

# Efficient Secure Three-Party Computation

Seung Geol Choi<sup>1</sup> and Jonathan Katz<sup>2</sup> and **Alex J. Malozemoff**<sup>2</sup>  
and Vassilis Zikas<sup>3</sup>

<sup>1</sup>United States Naval Academy

<sup>2</sup>University of Maryland, College Park

<sup>3</sup>University of California, Los Angeles



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## Prior Work

**Setting:** Malicious adversary, arbitrary # corruptions

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**2PC:** Many efficient constructions

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- Most based on Yao's garbled circuit approach [Yao82, Yao86]
  - *Boolean* circuits,  $\mathcal{O}(1)$  rounds
- Use inherently two-party techniques
  - E.g., cut-and-choose, oblivious transfer, authenticated bit shares, ...
- Fast in general (and only getting faster)

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**MPC:** SPDZ protocol [BDOZ11, DKL<sup>+</sup>12, DKL<sup>+</sup>13, DPSZ12, KSS13]

- *Arithmetic* circuits,  $\mathcal{O}(d)$  rounds
- Total running time slow, on-line running time fast

# MPC in Practice

Existing MPC deployments mostly utilize *three* parties

- The Danish sugar beet auction [BCD<sup>+</sup>09]
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Why is this?

- Increase in communication/computation cost as # parties increases
- Settings where three parties sufficient (and two is not)

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Since 2PC is fast and MPC is slow(er), but 3PC seems useful in practice. . .

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Can we achieve efficient *three*-party computation using two-party tools?

In particular, can we *lift* cut-and-choose-based 2PC protocols to the three-party setting?



# Contribution

## Main Contribution

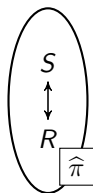
Constant-round maliciously-secure 3PC for boolean circuits at roughly *twice* the cost of underlying cut-and-choose-based 2PC used

- Tolerates arbitrary number of malicious parties
- Can lift [LP07, LP11] and [Lin13] to three-party setting
- Works in Random Oracle model
- Requires almost entirely two-party communication
  - Only three (three-party) broadcast calls needed
- Faster *start-to-finish* running time versus SPDZ
  - No implementation (yet...)
  - SPDZ has faster *on-line* running time

# High-level Idea

$\widehat{\pi}(S, R)$ : cut-and-choose 2PC protocol between sender  $S$  and receiver  $R$

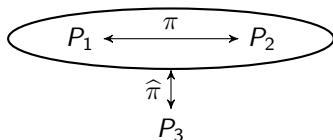
- $S$  generates many garbling circuits using a *circuit garbling scheme*
- $R$  does cut-and-choose on circuits



# High-level Idea

We emulate  $\widehat{\pi}$  using three parties as follows:

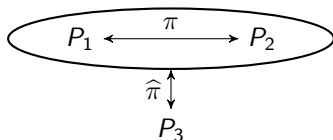
- $P_1$  and  $P_2$  run two-party protocol  $\pi$  emulating  $S$ 
  - In particular, the *circuit garbling scheme* of  $S$
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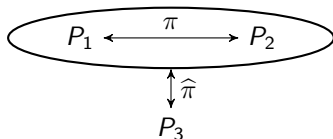
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- $P_3$  plays role of  $R$



**Note:** using “arbitrary” 2PC schemes for  $\widehat{\pi}$  and  $\pi$  won't be efficient!

# Outline of Rest of Talk

1. Distributing  $S$ 's circuit garbling scheme
  - 1.1 (Single party) circuit garbling scheme (i.e., garbling scheme for  $\hat{\pi}$ )
  - 1.2 Distributing the garbling scheme (i.e.,  $\pi$ )
2. Adapting 2PC protocols (i.e.,  $\hat{\pi}$ ) to three parties



# (Single-party) Circuit Garbling Scheme

## 1. Generate mask bits:

- For all wires  $w$ : Generate  $\lambda_w \xleftarrow{\$} \{0, 1\}$

## 2. Generate keys:

- For all wires  $w$ : Generate  $K_{w,0} \xleftarrow{\$} \{0, 1\}^k$  and  $K_{w,1} \xleftarrow{\$} \{0, 1\}^k$

## 3. Garble gates:

- For all gates  $G$  with input wires  $\alpha$  and  $\beta$  and output wire  $\gamma$ :

$$\text{Enc}_{K_{\alpha,0}, K_{\beta,0}} \left( K_{\gamma, G(\lambda_{\alpha}, \lambda_{\beta}) \oplus \lambda_{\gamma}} \parallel G(\lambda_{\alpha}, \lambda_{\beta}) \oplus \lambda_{\gamma} \right)$$

$$\text{Enc}_{K_{\alpha,0}, K_{\beta,1}} \left( K_{\gamma, G(\lambda_{\alpha}, \lambda_{\beta} \oplus 1) \oplus \lambda_{\gamma}} \parallel G(\lambda_{\alpha}, \lambda_{\beta} \oplus 1) \oplus \lambda_{\gamma} \right)$$

$$\text{Enc}_{K_{\alpha,1}, K_{\beta,0}} \left( K_{\gamma, G(\lambda_{\alpha} \oplus 1, \lambda_{\beta}) \oplus \lambda_{\gamma}} \parallel G(\lambda_{\alpha} \oplus 1, \lambda_{\beta}) \oplus \lambda_{\gamma} \right)$$

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(Note: This is standard Yao using point-and-permute)

# Distributing the Garbling Scheme

Desired properties:

1. Obliviousness
  - Parties cannot know output key/tag being encrypted
2. Correctness
  - If one party malicious, garbled circuit evaluation must either:
    - Compute correct answer
    - Abort, *independent* of honest party's input

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## Solution

Combine distributed garbling techniques [DI05] with authenticated bit shares [NNOB12]



# Distributing the Garbling Scheme: Outline

- Building blocks:
  - Authenticated bit shares
  - Sub-protocols on authenticated bit shares
  - Distributed encryption scheme
- Two-party distributed circuit garbling protocol

# Building Blocks: Authenticated Bit Shares [NNOB12]

- $\langle b \rangle = (\langle b \rangle^{(1)}, \langle b \rangle^{(2)})$ 
  - $\langle b \rangle^{(1)} = (b_1, T_1, K_2)$  and  $\langle b \rangle^{(2)} = (b_2, T_2, K_1)$
  - $b = b_1 \oplus b_2$

<b>P<sub>1</sub></b>	<b>P<sub>2</sub></b>
$b_1, T_1, K_1$	$b_2, T_2, K_2$
$T_1 = \text{MAC}_{K_2}(b_1)$	$T_2 = \text{MAC}_{K_1}(b_2)$

- Sharing is linear:
  - $\langle b \rangle \oplus \langle b' \rangle = (\langle b \oplus b' \rangle^{(1)}, \langle b \oplus b' \rangle^{(2)})$
  - $\langle b \oplus b' \rangle^{(i)} = (b_i \oplus b'_i, T_i \oplus T'_i, K_j \oplus K'_j)$

# Building Blocks: Sub-protocols on authenticated bit shares

Two-party sub-protocols:

- $\mathcal{F}_{\text{gate}}^G(\langle a \rangle, \langle b \rangle) \rightarrow \langle G(a, b) \rangle$
- $\mathcal{F}_{\text{oshare}}^i(\langle b \rangle, m_0, m_1) \rightarrow [m_b]$ 
  - Inputs  $m_0$  and  $m_1$  are *private* to party  $P_i$
- $\mathcal{F}_{\text{rand}}() \rightarrow \langle b \rangle$
- $\mathcal{F}_{\text{ss}}^i(b) \rightarrow \langle b \rangle$ 
  - Input  $b$  is *private* to party  $P_i$

**Note:** efficient maliciously secure constructions exist

- Use ideas from [NNOB12]; OT tricks

# Building Blocks: Distributed Encryption Scheme [DI05]

$$\begin{array}{ccc} & [m] = m_1 \oplus m_2 & \\ & K_1 = (s_1^1, s_1^2), K_2 = (s_2^1, s_2^2) & \\ \hline \mathbf{P}_1 & & \mathbf{P}_2 \\ \hline & m_1, s_1^1, s_2^1 & m_2, s_1^2, s_2^2 \\ \text{Enc}_{K_1, K_2}([m]) = & & \\ (m_1 \oplus F_{s_1^1}^1(0) \oplus F_{s_2^1}^2(0)), & & m_2 \oplus F_{s_1^2}^1(0) \oplus F_{s_2^2}^2(0)) \end{array}$$

- $F^1$  and  $F^2$  are PRFs
- Encryption is *local*

# Two-party Distributed Circuit Garbling Protocol

## 1. Generate mask bits:

- For all wires  $w$ : Generate  $\lambda_w \xleftarrow{\$} \{0, 1\}$

## 2. Generate keys:

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 $P_2$ 's input wires  $w$ :  $P_2$  sets  $\lambda_w \xleftarrow{\$} \{0, 1\}$ ; computes  $\langle \lambda_w \rangle \leftarrow \mathcal{F}_{ss}^2(\lambda_w)$   
All other wires  $w$ :  $P_1$  and  $P_2$  compute  $\langle \lambda_w \rangle \leftarrow \mathcal{F}_{rand}$

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## 2. Generate keys:

- For all wires  $w$ :  
 $P_i$ , for  $i \in \{1, 2\}$ , sets  $s_{w,0}^i \xleftarrow{\$} \{0, 1\}^k$  and  $s_{w,1}^i \xleftarrow{\$} \{0, 1\}^k$   
Let  $K_{w,0} = (s_{w,0}^1, s_{w,0}^2)$  and  $K_{w,1} = (s_{w,1}^1, s_{w,1}^2)$

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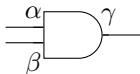
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## Example: Garbling an AND Gate



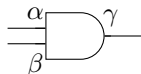
$$\lambda_\alpha = 1, \lambda_\beta = 0, \lambda_\gamma = 1$$

### Standard (single-party) garbling:

**Step 1:** Compute tags:

$i$	$j$	$AND(\lambda_\alpha \oplus i, \lambda_\beta \oplus j) \oplus \lambda_\gamma$
0	0	$AND(1 \oplus 0, 0 \oplus 0) \oplus 1 = 1$
0	1	$AND(1 \oplus 0, 0 \oplus 1) \oplus 1 = 0$
1	0	$AND(1 \oplus 1, 0 \oplus 0) \oplus 1 = 1$
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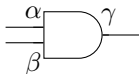
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**Standard (single-party) garbling:**

**Step 2: Encrypt:**

$i$	$j$	
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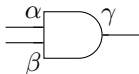
**Distributed garbling:**

**Step 1:** Compute *oblivious sharings* of tags:

$i$	$j$	$\langle \text{AND}(\lambda_\alpha \oplus i, \lambda_\beta \oplus j) \oplus \lambda_\gamma \rangle$
0	0	$\mathcal{F}_{\text{gate}}^{\text{AND}}(\langle 1 \rangle \oplus \langle 0 \rangle, \langle 0 \rangle \oplus \langle 0 \rangle) \oplus \langle 1 \rangle = \langle 1 \rangle$
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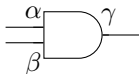
$$\langle \lambda_\alpha \rangle = 1, \langle \lambda_\beta \rangle = 0, \langle \lambda_\gamma \rangle = 1$$

**Distributed garbling:**

**Step 2:** Compute *oblivious sharings* of each party's output sub-keys:

$i$	$j$				
0	0	$\mathcal{F}_{\text{oshare}}^1(\langle 1 \rangle, s_{\gamma,0}^1, s_{\gamma,1}^1) =$	$\begin{bmatrix} s_{\gamma,1}^1 \end{bmatrix}$	$\mathcal{F}_{\text{oshare}}^2(\langle 1 \rangle, s_{\gamma,0}^2, s_{\gamma,1}^2) =$	$\begin{bmatrix} s_{\gamma,1}^2 \end{bmatrix}$
0	1	$\mathcal{F}_{\text{oshare}}^1(\langle 0 \rangle, s_{\gamma,0}^1, s_{\gamma,1}^1) =$	$\begin{bmatrix} s_{\gamma,0}^1 \end{bmatrix}$	$\mathcal{F}_{\text{oshare}}^2(\langle 0 \rangle, s_{\gamma,0}^2, s_{\gamma,1}^2) =$	$\begin{bmatrix} s_{\gamma,0}^2 \end{bmatrix}$
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$$\langle \lambda_\alpha \rangle = 1, \langle \lambda_\beta \rangle = 0, \langle \lambda_\gamma \rangle = 1$$

**Distributed garbling:**

**Step 3:** Use *distributed* encryption to encrypt:

$i$	$j$	
0	0	$\text{Enc}_{K_{\alpha,0}, K_{\beta,0}} \left( \begin{bmatrix} s_{\gamma,1}^1 \end{bmatrix} \parallel \begin{bmatrix} s_{\gamma,1}^2 \end{bmatrix} \parallel \langle 1 \rangle \right)$
0	1	$\text{Enc}_{K_{\alpha,0}, K_{\beta,1}} \left( \begin{bmatrix} s_{\gamma,0}^1 \end{bmatrix} \parallel \begin{bmatrix} s_{\gamma,0}^2 \end{bmatrix} \parallel \langle 0 \rangle \right)$
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## 3PC Using Distributed Garbled Circuits

### High-level Idea

- Take existing cut-and-choose protocol (e.g., [LP07, LP11, Lin13])
- Replace sender's circuit generation by distributed circuit generation

(Many details ignored here. . .)

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(Many details ignored here. . .)

## Security Intuition

- Exactly one of  $P_1$  or  $P_2$  malicious: garbled circuits either correct or abort independent of input, even with malicious  $P_3$
- Both  $P_1$  and  $P_2$  malicious: cut-and-choose by  $P_3$  detects cheating

## 3PC Using Distributed Garbled Circuits

Efficiency versus underlying 2PC protocol:

- Roughly *two times* more expensive in computation
- Roughly *three times* more expensive in communication

Approach works for several cut-and-choose-based 2PC protocols:

- ✓: Combination of [LP07, LP11] (probably [SS11, KsS12] as well)
- ✓: [Lin13]
- ✗: [HKE13] and [MR13], due to symmetry between  $P_1$  and  $P_2$

# Summary

Can “lift” cut-and-choose-based 2PC to 3PC setting

- Only *twice* as slow as underlying 2PC protocol
- Only three broadcast calls needed
  - Important since broadcast expensive in WAN setting

Work still needs to be done to determine *empirical* efficiency

- Free-XOR? (**very important in practice!**)
- Implementation? Many engineering issues to consider

Paper to be published on ePrint shortly!

**Thank you**

**Extra slides...**



## 3PC Using Distributed Garbled Circuits

- Two main challenges of cut-and-choose:
  - Input Inconsistency*
    - Malicious generator (either  $P_1$  or  $P_2$ ) inputs inconsistent sub-keys in two different circuits;  $P_3$  evaluates on different inputs
    - Solution*: apply Diffie-Hellman pseudorandom synthesizer trick [LP11, MF06]
  - Selective Failure*
    - Sender in OT can input invalid keys, potentially learning bit of  $P_3$ 's input
    - Solution*: "XOR-tree" approach [LP07, Woo07]

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9. For evaluation circuits:  $P_1/P_2$  send sub-keys and selector bits to  $P_3$ ;  $P_3$  checks input consistency using ZKPoK as in [LP11]; evaluates circuits, outputting majority output