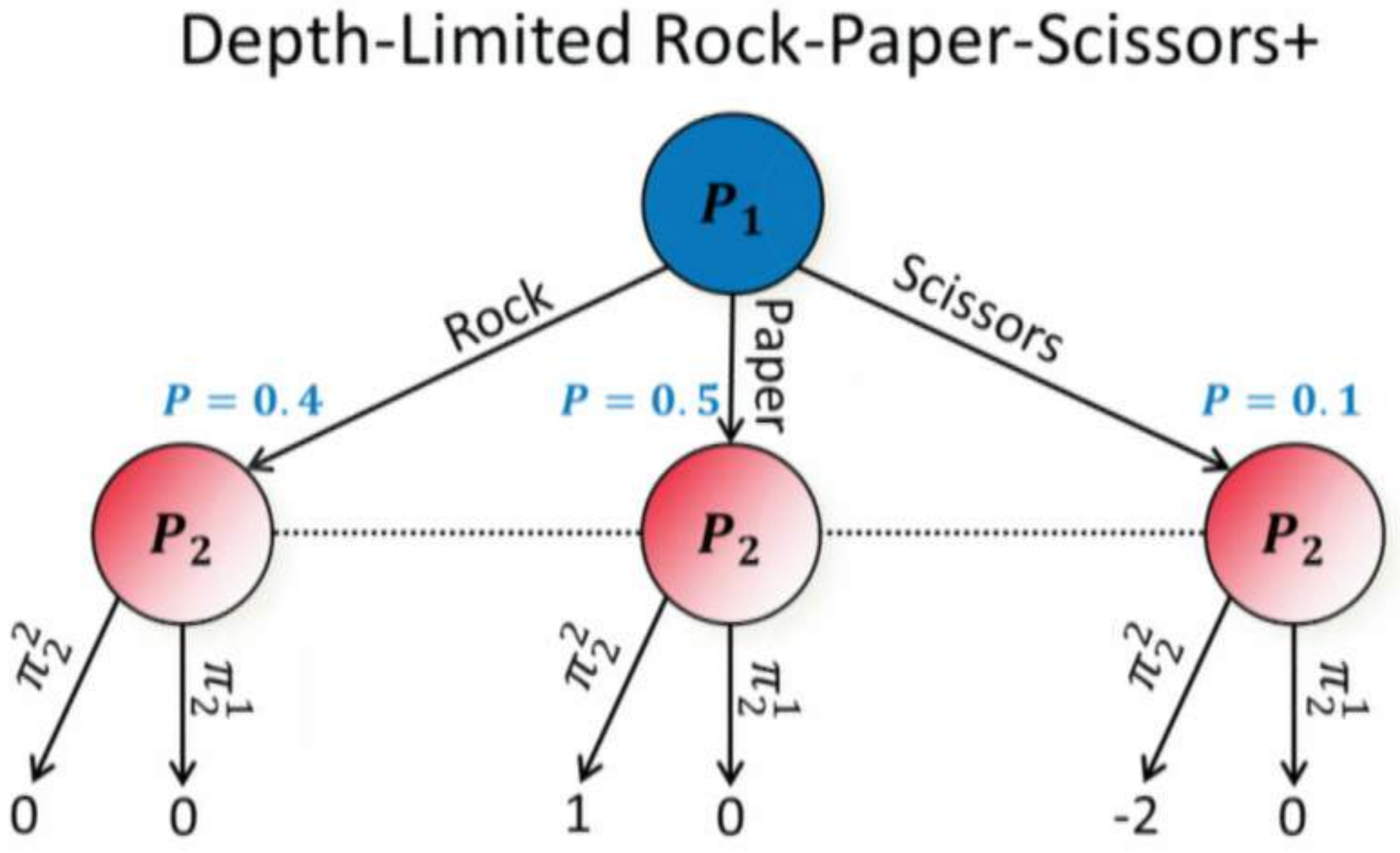
[Brown et al. NIPS-18]





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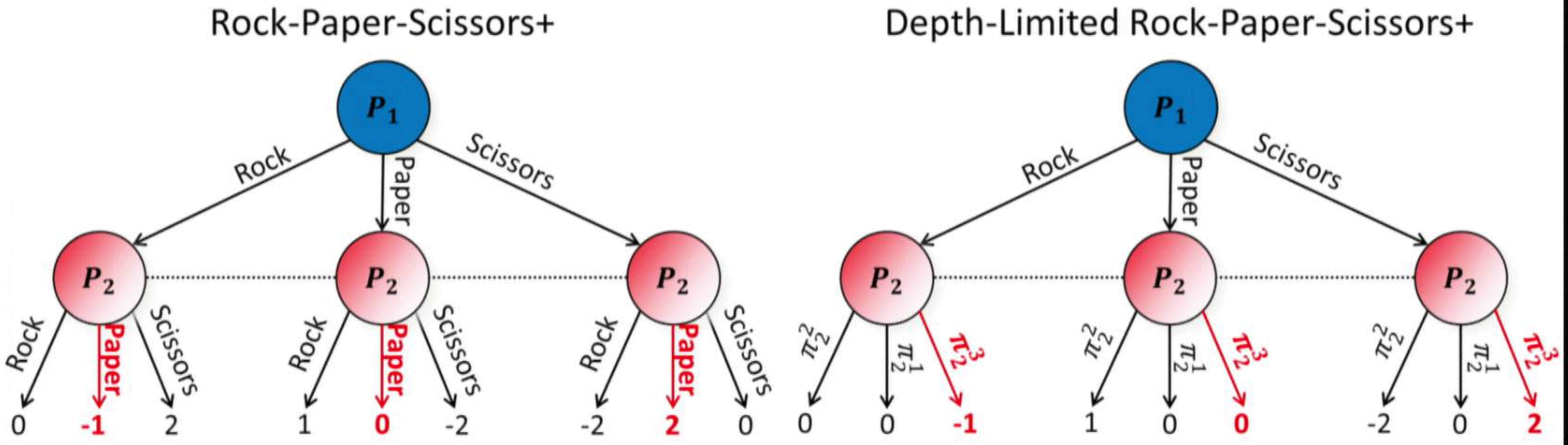

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Depth-Limited Solving in *Modicum*

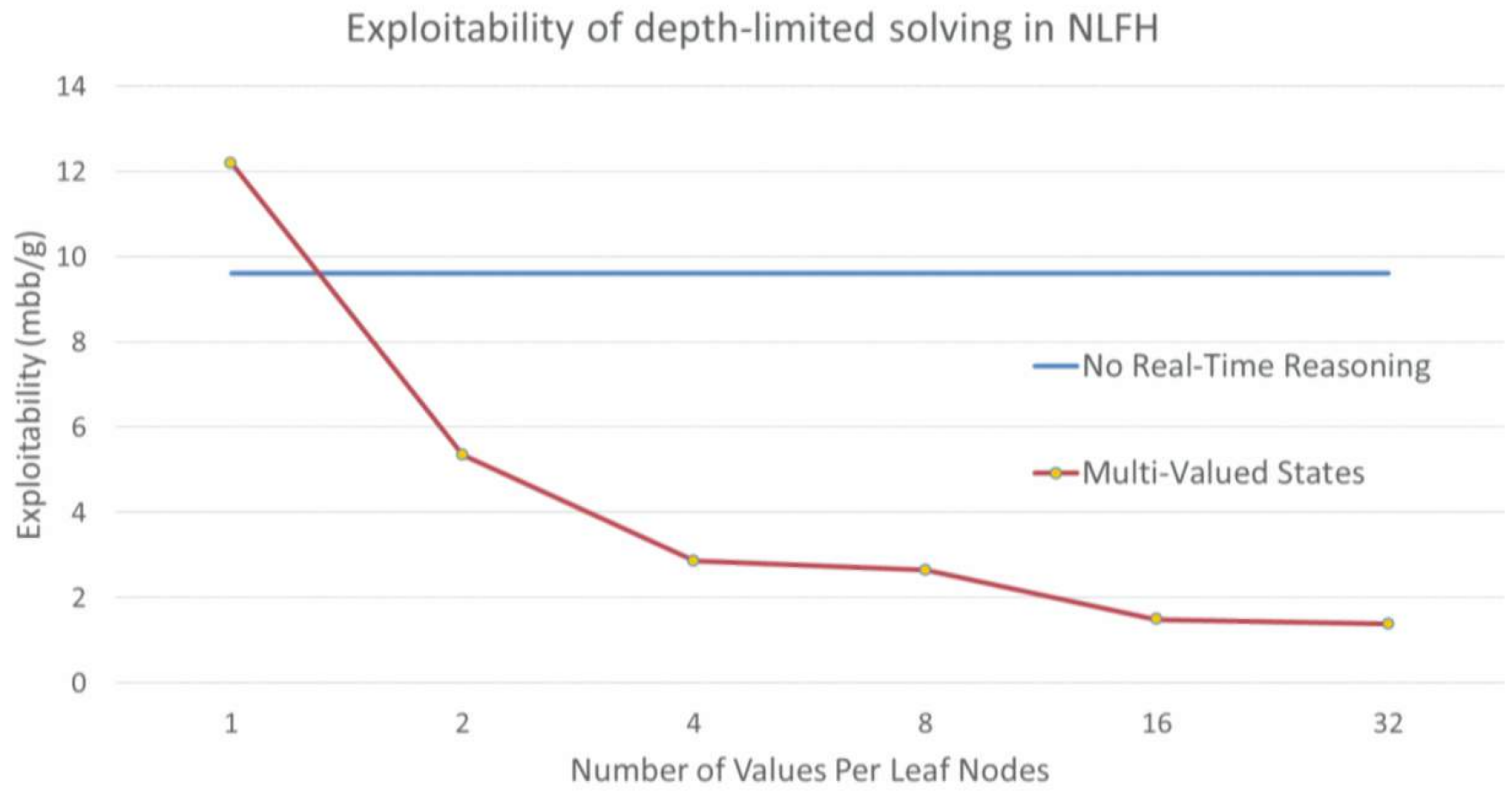
[Brown et al. NIPS-18]

- P_2 decision is made at each leaf information set *separately*
  - 100 leaf infosets and 10 policies to choose from means 10^{100} choices

- For P_1 decisions below the depth limit, we assume P_1 plays according to the pre-computed approximate equilibrium

- The set of P_2 policies is pre-computed, not decided in real time

Exploitability Measurements



Head-to-head performance of *Modicum*

- Tartanian8 [2016 champion]
  - 2 million core hours
  - 18 TB of memory
  - No real-time reasoning
- Slumbot [2018 champion]
  - 250,000 core hours
  - 2 TB of memory
  - No real-time reasoning
- Modicum
  - 700 core hours
  - 16 GB of memory
  - Plays in real time with a 4-core CPU in 20 seconds per hand

	Tartanian8	Slumbot
Modicum (no real-time reasoning)	-57 ± 13	-11 ± 8
Modicum (just one value per leaf node)	-10 ± 8	-1 ± 15
Modicum	6 ± 5	11 ± 9

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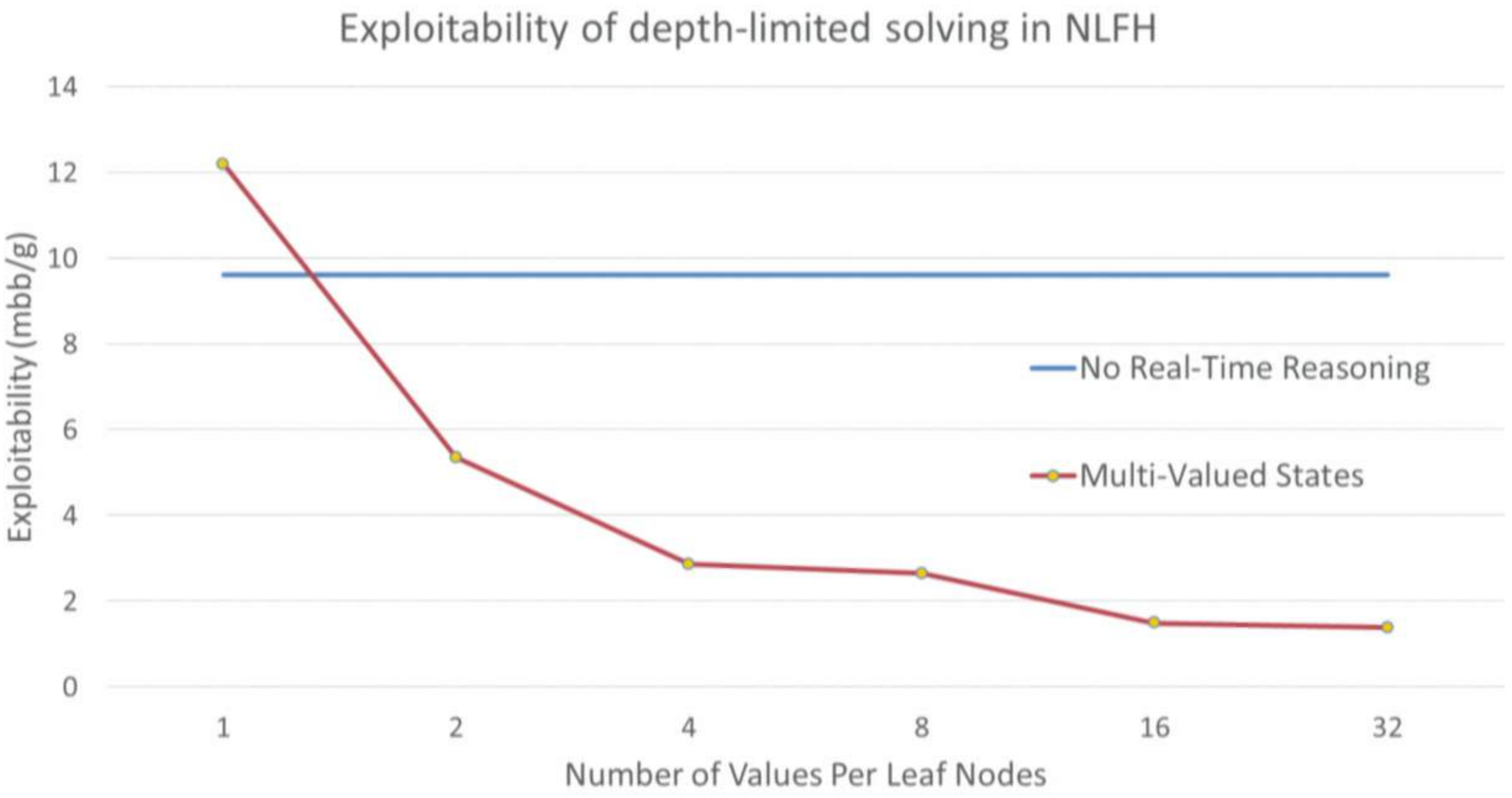

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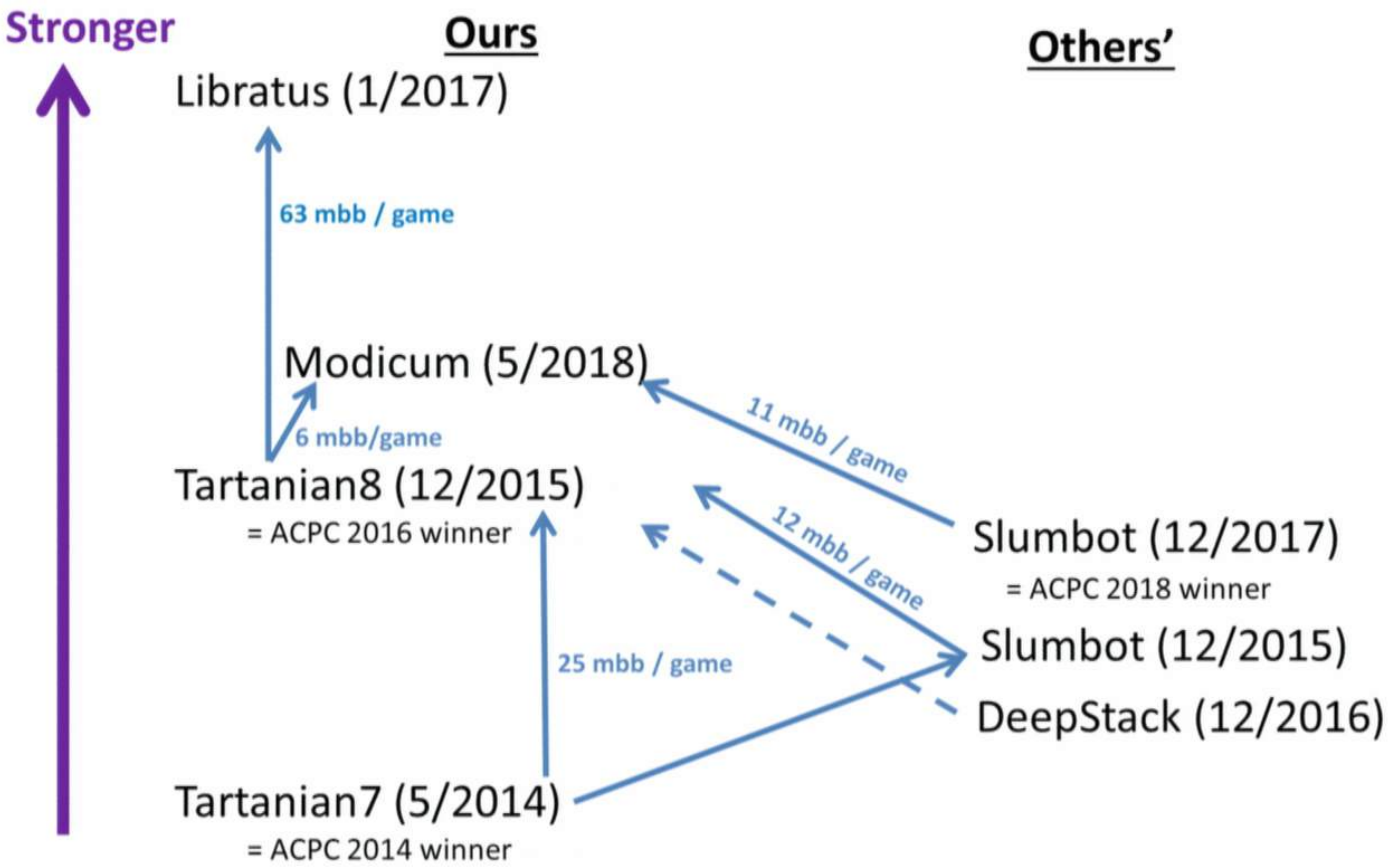
Head-to-head strength of top AIs



Key Takeaways

- In real-time planning, you must always consider how the opponent can **adapt** to changes in your policy
  - Except in perfect-information games
- Imperfect-information subgames cannot be solved in isolation
- States do not have a single well-defined value in imperfect-information games

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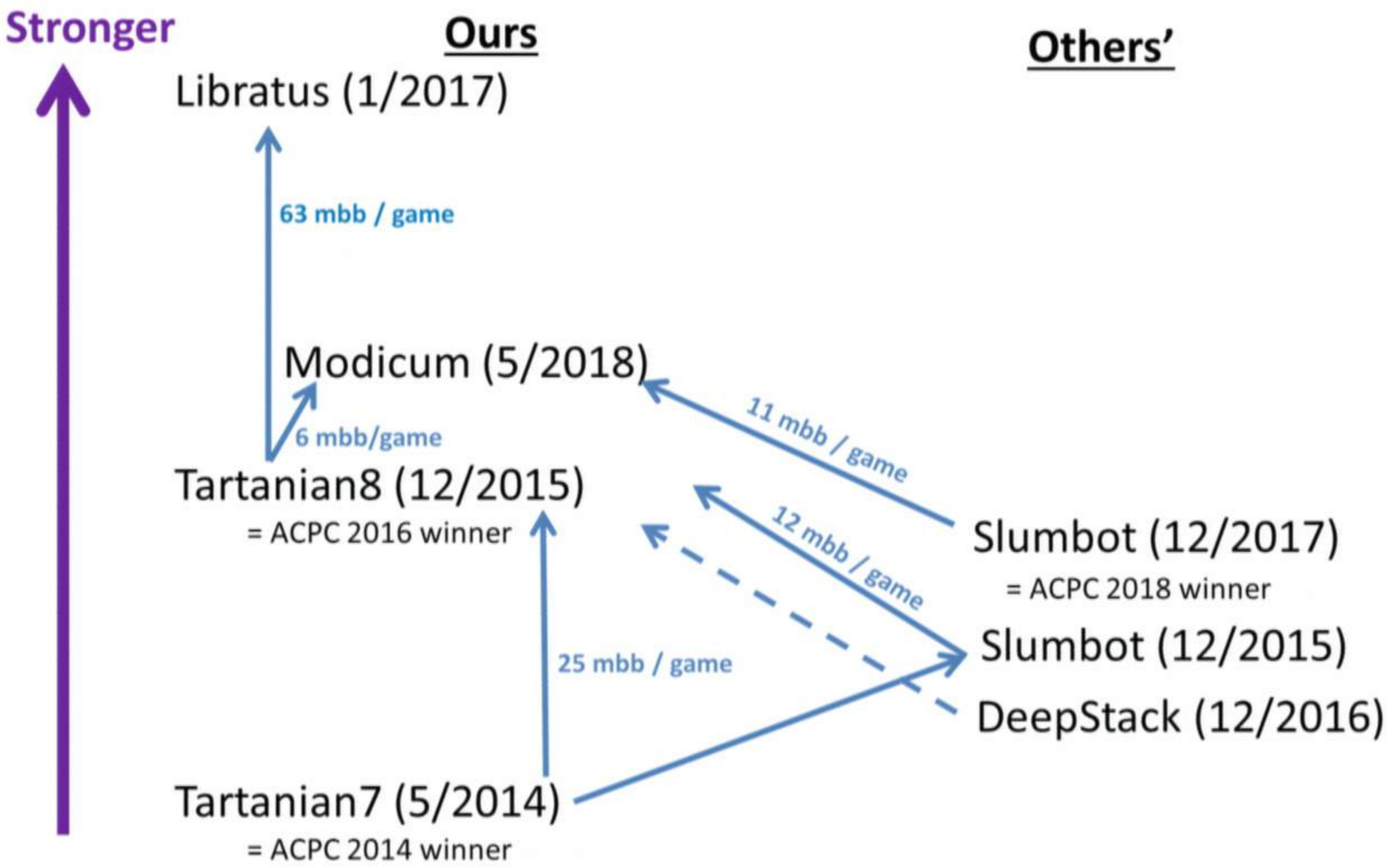

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Other work

- How do we actually solve these games? Answer: CFR
  - Developed a form of CFR that is faster than the prior best by 3x
- Pruning in CFR (and Fictitious Play) [Brown & Sandholm NIPS-15, ICML-17]
  - Provably reduces computing and memory requirements
  - In practice, can speed up convergence by orders of magnitude
- Determining the optimal action(s) in a continuous action space [Brown & Sandholm AAAI-14]

Future Directions

- Bringing together techniques for perfect-information and imperfect-information games

- Semi-cooperative (general-sum) games, emergent communication

- Real-world applications: negotiations, security, auctions

Thank You!

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