Mark Howard
Sheffield

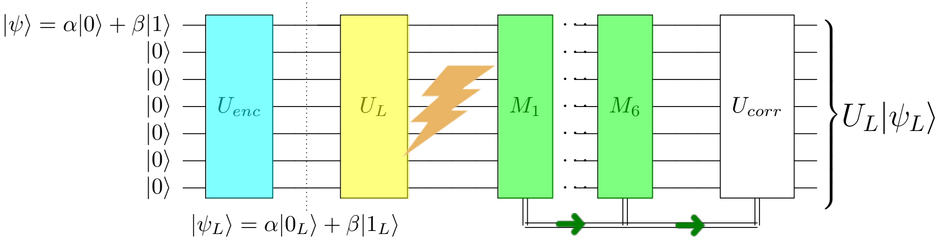
APPLICATION OF A RESOURCE THEORY FOR MAGIC STATES TO FAULT-TOLERANT QUANTUM COMPUTING

Howard & Campbell

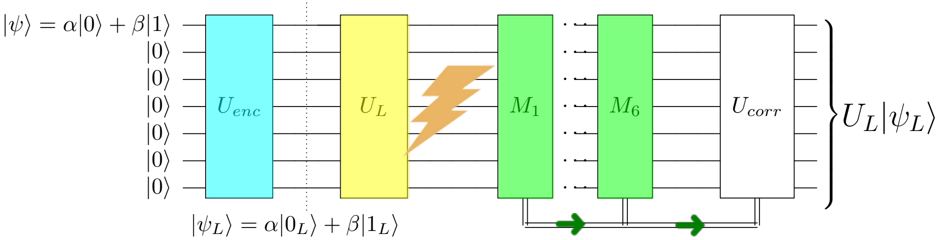
arXiv:1609.07488

Earl Campbell
Sheffield

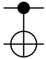
- Need error-correcting code to protect our quantum computation



- Need error-correcting code to protect our quantum computation



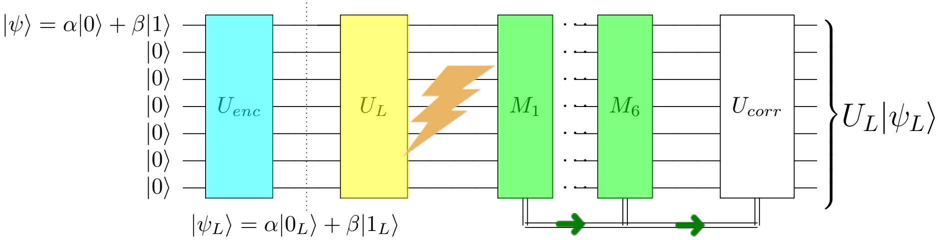
- Stabilizer codes $\rightsquigarrow$ gates from finite Clifford group $\mathcal{C}$

$\mathcal{C} = \Big\langle$  , Symmetry group of  $\Big\rangle = \Big\langle CNOT, H, S = \sqrt{Z} \Big\rangle$

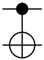
Encoding/correction are Clifford. Typically U_L gates too.

Problem: Can't get transversal & universal (dense) set of gates U_L (e.g. Toric)

- Need error-correcting code to protect our quantum computation



- Stabilizer codes $\rightsquigarrow$ gates from finite Clifford group $\mathcal{C}$

$\mathcal{C} = \Big\langle$  , Symmetry group of 

$\Big\rangle = \Big\langle CNOT, H, S = \sqrt{Z} \Big\rangle$

Encoding/correction are Clifford. Typically U_L gates too.

Problem: Can't get transversal & universal (dense) set of gates U_L (e.g. Toric)

Solution: Supplement transversal gates with supply of "Magic States"

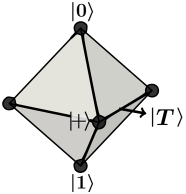
Q: What's a magic state?

A: A state that enables a non-Clifford gate e.g., $|T\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi/4}|1\rangle)$

Q: What's a magic state?

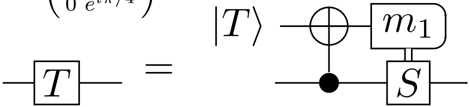
A: A state that enables a non-Clifford gate e.g., $|T\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi/4}|1\rangle)$

- ☐ It is **not** a Pauli eigenstate (stabilizer state)



$|T\rangle$-type magic state

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

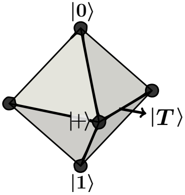


$|T\rangle$ states (+Cliffords) enable T gates

Q: What's a magic state?

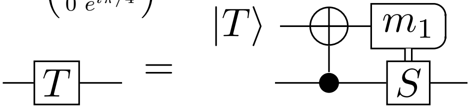
A: A state that enables a non-Clifford gate e.g., $|T\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi/4}|1\rangle)$

- ☐ It is **not** a Pauli eigenstate (stabilizer state)



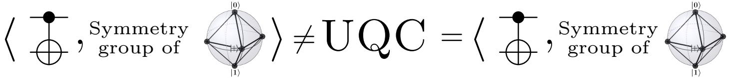
$|T\rangle$-type magic state

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

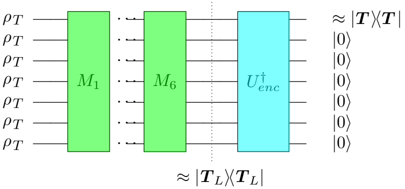


$|T\rangle$ states (+Cliffords) enable T gates

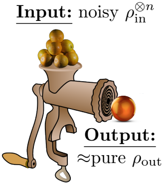
Adding the ability to do $U \notin \mathcal{C}$ promotes Cliffords to Universal QC



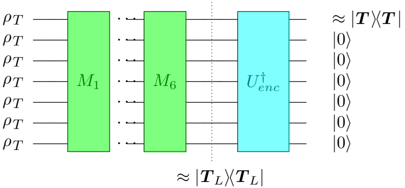
$$, T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \Big\rangle$$



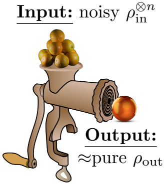
Magic state distillation circuit



MSD schematic

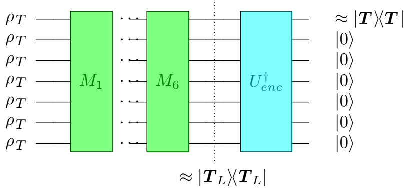


Magic state distillation circuit

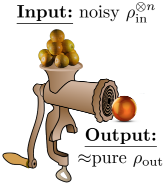


MSD schematic

- Overhead associated with MSD is polynomial in the number of T gates used
  - Logical Cliffords (2% – 10% overhead)
  - Logical Non-Clifford e.g. T gate (90% – 98% overhead **including MSD**)

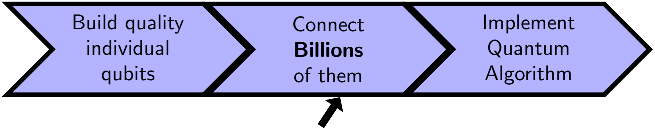


Magic state distillation circuit

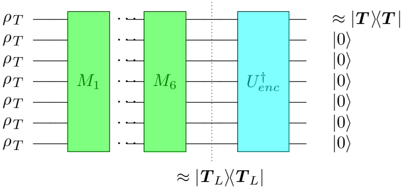


MSD schematic

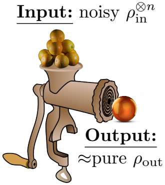
- Overhead associated with MSD is polynomial in the number of T gates used
  - Logical Cliffords (2% – 10% overhead)
  - Logical Non-Clifford e.g. T gate (90% – 98% overhead **including MSD**)



Let's try to convert **Billions**→**Millions** (i.e, reduce overhead)

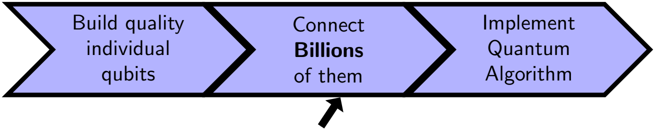


Magic state distillation circuit



MSD schematic

- Overhead associated with MSD is polynomial in the number of T gates used
  - Logical Cliffords (2% – 10% overhead)
  - Logical Non-Clifford e.g. T gate (90% – 98% overhead **including MSD**)

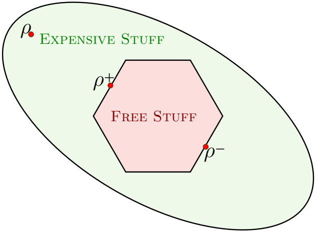


Let's try to convert **Billions**→**Millions** (i.e, reduce overhead)

- Clifford Z and $S = Z^{\frac{1}{2}}$ rotations "easy" but $T = Z^{\frac{1}{4}}$ gates hard since **Encoded**

Natural partition into easy/hard operations ⇒ Resource Theory.

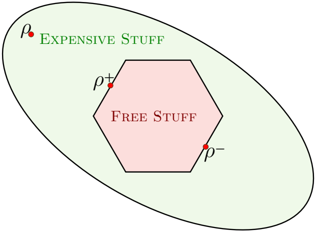
Expense quantifier must obey certain reasonable properties to be useful!



Generic Resource

Natural partition into easy/hard operations ⇒ Resource Theory.

Expense quantifier must obey certain reasonable properties to be useful!



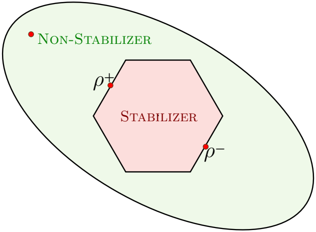
Generic Resource

$$\mathcal{R}(\rho) = \min_{\rho^+,\rho^- \in \mathcal{P}_{\text{FREE}}} \{2p+1 | \rho = (p+1)\rho^+ - p\rho^-\}$$

Robustness quantifies expensive stuff.

Defn: How much free stuff must be mixed in to make your expensive stuff become free.

Natural partition into easy/hard operations ⇒ Resource Theory.
Expense quantifier must obey certain reasonable properties to be useful!



Multi-qubit QC

$$\mathcal{R}(\rho) = \min_{\rho^+,\rho^- \in \mathcal{P}_{\text{FREE}}} \{2p+1 | \rho = (p+1)\rho^+ - p\rho^-\}$$

Robustness quantifies expensive stuff.
Defn: How much free stuff must be mixed in to make your expensive stuff become free.

$$\mathcal{R}(\rho) = \min_{\rho^+,\rho^- \in \mathcal{P}_{\text{STAB}}} \{2p+1 | \rho = (p+1)\rho^+ - p\rho^-\}$$

Resource Desiderata

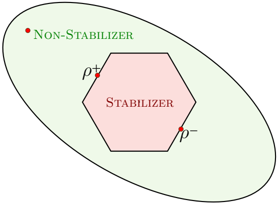
- $\mathcal{R}(\rho) \geq 1, \ (\mathcal{R}(\rho \in \mathcal{P}_{\text{STAB}}) = 1)$
- $\mathcal{R}(\rho_1 \otimes \rho_2) \leq \mathcal{R}(\rho_1)\mathcal{R}(\rho_2)$
- $\mathcal{R}\left(\mathcal{E}_{\text{STAB}}(\rho)\right) \leq \mathcal{R}\left(\rho\right)$

... ⇒ Well-behaved quantifier

... **or take** $\log \mathcal{R}$

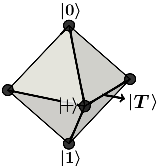
- $\log \mathcal{R}(\rho) \geq 0,$
- $\log \mathcal{R}(\rho_1 \otimes \rho_2) \leq \log \mathcal{R}(\rho_1) + \log \mathcal{R}(\rho_2)$
- $\log \mathcal{R}\left(\mathcal{E}_{\text{STAB}}(\rho)\right) \leq \log \mathcal{R}\left(\rho\right)$

To solve the geometrical problem for $\mathcal{R}(\rho)$ rewrite as Linear Program:



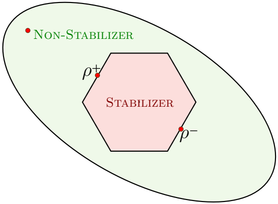
$$\mathcal{R}(\rho) = \min ||x||_1 \text{ subject to } Ax = b$$

where columns of A are vertices of $\mathcal{P}_{\text{STAB}}$



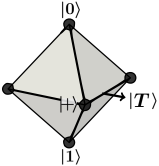
Prob: $$A_{\text{STAB}} = \begin{matrix} \langle I\rangle \\ \langle X\rangle \\ \langle Y\rangle \\ \langle Z\rangle \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

To solve the geometrical problem for $\mathcal{R}(\rho)$ rewrite as Linear Program:



$$\mathcal{R}(\rho) = \min ||x||_1 \text{ subject to } Ax = b$$

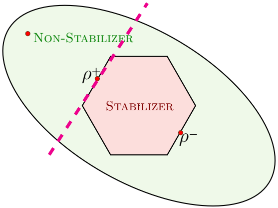
where columns of A are vertices of $\mathcal{P}_{\text{STAB}}$



Prob: $$A_{\text{STAB}} = \begin{matrix} \langle I \rangle \\ \langle X \rangle \\ \langle Y \rangle \\ \langle Z \rangle \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

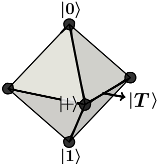
Soln: $$\mathcal{R}(|T\rangle) = ||x||_1 = \sqrt{2}: \quad x = \left(0, 0, \frac{1}{\sqrt{2}}, 0, \frac{1}{2}, \frac{1-\sqrt{2}}{2}\right)$$

To solve the geometrical problem for $\mathcal{R}(\rho)$ rewrite as Linear Program:



$$\mathcal{R}(\rho) = \min ||x||_1 \text{ subject to } Ax = b$$

where columns of A are vertices of $\mathcal{P}_{\text{STAB}}$

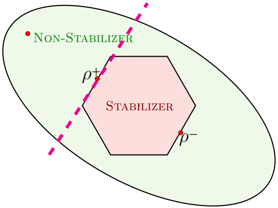


Prob: $A_{\text{STAB}} = \begin{matrix} \langle I \rangle \\ \langle X \rangle \\ \langle Y \rangle \\ \langle Z \rangle \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$

Soln: $\mathcal{R}(|T\rangle) = ||x||_1 = \sqrt{2}: \quad x = \left(0, 0, \frac{1}{\sqrt{2}}, 0, \frac{1}{2}, \frac{1-\sqrt{2}}{2}\right)$

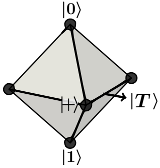
Dual: $\min\limits_{Ax=b} ||x||_1 = \max\limits_{||A^T y||_\infty \leq 1} -b^T y$ gives Witness

To solve the geometrical problem for $\mathcal{R}(\rho)$ rewrite as Linear Program:



$$\mathcal{R}(\rho) = \min ||x||_1 \text{ subject to } Ax = b$$

where columns of A are vertices of $\mathcal{P}_{\text{STAB}}$



Prob: $A_{\text{STAB}} = \begin{matrix} \langle I \rangle \\ \langle X \rangle \\ \langle Y \rangle \\ \langle Z \rangle \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$

Soln: $\mathcal{R}(|T\rangle) = ||x||_1 = \sqrt{2}: \quad x = \left(0, 0, \frac{1}{\sqrt{2}}, 0, \frac{1}{2}, \frac{1-\sqrt{2}}{2}\right)$

Dual: $\min_{Ax=b} ||x||_1 = \max_{||A^T y||_\infty \leq 1} -b^T y$ gives Witness

Use CVX with MATLAB... Guaranteed to converge

Syntax: `variable x(n);minimize(norm(x,1));subject to A*x == b`

Only downside is n (vertices) grows rapidly $\{6, 60, 1080, 36720, 2423520, \ldots\}$

Three immediate applications of Robustness of Magic

1. Simulation of quantum circuits

Clifford gates **and** T gates $\Rightarrow$ Universal QC (shouldn't be efficiently simulable)

Result: Robustness gives an exponential simulation protocol with **small exponent.**
(Not just T gates . . . any third level hierarchy gate U works)

Three immediate applications of Robustness of Magic

1. Simulation of quantum circuits

Clifford gates **and** T gates $\Rightarrow$ Universal QC (shouldn't be efficiently simulable)
Result: Robustness gives an exponential simulation protocol with **small exponent.**
(Not just T gates ... any third level hierarchy gate U works)

2. Lower bounds on number of T gates and proving optimality

Result: At least τ T gates are required to implement interesting non-Clifford U.

Three immediate applications of Robustness of Magic

1. Simulation of quantum circuits

Clifford gates **and** T gates $\Rightarrow$ Universal QC (shouldn't be efficiently simulable)
Result: Robustness gives an exponential simulation protocol with **small exponent.**
(Not just T gates ... any third level hierarchy gate U works)

2. Lower bounds on number of T gates and proving optimality

Result: At least τ T gates are required to implement interesting non-Clifford U.

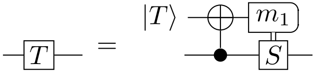
3. Identify new circuit identities

Result: Surprising Clifford-equivalence of Magic states

Two most important steps toward simulation scheme:

1. Realize that

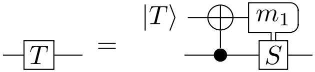
   a quantum circuit with τ T gates is equivalent to
   a purely Clifford circuit acting on τ magic states $|T\rangle$



Two most important steps toward simulation scheme:

1. Realize that

   a quantum circuit with τ T gates is equivalent to
   a purely Clifford circuit acting on τ magic states $|T\rangle$

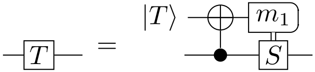


2. Adapt the efficient classical simulation schemes for Clifford circuits to allow magic state inputs e.g., input$= |0\rangle^{\otimes n-\tau}|T\rangle^{\otimes\tau}$

Two most important steps toward simulation scheme:

1. Realize that

   a quantum circuit with τ T gates is equivalent to
   a purely Clifford circuit acting on τ magic states $|T\rangle$



2. Adapt the efficient classical simulation schemes for Clifford circuits to allow magic state inputs e.g., input$= |0\rangle^{\otimes n-\tau}|T\rangle^{\otimes \tau}$

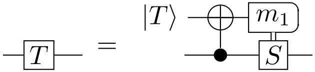
Robustness gives a quasiprobability distribution over stabilizer states:

$$\mathcal{R}(\rho) = \min_x \left\{ \sum_i |x_i|; \rho = \sum_i x_i \,(\text{Stabilizer State})_i \right\} \quad \sum_i x_i = 1$$

Two most important steps toward simulation scheme:

1. Realize that

   a quantum circuit with τ T gates is equivalent to
   a purely Clifford circuit acting on τ magic states $|T\rangle$



2. Adapt the efficient classical simulation schemes for Clifford circuits to allow magic state inputs e.g., input$=|0\rangle^{\otimes n-\tau}|T\rangle^{\otimes\tau}$

probability: $q_i = |x_i|/\sum_i |x_i|$

Robustness gives a quasiprobability distribution over stabilizer states:

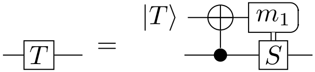
$$\mathcal{R}(\rho) = \min_x \left\{ \sum_i |x_i|; \rho = \sum_i x_i \, (\text{Stabilizer State})_i \right\} \quad \sum_i x_i = 1$$

Simulation takes longer to converge to desired accuracy (Chernoff-Hoeffding)
Require $\frac{2}{\delta^2}(\sum_i |x_i|)^2 \ln\left(\frac{2}{\epsilon}\right)$ samples to get δ-close to real dist. with prob $1-\epsilon$

Two most important steps toward simulation scheme:

1. Realize that

   a quantum circuit with τ T gates is equivalent to
   a purely Clifford circuit acting on τ magic states $|T\rangle$



2. Adapt the efficient classical simulation schemes for Clifford circuits to allow magic state inputs e.g., input$= |0\rangle^{\otimes n-\tau}|T\rangle^{\otimes\tau}$

Robustness gives a quasiprobability distribution over stabilizer states:

$$\mathcal{R}(\rho) = \min_x \left\{ \sum_i |x_i|; \rho = \sum_i x_i \,(\text{Stabilizer State})_i \right\} \quad \sum_i x_i = 1$$

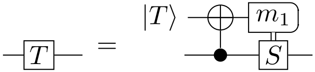
Simulation takes longer to converge to desired accuracy (Chernoff-Hoeffding)
Require $\frac{2}{\delta^2}(\sum_i |x_i|)^2 \ln\left(\frac{2}{\epsilon}\right)$ samples to get δ-close to real dist. with prob $1-\epsilon$

$\Rightarrow$ Robustness has operational meaning as the classical simulation overhead

Two most important steps toward simulation scheme:

1. Realize that

   a quantum circuit with τ T gates is equivalent to
   a purely Clifford circuit acting on τ magic states $|T\rangle$



2. Adapt the efficient classical simulation schemes for Clifford circuits to allow magic state inputs e.g., input$= |0\rangle^{\otimes n-\tau}|T\rangle^{\otimes \tau} \ldots \mathcal{R}(|T\rangle^{\otimes \tau}) = 1.685^{\tau}$

Robustness gives a quasiprobability distribution over stabilizer states:

$$\mathcal{R}(\rho) = \min_{x} \left\{ \sum_i |x_i|; \rho = \sum_i x_i \, (\text{Stabilizer State})_i \right\} \quad \sum_i x_i = 1$$

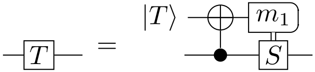
Simulation takes longer to converge to desired accuracy (Chernoff-Hoeffding)
Require $\frac{2}{\delta^2}(\sum_i |x_i|)^2 \ln\left(\frac{2}{\epsilon}\right)$ samples to get δ-close to real dist. with prob $1-\epsilon$

$\Rightarrow$ Robustness has operational meaning as the classical simulation overhead

Two most important steps toward simulation scheme:

1. Realize that

   a quantum circuit with τ T gates is equivalent to
   a purely Clifford circuit acting on τ magic states $|T\rangle$



2. Adapt the efficient classical simulation schemes for Clifford circuits to allow magic state inputs e.g., input$= |0\rangle^{\otimes n-\tau}|T\rangle^{\otimes \tau} \ldots \mathcal{R}(|T\rangle^{\otimes \tau}) = 1.685^{\tau}$

[BSS'16] $\chi(|T\rangle^{\otimes \tau}) \rightsquigarrow 1.919^{\tau}$

Robustness gives a quasiprobability distribution over stabilizer states:

$$\mathcal{R}(\rho) = \min_{x} \left\{ \sum_i |x_i|; \rho = \sum_i x_i \left(\text{Stabilizer State}\right)_i \right\} \quad \sum_i x_i = 1$$

Simulation takes longer to converge to desired accuracy (Chernoff-Hoeffding)
Require $\frac{2}{\delta^2}(\sum_i |x_i|)^2 \ln\left(\frac{2}{\epsilon}\right)$ samples to get δ-close to real dist. with prob $1-\epsilon$

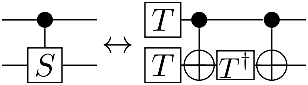
$\Rightarrow$ Robustness has operational meaning as the classical simulation overhead

Circuit using n copies of resource state ρ has simulation cost $\mathcal{R}(\rho)^{2n}$
Cost of simulating a circuit with $|T\rangle^{\otimes\tau}$ ancilla?
Submultiplicativity gives savings here:

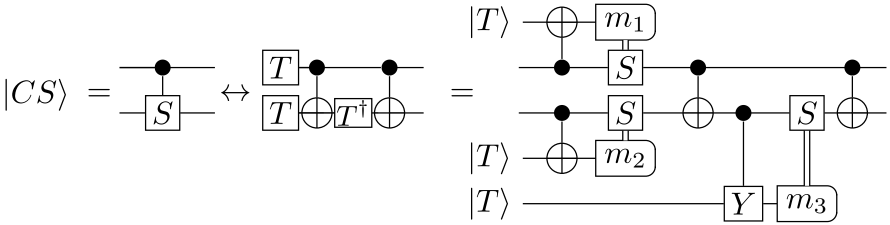
$$\begin{aligned}
\mathcal{R}\left(|T\rangle\right)^{2\tau} &\sim 2^{\tau}\\
\mathcal{R}\left(|T,T\rangle\right)^{\frac{2\tau}{2}} &\sim 1.748^{\tau}\\
\mathcal{R}\left(|T,T,T\rangle\right)^{\frac{2\tau}{3}} &\sim 1.701^{\tau}\\
\mathcal{R}\left(|T,T,T,T\rangle\right)^{\frac{2\tau}{4}} &\sim 1.692^{\tau}\\
\mathcal{R}\left(|T,T,T,T,T\rangle\right)^{\frac{2\tau}{5}} &\sim 1.685^{\tau}\\
\vdots \quad & \quad \vdots\\
\lim_{n\to\infty}\mathcal{R}\left(|T^{\otimes n}\rangle\right)^{\frac{2\tau}{n}} &\sim \quad \in [1.457^{\tau}, 1.685^{\tau}]
\end{aligned}$$

Thm: Regularized robustness of $\rho = (r_x, r_y, r_z)$ lower-bounded by $\frac{1+r_x+r_y+r_z}{2}$

A quantum algorithm will require a sequence of unitaries and measurements.
These unitaries will not be Clifford$+T$ in general so we must compile.
We must not waste hard-earned T gates

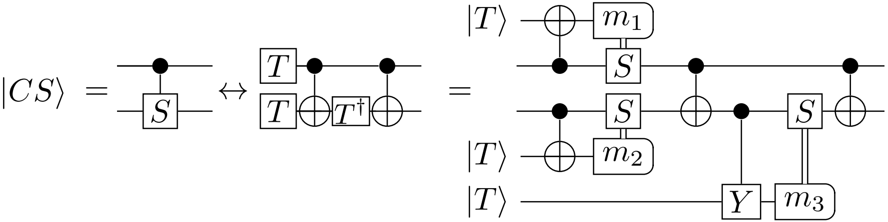


A quantum algorithm will require a sequence of unitaries and measurements.
These unitaries will not be Clifford$+T$ in general so we must compile.
We must not waste hard-earned T gates



Idea: Compare robustness of target gate $|U\rangle := U|+\rangle$ with $\mathcal{R}(|T\rangle^{\otimes\tau})$

A quantum algorithm will require a sequence of unitaries and measurements.
These unitaries will not be Clifford+T in general so we must compile.
We must not waste hard-earned T gates

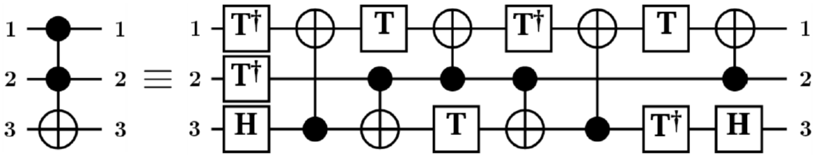


Idea: Compare robustness of target gate $|U\rangle := U|+\rangle$ with $\mathcal{R}(|T\rangle^{\otimes\tau})$

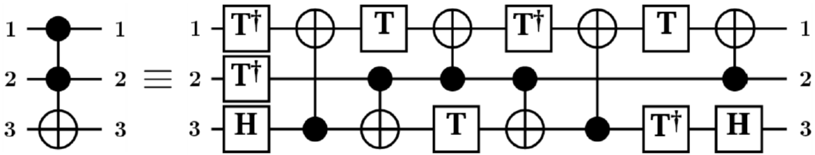
Calculate and find: $$\begin{array}{ccccc} \mathcal{R}(|T\rangle^{\otimes 2}) & < & \mathcal{R}(|CS\rangle) & < & \mathcal{R}(|T\rangle^{\otimes 3}) \\ 1.747 & < & 2.2 & \lesssim & 2.219 \end{array}$$

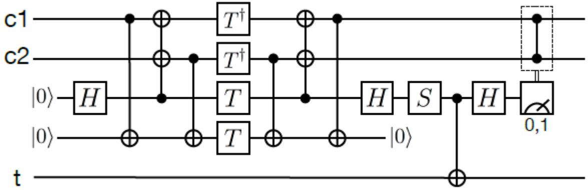
meaning no possible scheme could implement CS using fewer than 3 T gates.

Unitary synthesis is understood **OK** but ancilla-assisted is **HARD**



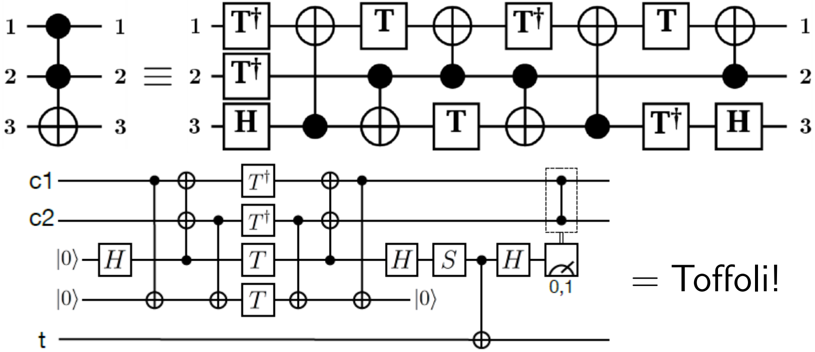
Unitary synthesis is understood **OK** but ancilla-assisted is **HARD**





$=$ Toffoli!

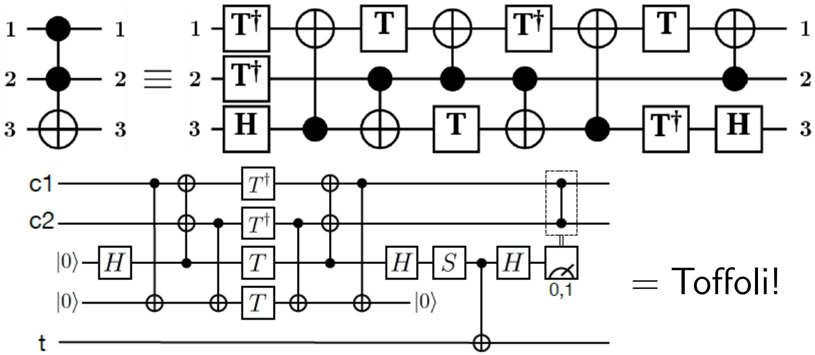
Unitary synthesis is understood **OK** but ancilla-assisted is **HARD**



= Toffoli!

How do we know there is no way of doing Toffoli with 3 T gates (or 2 or 1) ?

Unitary synthesis is understood **OK** but ancilla-assisted is **HARD**

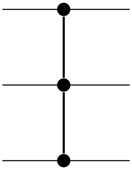


= Toffoli!

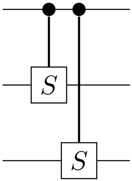
How do we know there is no way of doing Toffoli with 3 T gates (or 2 or 1) ?
Once again, our resource theory allows us to say

$$\mathcal{R}(|T\rangle^{\otimes 3}) < \mathcal{R}(|\text{Toffoli}\rangle) < \mathcal{R}(|T\rangle^{\otimes 4}).$$

and so 4 T gates is the minimum possible.
We have shown (non-)optimality of several important circuits

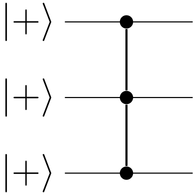


~~$\overset{\sim}{\text{Clifford}}$~~

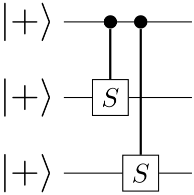


T cost $= 7$

T cost $= 4$



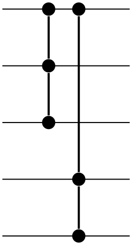
$\underset{\text{Clifford}}{\sim}$



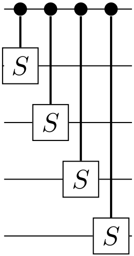
T cost $= \not{7}\ 4$

T cost $= 4$

$$\mathcal{R}(|CCZ\rangle) = 2.555 = \mathcal{R}(|CS_{12,13}\rangle)$$

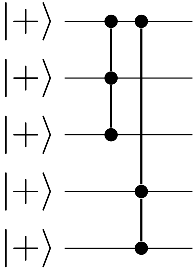


$\not\sim_{\text{Clifford}}$

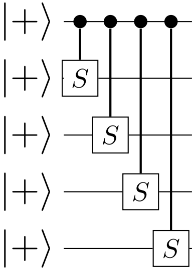


T cost $= 11$

T cost $= 8$



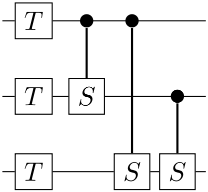
$\sim$
Clifford



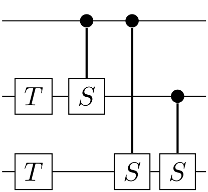
T cost $=$ ~~11~~ 8

T cost $= 8$

$$\mathcal{R}(|CCZ_{123,145}\rangle) = 4.077 = \mathcal{R}(|CS_{12,13,14,15}\rangle)$$

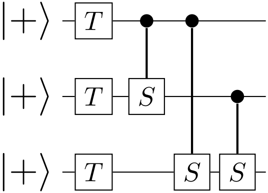


$\not\sim_{\text{Clifford}}$

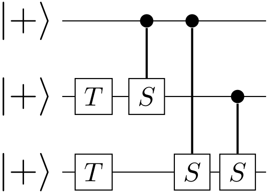


T cost $= 6$

T cost $= 5$



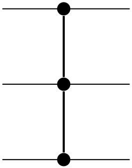
$\underset{\text{Clifford}}{\sim}$



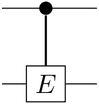
T cost $=$ ~~6~~ 5

T cost $= 5$

$$\mathcal{R}(|T_{1,2,3}CS_{12,13,23}\rangle) = 3.121 = \mathcal{R}(|T_{2,3}CS_{12,13,23}\rangle)$$

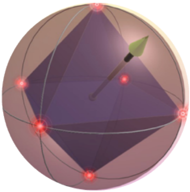


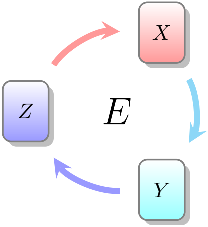
$\underset{\text{Clifford}}{\not\sim}$

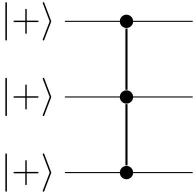


T cost $= 7$

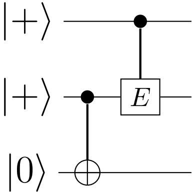
T cost $= 4$







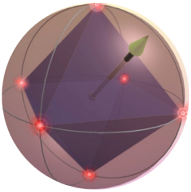
$\underset{\text{Clifford}}{\sim}$

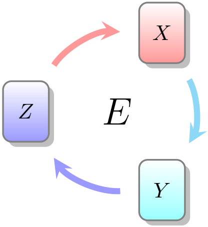


T cost $=$ ~~7~~ 4

T cost $= 4$

$$\mathcal{R}(|CCZ\rangle) = 2.555 = \mathcal{R}(|CE\rangle)$$





In Summary

Protecting qubits from errors incurs an overhead:

While Clifford gates (CNOT,H etc) are easily implementable, T gates are costly ...This suggests a resource theory picture

- We used Robustness of Magic as the resource quantifier.
  1. Simulate quantum circuits with modestly exponential (in T gates) samples
  2. Identify (non-)optimality of circuit synthesis (compilation)...prevent T wastage
  3. Find Clifford-equivalent magic states

Bonus: This norm-minimization approach ($Ax = b$) encompasses all similar known results (ℓ_0 norm, kets, Wigner polytope)...different quantifiers

In Summary

Protecting qubits from errors incurs an overhead:

While Clifford gates (CNOT,H etc) are easily implementable, T gates are costly …This suggests a resource theory picture

- We used Robustness of Magic as the resource quantifier.
    1. Simulate quantum circuits with modestly exponential (in T gates) samples
    2. Identify (non-)optimality of circuit synthesis (compilation)…prevent T wastage
    3. Find Clifford-equivalent magic states

Bonus: This norm-minimization approach ($Ax = b$) encompasses all similar known results (ℓ_0 norm, kets, Wigner polytope)...different quantifiers

Open Questions

Q: Is there a scalable way of calculating a Magic Resource?

Q: Is a measure that combines different quantifiers possible/preferable?

Q: Can we establish interconvertability results a la Entanglement?

Q: Algorithms to calculate the T cost for ancilla states $|U\rangle$?