LIGHT REFLECTION FUNCTIONS FOR SIMULATION OF CLOUDS AND DUSTY SURFACES

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ABSTRACT

The study of the physical process of light interacting with matter is an important part of computer image synthesis since it forms the basis for calculations of intensities in the picture. The simpler models used in the past are being augmented by more complex models gleaned from the physics literature. This paper is another step in the direction of assimilating such knowledge. It concerns the statistical simulation of light passing through and being reflected by clouds of similar small particles. (It does not, however, address the cloud structure modeling problem). By extension it can be applied to surfaces completely covered by dust and is therefore a physical basis for various theories of diffuse reflection.

CR Categories and Subject Descriptors: I.3.3 [Computer Graphics]: Picture/Image Generation - display algorithms; I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism - Visible line/surface algorithm

General Terms: Algorithms, Theory

1. INTRODUCTION

Computer image synthesis requires the calculation of intensities of light reflecting from an object. Such calculations are based on the physics of light interaction with the surface and on the geometry of the light sources. Intensity functions are generally expressed in terms of the vector quantities shown in figure 1.

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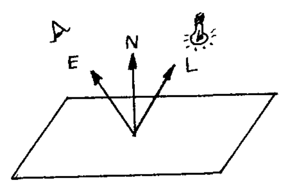


N = Vector perpendicular to the reflecting surface
L = Vector in the direction of the light source
E = Vector in the direction of the observer

Figure 1 - Lighting Geometry

Care must be taken in choosing the direction of the vector N. Two opposite directions will equally well describe the orientation of the surface. It usual to choose the direction which forms a positive dot product with E.

Computer graphics first used the simplest of models, Lambert's law, where the observed intensity is assumed proportional to the projected areas of the light source on the surface, thus:

$$I = a\,(N.L) \quad \text{for } N.L > 0$$
$$= 0 \quad \text{for } N.L < 0$$

Later models [1,2] enhanced this by adding a specular component which is a function of all three vectors:

$$I = a\,(N.L) + b\,\mathrm{Spec}(N,L,E)$$

These models were applied to solid surfaces and produced good simulations of both matte (diffuse reflecting) and shiny (specularly reflecting) surfaces.

One of the current frontiers in computer image synthesis is the simulation of fuzzy or cloudy surfaces [14]. Some initial efforts on this problem have been undertaken by Nelson Max at Lawrence Livermore Laboratory [13] and Roger Wilson at Ohio State University. The problem has two components, the modeling of the cloud density as a function of position in space, and the simulation of how light interacts with this density function. The modeling problem will not be addressed here.

This paper presents some simple simulations relevant to the second part of the problem. This work was motivated by the need to synthesize images of another type of cloudy object, the rings of the planet Saturn. This problem has the advantage that it is a fairly simple geometric situation and attention can be paid to the light reflection portion of the problem.

2. SINGLE SCATTERING CLOUD MODEL

The exact simulation of light interacting with clouds of particles is, in general, a very complex problem. It is extensively studied by the discipline known as Radiative Transport Theory. The classic work in the field is by Chandrasekhar [3]. Unfortunately this book is somewhat inaccessible for all but the mathematically sophisticated. Various of the simpler results have been presented elsewhere, however [5,8,10] and it is to these that we turn.

2.1 Geometry of the Model

The basic model, shown in figure 2, assumes a cloud of spherical reflecting particles of radius p, positioned randomly in a layer of thickness T, and having number density n (That is, there are n particles per unit volume). The proportional volume of the cloud occupied by particles is then the number density times the volume of one particle:

$$D = n\,(4/3)\pi\,p^3$$

This will have values between 0 and 1 but will presumably be small for fairly diffuse clouds.

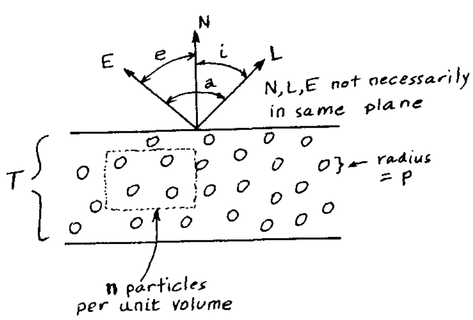


Figure 2 - Geometry of Cloud Layer

We wish to find the amount of light escaping from the layer in the direction of E after entering from the direction L and reflecting off one or more particles. We would also like to find the transparency of the layer, Tr, the amount of light showing through the layer from behind, i.e. from the direction -E.

In the literature on the subject the following notation is used for the angles between the three lighting vectors N, L and E.

incident angle = i	$\cos(i) = \mu_0 = N.L$
emission angle = e	$\cos(e) = \mu = N.E$
phase angle = a	$\cos(a) = L.E$

The angle between L and E is called the phase angle. A phase angle of zero occurs when light is coming from directly behind the viewer. When observing the moon, for example, this situation corresponds to a full moon. As the moon goes through its various phases the phase angle cycles through 360 degrees.

We will make a series of simplifying assumptions about the physical system which will make the problem analytically tractable. The result will be a brightness function of form:

$$B = w/\mu\ \varphi(a)\ S$$

where
- w = albedo of individual particles (section 2.3)
- u = cosine of emission angle = N.E
- φ = Phase function of a (section 2.2)
- S = Scattering probability (section 2.4)

The brightness calculations to follow will determine a brightness per unit area of the surface. An observer viewing the surface at a shallow angle e sees a greater portion of the surface projected into one pixel than an observer viewing it perpendicularly. This projected area is taken into consideration by the division by N.E. The remaining terms are discussed in sections below.

2.2 Phase Function

Each particle is too small to be seen individually and so its observed brightness is the integral of the contributions over its observed area. Due to the symmetry of the particles this net brightness can be assumed to vary only as a function of the lighting direction from the point of view of the observer. This is the angle between E and L, the phase angle a. A function characterizing the total brightness of a particle as a function of this angle is called a phase function, $\varphi(a)$. For example, if the particles are substantially larger than the wavelength of light, diffraction at the edges of the particles is negligible and the phase function will be as in figure 3. The exact form of various useful functions for φ will be discussed in section 3.

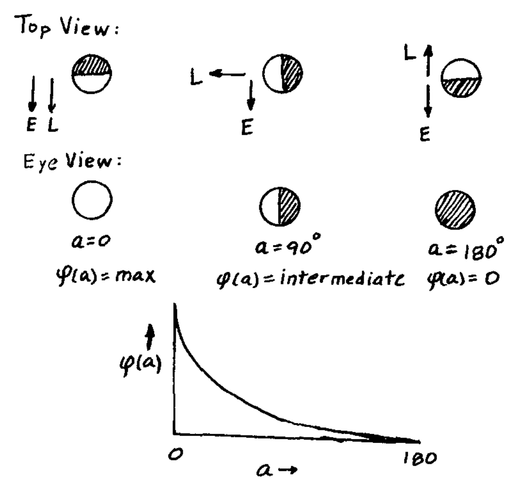


Figure 3 - Definition of Phase Function

2.3 Albedo

The next simplifying assumption is that the primary effect is from the interaction of a ray of light with a single particle, i.e. multiple reflections will be considered negligable. This will be true if the albedo, w, of each particle is small. (Albedo is the proportion of light reflected from a particle vs. light impinging on it, i.e. the reflectivity of the particle). The net brightness will be proportional to w.

2.4 Scattering

The final effect on the brightness will be due to the shadowing and blocking effect of other particles on the light ray as it enters and exits the cloud layer. The model asserts that a ray of light will be visible if there are no other particles in the way along this path. For example; for a particle at depth T' in the layer, illuminated from above, a ray of light will bounce off it and be visible if no other particles encroach on the volume formed by the two cylinders Vin and Vout having radius p in figure 4.

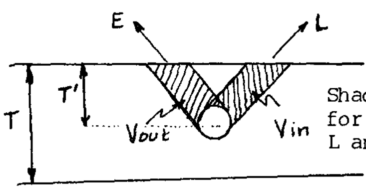


Figure 4 - Scattering Conditions

All other particles are assumed to be of radius p also. For such a particle to be completely outside the cylinder its center must be outside this volume. Statistically, then, the attenuation of light traversing a cylinder of radius p and volume V is P(0;V) = the probability of 0 particles in volume V. Now the expected number of particles in a given volume V is nV. If n is small this can be modelled as a Poisson process and

$$P(0;V) = \exp(-nV)$$

An approximation is made here that two particles being inside V are independent events. In reality, the impossibility of mutual overlap makes them slightly dependant. We can neglect this if D is small.

The brightness due to a given layer dT' within the cloud is the product of

projected viewing area = $\pi p^2/\mu$
brightness of particle = $w\,\varphi(a)$
expected number of particles/unit area = n dT'
probability particle will be illuminated = p(0;V)

The net brightness of the cloud will be the integral of this function over T. This reduces to

$$B = w/\mu\ \varphi(a)\ n\ \pi\ p^2 \int_0^T P(0;V)\ dT'$$

There are two cases to be considered here. The top lit case, where N.L > 0, is shown in figure 5.

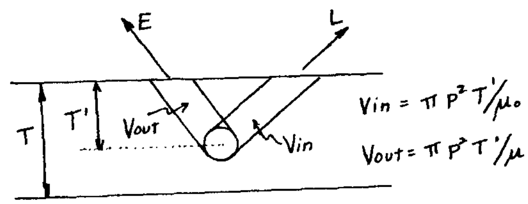


Figure 5 - Top Lit

Note that there is an approximation introduced here in that the overlap between cylinders Vin and Vout is being neglected. That is, some of the volume is being counted twice. This becomes significant only when E≈L and will be addressed later.

$$B = \frac{w}{\mu}\varphi(a)\,n\pi p^2 \int_0^T e^{-n\left(\frac{\pi p^2 T'}{\mu_0} + \frac{\pi p^2 T'}{\mu}\right)}\,dT'$$

$$= w\,\varphi(a)\frac{\mu_0}{\mu_0+\mu}\left\{1 - e^{-n\pi p^2 T\left(\frac{1}{\mu_0}+\frac{1}{\mu}\right)}\right\}$$

The bottom lit case, where N.L < 0, is illustrated in figure 6.

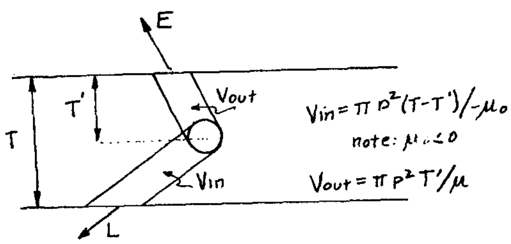


Figure 6 - Bottom Lit

$$B = \frac{w}{\mu}\varphi(a)\,n\pi p^2 \int_0^T e^{-n\pi p^2\left(\frac{T'-T}{\mu_0} + \frac{T'}{\mu}\right)}\,dT'$$

$$= w\,\varphi(a)\frac{\mu_0}{\mu_0+\mu}\left\{e^{\frac{n\pi p^2 T}{\mu_0}} - e^{\frac{-n\pi p^2 T}{\mu}}\right\}$$

Note that the variables T, n and p occur in the above expressions only in the form of

$$n\ \pi\ p^2\ T$$

This dimensionless quantity is called the "optical depth" and is written . Light traveling through a cloud of optical depth τ is attenuated by the factor $\exp(-\tau)$. In terms of this quantity the final brightness function is

Top lit:

$$B = w\,\varphi(a)\ \mu0/(\mu0+\mu)\ (1 - e^{-\tau(1/\mu0 + 1/\mu)})$$

Bottom lit:

$$B = w\,\varphi(a)\ \mu0/(\mu0+\mu)\ (e^{\tau/\mu0} - e^{-\tau/\mu})$$

Note that in this latter case we may have a singularity if $\mu0=-\mu$. Here we must perform the substitution before the integration and get

$$B = w\,\varphi(a)\ \tau/\mu\ e^{-\tau/\mu}$$

2.5 Transparency

The transparency of the cloud layer is the amount of light coming from directly behind the cloud which is not blocked off by particles. This will be the probability that a light ray does not encounter a particle in travelling through the layer. See figure 7. For arguments similar to the above it will be the probability that a cylinder of radius p extending through the layer has no particles in it. That is:

$$Tr = e^{-\tau/\mu}$$

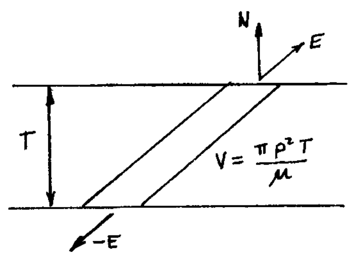


Figure 7 - Transparency

One may, in addition add a term for the forward scattering of the background through the cloud

$$Tr = e^{-\tau/\mu} + w\,\varphi(\pi)$$

2.6 Net Intensity calculation

The new brightness to be displayed at a given pixel for a cloud overlayed on a background color Bkg is

$$Bnew = B + Tr * Bkg$$

2.7 The Hapke-Irvine Function

An important special case of this function was presented by Hapke [8] and Irvine [10]. They proposed to model a dust covered surface or a dense atmosphere as a cloud with an infinite optical depth. In this case the top lit brightness function becomes

$$B = w\,\varphi(a)\ (N.L)/(N.L+N.E)$$

Note that for the case where L=E (i.e. where the light is directly behind the observer) this reduces to

$$B = 1/2\ w\,\varphi(a)$$

i.e.. a constant for any value of N. Recall that L=E corresponds to the situation during a full moon. This result, then, says that the entire disk of the full moon should have a uniform intensity, independant of the variation in normal vector from center to edges.

3. PHASE FUNCTIONS

We now turn to the phase function used in the above models. The form of the phase function depends on the physical structure of the individual particles. Several functions have been proposed in the literature. The simplest are motivated more due to their mathematical simplicity than for physical reasons.

3.1 Constant Function

The simplest function used in the earliest of models assumes isotropic scattering [4] and is simply a constant.

$$\varphi(a) = 1$$

This function corresponds to the situation where the size of the scattering particles is substantially less than the wavelength of the light.

3.2 Anisotropic

The next most complex form takes account of the fact that more light should be reflected back toward the light source than forward. Its form has been chosen to be simple algebraically and of roughly the correct shape for this effect.

$$\varphi(a) = F(a) = 1 + x\cos(a)$$

where x = adjustable property of material.

3.3 Lambert surfaces

The first really physically motivated function assumes each sphere to reflect light according to Lambert's Law. Integrating the brightness of the visible disk of a particle for a given viewing direction yields [4]

$$\varphi(a) = L(a) = (8/3\pi)\ (\sin(a) + (\pi - a)\cos(a))$$

3.4 Rayleigh Scattering

For particles which are small compared to the wavelength of the light, diffraction effects predominate. This situation was first discussed by Lord Rayleigh [7] yielding the function

$$\varphi(a) = 3/4\ (1 + \cos^2 a)$$

Note that with this function, as much light is scattered in the forward direction as in the backward direction.

3.5 Henyey-Greenstein

The analytic function which seems most popular in the literature is due to Henyey and Greenstein [9]

$$\varphi(a) = HG(g,a) = (1-g^2) / (1+g^2-2g\cos(a))^{3/2}$$

This is just the equation of an ellipse in polar coordinates, centered at one focus. The parameter g is the eccentricity of the ellipse and is a property of the material. It can be used to generate a primarily forward scattering function (g<0) or a primarily backward scattering function (g>0) or an isotropic function (g=0). The physical relevance of this function is confirmed by its very good fit, using a value of g=.325, to empirical data of dark rough surfaces such as furnace slag [16].

3.6 Empirical Measurements

Some surfaces have been empirically measured and the resulting phase function tabulated. One example is measurements of the surfaces of the two moons of mars, Deimos and Phobos. The results are listed in [12]

3.7 Sums of Functions

Some approximate simulation can be made of clouds of non-equal-sized particles by using a net phase function that is a weighted average of several functions, each applicable to a different size of particle. This was done originally by Hapke in approximating the phase function for particles on the surface of the moon. He added a forward scattering function to a Lambert function to get:

$$(a) = w1\ L(a) + w2\ F(a)$$

The first term accounts for the back scattering of the rough particles and the second term accounts for the forward scattering of glass-like spherical particles.

This technique was later employed by Esposito and Lumme [4] for the rings of Saturn, using two Henyey-Greenstein functions

$$\varphi(a) = w1\ HG(g1,a) + w2\ HG(g2,a)$$

They achieved a fair match to earth based observations with

$$w1 = .596 \qquad g1 = .5$$
$$w2 = .404 \qquad g2 = -.5$$

The first term accounts for the back-scattering of the large particles, the second term accounts for the forward scattering of the smaller particles.

4. RESULTS OF SINGLE SCATTERING MODEL

4.1 Variation with Incident/Emission Angle

Figure 8 shows a plot of the amount of light reflected from a surface as a result of an incident ray at four different angles from the surface normal (36, 72, 108, and 144 degrees). Note that the brightness function for an incident angle of 144 degrees is the same as the function at 36 degrees viewed from the other side of the cloud layer.

The rectangular shape represents the cloud layer and has two embedded coordinate axes to emphasize the location of the incident ray, coming from the right. The distance from this point to the surface in a given direction represents the amount of light scattered in that direction. This is the value of the brightness function, omitting the division by the viewing angle term N.E. In addition, to minimize confusion, the phase function is taken to be unity. Any given phase function will be symmetrical about the incident ray and will scale the brightness values plotted here.

Note that most of the light is reflected in directions perpendicular to the cloud layer. This is to be expected since this represents the shortest emission path and thus the least likelihood of encountering a blocking particle. Also note that the reflected light diminishes with increasing incidence angle since this represents a longer incident path and thus a greater likelihood of encountering a blocking particle.

4.2 Variations With Optical Depth

Figure 8 also shows the function for four different values of τ = 1000, 2., 0.5 and 0.1.

For light reflected from the top of the cloud the brightness increases as τ increases since there are more and more scattering particles. The brightness reaches a finite limit (the Hapke-Irvine function) as τ approaches infinity.

Light scattered through the cloud appears as a blob on the opposite side of the rectangle than the incident light ray. Note that for low values of τ an appreciable amount of light comes out the bottom of the cloud. As τ increases the scattered brightness reaches a maximum and then begins to decreased as the shadowing and blocking effect predominates. In the limit of τ=infinity the cloud layer is opaque and no light gets through.

4.3 Application to the rings of Saturn

The rings of Saturn consist of a cloud of reflective ice particles in orbit about the planet. The optical depth, albedo and size distribution of scattering particles in the rings varies with radius from the planet. A phase function which can account for this is

$$w(r)\ \varphi(a,r) = w1(r)\ L(a) + w2(r)\ HG(-.5,a)$$

Note that we have merged the albedo and phase function proportions into just the two coefficients w1 and w2. Tabulated values for w1(r), w2(r), and τ(r) were derived from Voyager 1 photographs taken at a few known viewing geometries by substituting observed brightnesses into the above equations and solving for w1,w2, and τ. The database was then used to synthesize

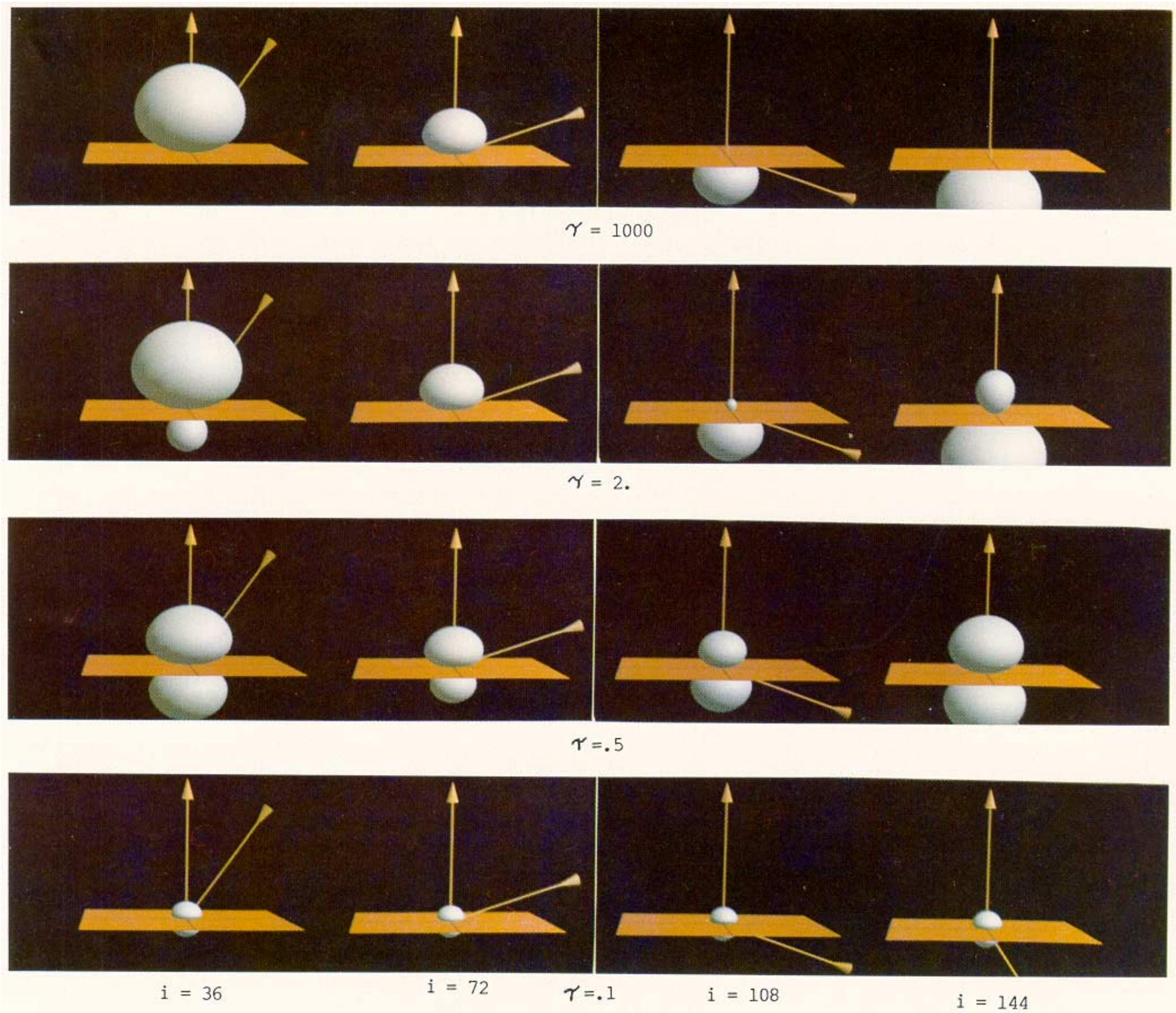


Figure 8 - Brightness Function

views of the rings at other viewing geometries. Figures 9a and 9b show two synthesized views of Saturn's rings looking down on the sunlit side and up at the unlit side respectively.

4.4 Atmosphere Simulation

Figure 10 shows a simple usage of the cloud function to simulate a cloud layer over a hypothetical planet. For ease of interpretation, the surface feature map on the planet is a collection of randomly colored squares seen unclouded in the image on the left. The overlying cloud layer is modelled with a random texture pattern specifying the value of Υ. The image in the center shows this in front of a uniformly colored background to illustrate the brightness and opacity variations. Note the increase in brightness and opacity on the left limb. The image on the right shows the combined planet and cloud layer.

5. EXTENSIONS TO THE MODEL

Extensions to the classical single scattering model generally consist of finding ways to correct for the various simplifying assumptions made above, thus making the model applicable to a wider range of situations.

5.1 Extension to Greater Density.

The probability of finding zero particles in volume V was taken to be due to a Poisson process, exp(-nV). This, however, admits to the possibility of more than one particle occupying the same volume. Esposito [6] derived a fairly simple correction term by using a somewhat altered number density

$$n' = n/(1-D)$$

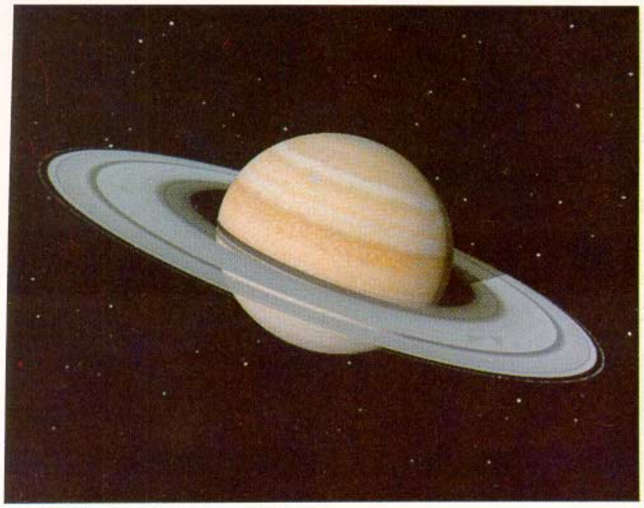


Figure 9a - Saturn Rings (Illuminated side)

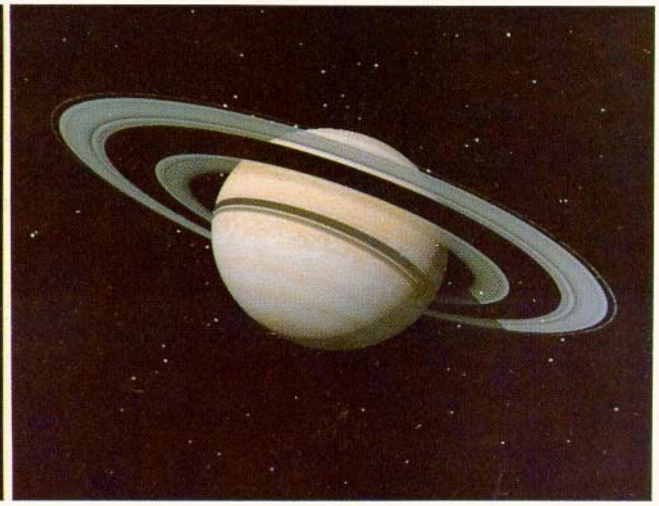


Figure 9b - Saturn Rings (Un-Illuminated side)

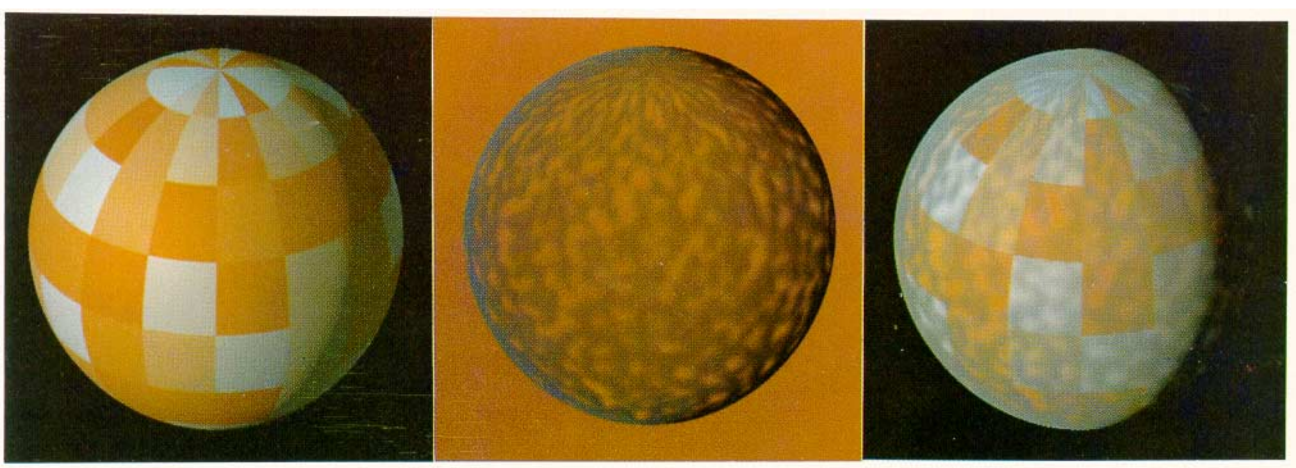
This results in an effective optical depth

$$t' = t/(1-D)$$

For the very small values of D for which the approximation was valid this reduces to the classical result. When D approaches 1 (i.e. a solid packing of scattering particles) the effective optical depth approaches infinity, as would be expected. Note that this extension is particularly nice in that it only alters the value of the input parameter to the brightness function but does not otherwise alter the properties of that function.

5.2 Shadowing Effect

The scattering function was derived from considering the volume of two cylinders for entering and exiting rays of light. At that time is was mentioned that there was a small overlap between the cylinders Vin and Vout which was neglected. This overlap actually becomes quite significant when L=E ($\mu=\mu_0$). The two cylinders, in fact, coincide and the entire volume is erroneously counted twice. This geometrical situation will yield a brighter observed intensity than that predicted by the simple model. The correct value will be produced by counting only



Planet Surface Cloud Layer Cloud Covered Planet

Figure 10 - Simulation of Cloudy Atmosphere

Vin (=Vout). Substituting $\mu=\mu_0$ into the old expression

$$Iold = w\,\varphi(a)\ .5(1 - e^{-2\tau/\mu})$$

Substituting V=Vin into the original integral equation and re-integrating

$$Inew = w\,\varphi(a)\ (1 - e^{-\tau/\mu})$$

The overlap for more general values of a is a bit more tricky. Solutions to this problem have been given by Hapke [8], Irvine [10], and Esposito [6]. They each conclude that the correction to compensate for this overlap effect depends on both the phase angle, a, and on the volume density, D. The mathematical form of these various corrections is complex. We can, however, generate a correcton factor with approximately the right shape in a more simple manner. At a=0 the factor is Inew/Iold. As a increases the effect of the overlap becomes more and more unimportant and the correction factor drops back to 1.0. The rate of decrease of the correction is a function of D; for small values of D the correction drops off quickly, for larger values it drops off slowly. Such a function can be constructed in a similar manner to the bump functions of [1].

5.3 Multiple Scattering

If the individual particles have an appreciable albedo the effects of second and higher order scattering cannot be ignored. Veverka [16] has shown experimentally that this happens at values of w above 0.3. Above this value, multiple scattering effects must be accounted for. The basic idea of simulations of this effect is to expand the net intensity in what is called a Neumann series.

$$I(\tau,N,E,L) = \sum_{n=0}^{\infty} w^n\ In(\tau,N,E,L)$$

where
- w = single scattering albedo
- $I(\tau,N,E,L)$ = intensity at optical depth in direction E
- $In(\tau,N,E,L)$ = intensity of photons scattered exactly n times

With the single scattering theory we have effectively calculated only $I1(\tau,N,E,L)$.

Various approaches have been used to find the total intensity I. Chandrasekhar [3] has reduced the solution to

$$I \propto H(i,a)\ H(e,a)$$

where the H functions are defined by some quite complex simultaneous integral equations. Another approach [6] uses Markov chains to numerically simulate the possible random scattering sequences. Yet another approach, called the doubling method, constructs a total picture of reflected intensity by building up layers of increasing optical depth. The intensity is discretized over a finite number of angular directions. The intensity for a layer of depth 2τ is found from two layers of depth τ. This is done by processing all sets of interactions for all possible combinations of the discretized directions between the two layers. This algorithm is described more fully in [7].

As might be inferred from the above discussion, the accurate simulation of multiple scattering requires a substantial amount of computation time, certainly more than would be practical on a pixel by pixel basis for the purposes of image synthesis. About the only practical approach would be to pre-evaluate such a function for various input parameters and generate a large look-up table for use in graphics applications. Certainly more work needs to be done here.

The importance of multiple scattering is alluded to by Veverka when he points out that in the limit the multiple scattering law should provide a physical basis for the, so far, purely empirical Lambert's law. In fact, the surface of Jupiter, which is all clouds, follows the ideal Lambert law very closely.

6. CONCLUSIONS

Computer graphics can benefit greatly from examination of the existing literature on light interacting with matter. Early models used for image synthesis, crude but effective, are being replaced by more accurate models. Earlier efforts in this regard have done extremely well for specular reflection. The models presented here are the beginnings of a more complete simulation of diffuse reflection.

The problem of clouds is still not solved. The extension of this model to the simulation of an arbitrarily varying spatial density function of scatterers, with multiple reflections and different amounts of shadowing from any direction is not straightforward. The models provided here do, however, represent some initial steps in that direction.

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