Energy as a detector of nonlocality of many-body spin systems

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Seattle, Washington

MAX PLANCK INSTITUTE OF QUANTUM OPTICS

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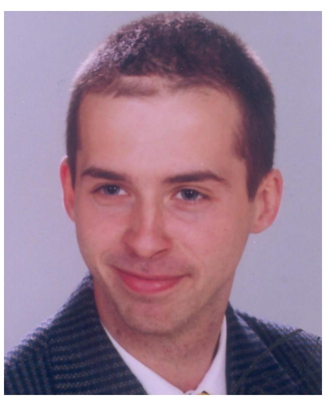
joint work with

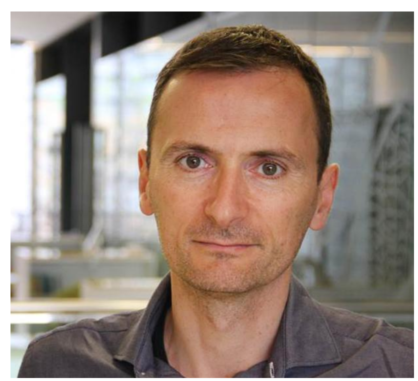
Gemma de las Cuevas

Maciej Lewenstein

J. Ignacio Cirac



Remigiusz Augusiak



Antonio Acín

The paper is available on [arXiv:1607.06090] (with referees in Phys. Rev. X)

Outline

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- Motivation

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- The idea, the setting

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- Quantum optimization

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- Conclusions and outlook

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- Bell correlations are stronger than entanglement

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    [Tura et al, Science **344** 1256 (2014), Schmied et al, Science **352** 441(2016)]

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- This talk: spin systems in one spatial dimension

A crash course on nonlocality

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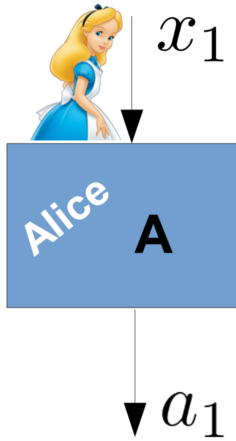

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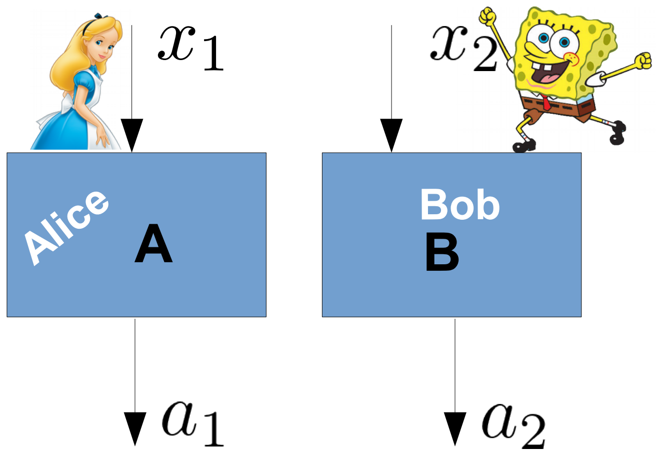

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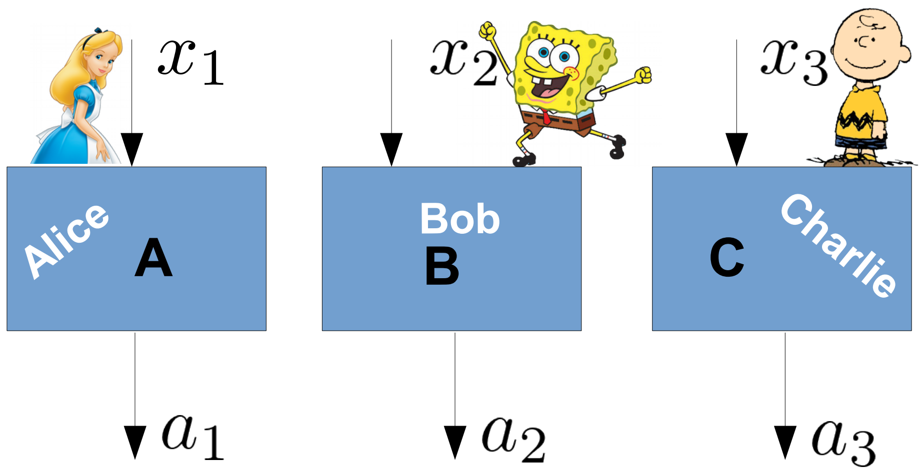

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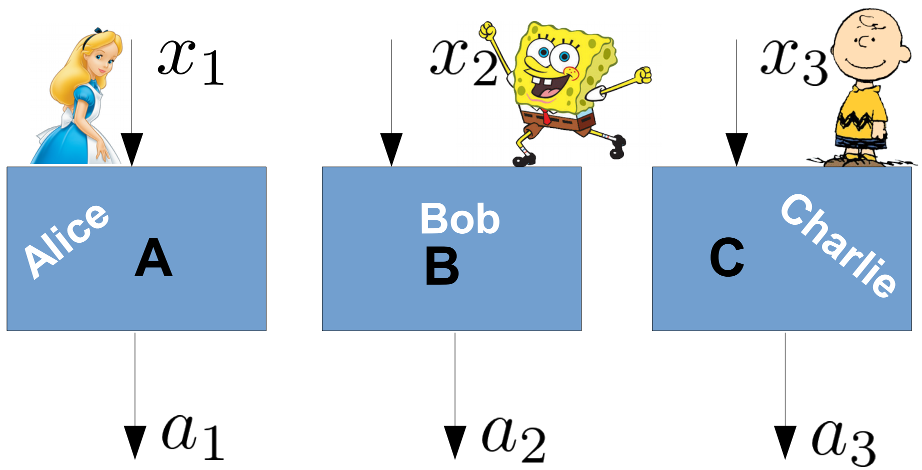

$$P(a_1 \dots a_n | x_1 \dots x_n)$$

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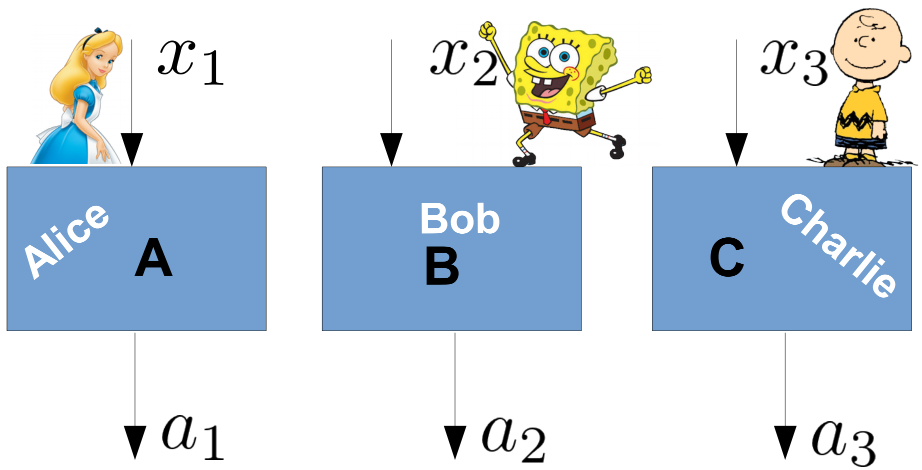

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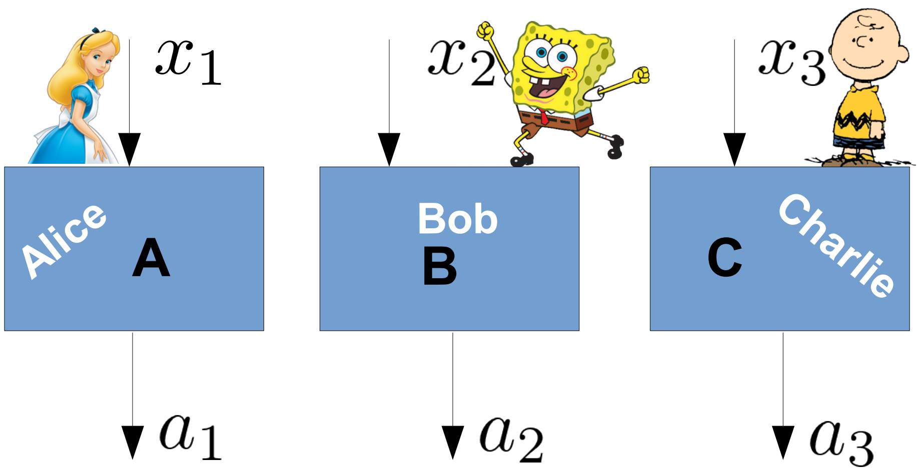

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$$\vec{v} = \{P(\vec{a}|\vec{x}) \quad \forall \vec{a}, \vec{x}\}$$

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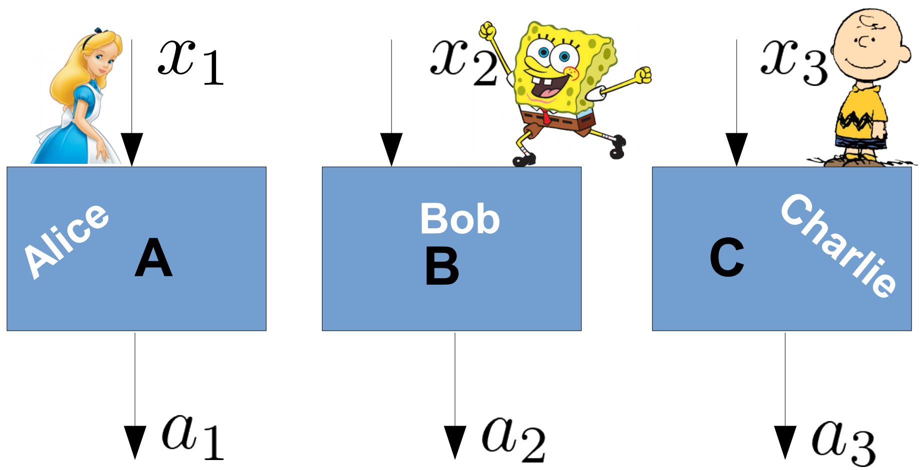

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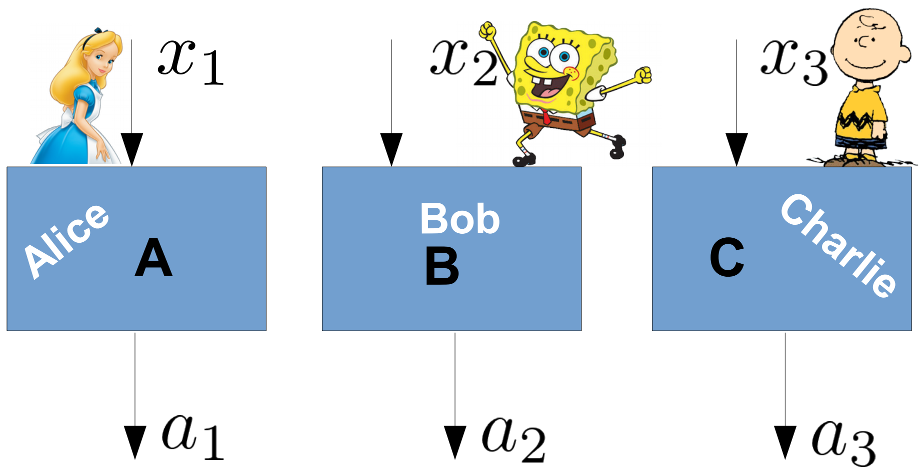

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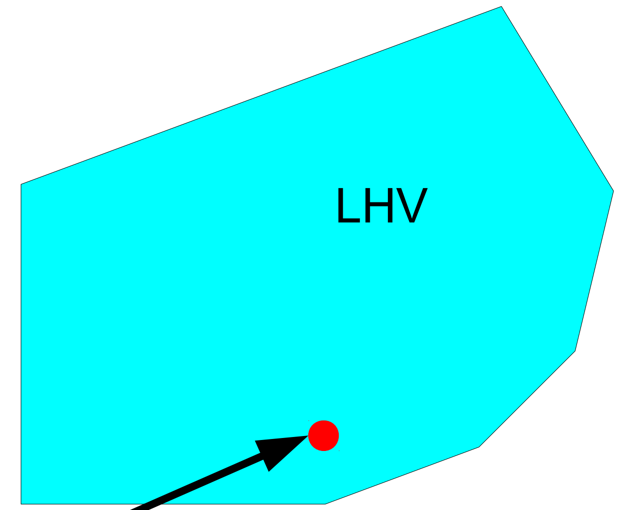

Local Polytope

$$\mathbb{P}_L$$

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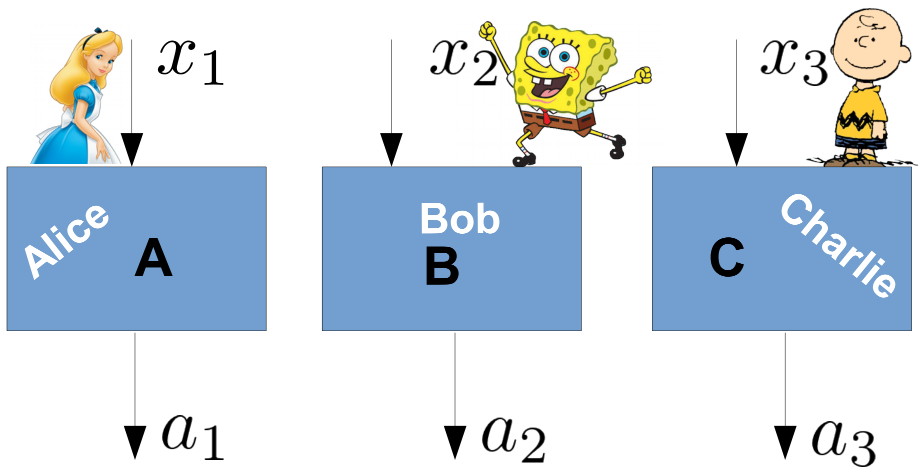

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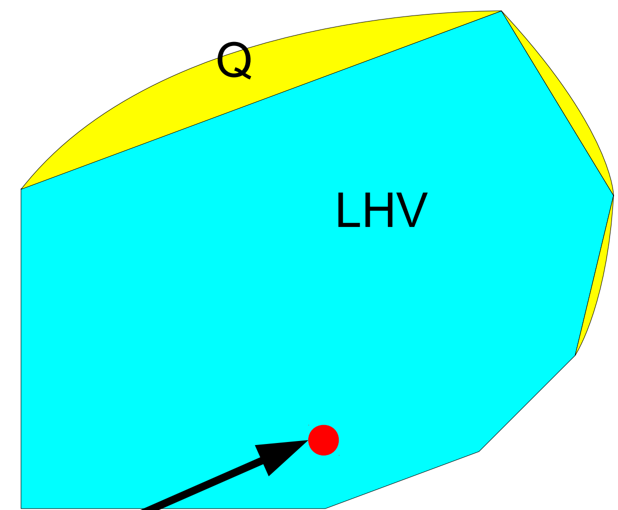

Local Polytope $\subset$ Quantum Set

$\mathbb{P}_L$ $\mathcal{Q}$

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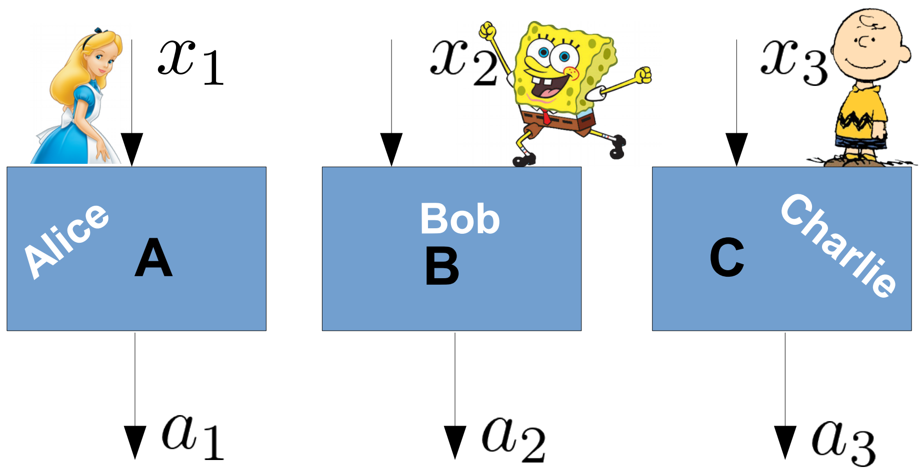

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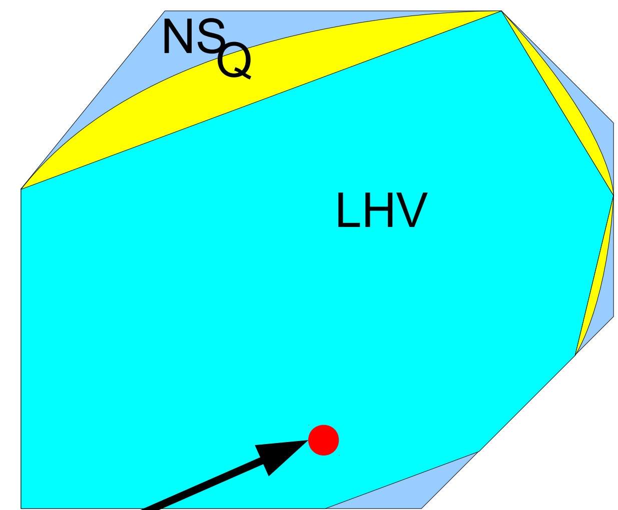

Local Polytope $\subset$ Quantum Set $\subset$ NS Polytope

$\mathbb{P}_L \qquad \mathcal{Q} \qquad \mathbb{P}_{NS}$

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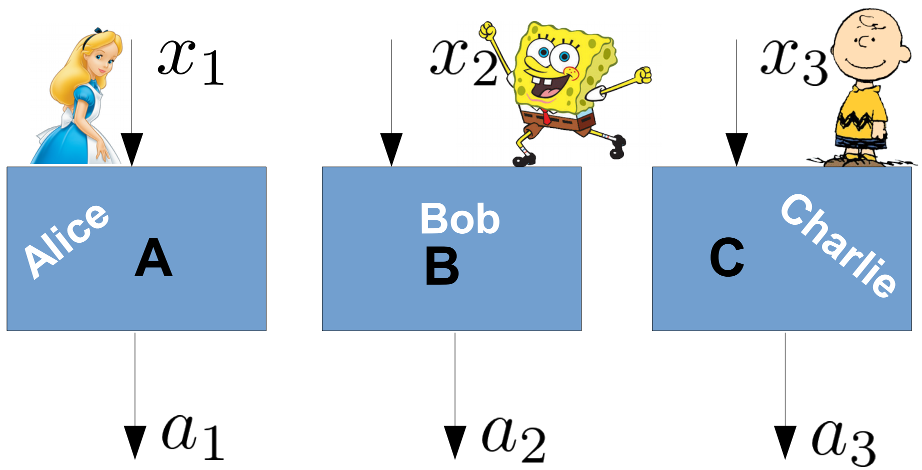

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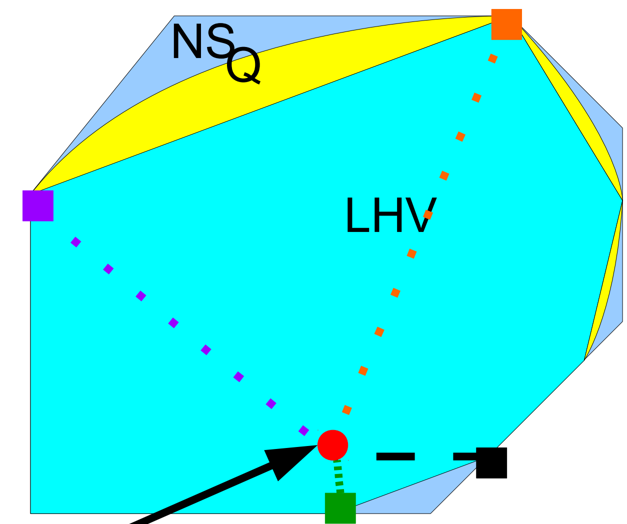

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Example:

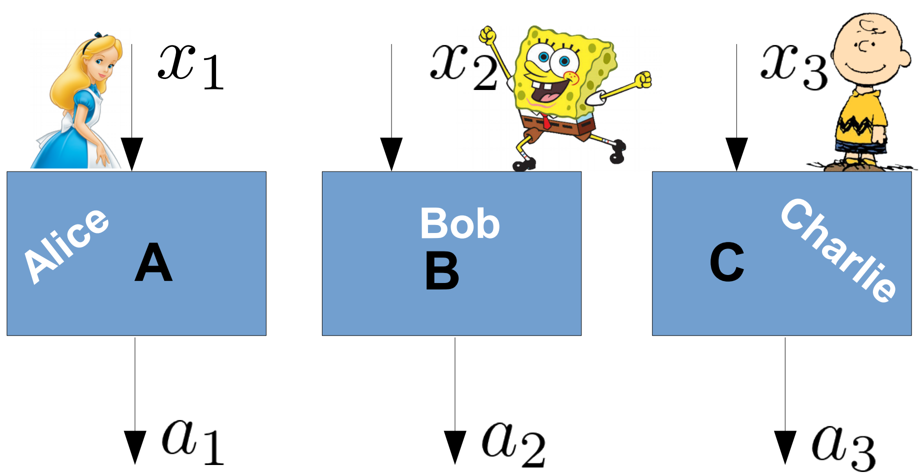
Charlie's Instructions

$\lambda = \{1, 3, 1, 2, 4, 3, 1, 1 \ldots\}$

Output

$0, x_3, 0, 1, \overline{x_3}, x_3, 0, 0, \ldots$

A crash course on nonlocality



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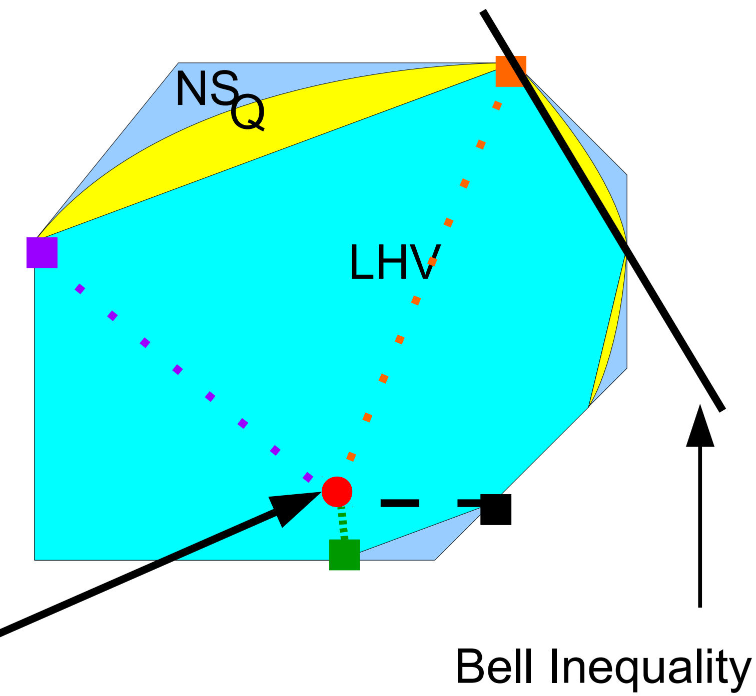

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The idea

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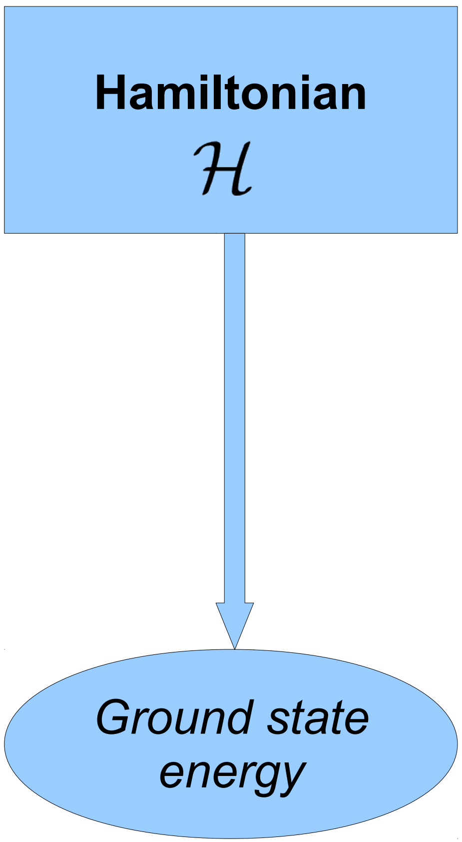
Hamiltonian

$\mathcal{H}$

The idea



The idea

Hamiltonian

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- Jordan-Wigner
- MPS, DMRG

Ground state energy

The idea

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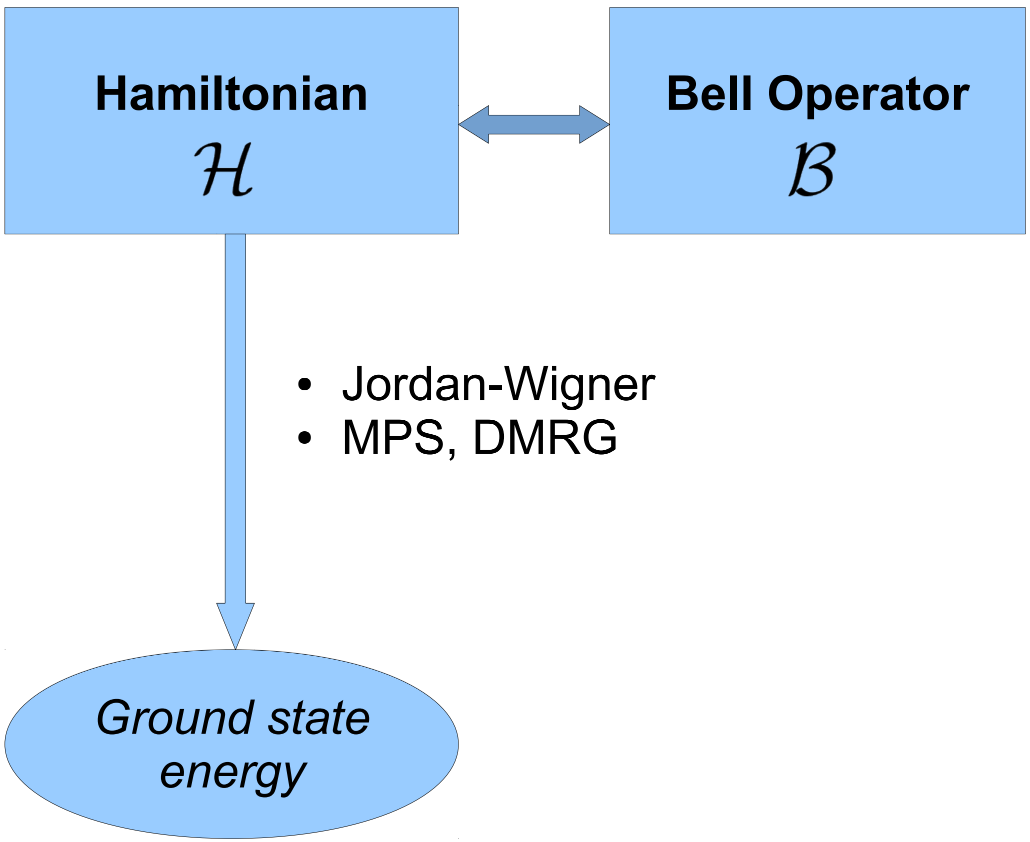
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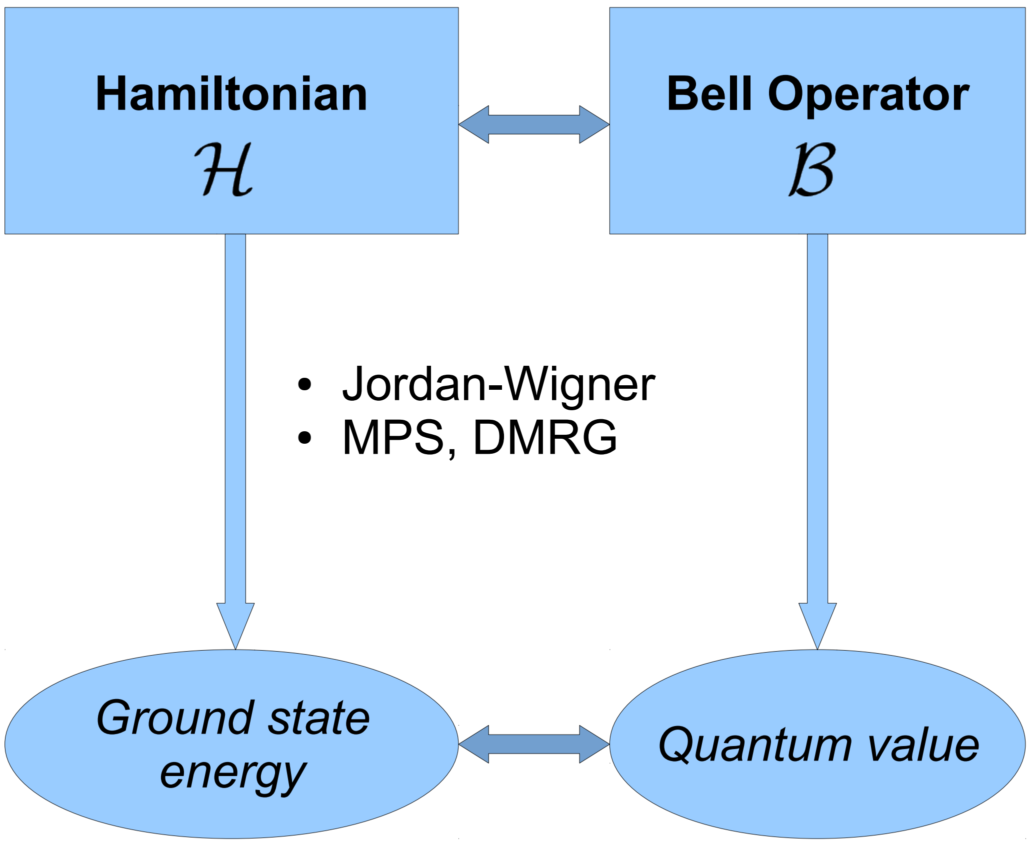
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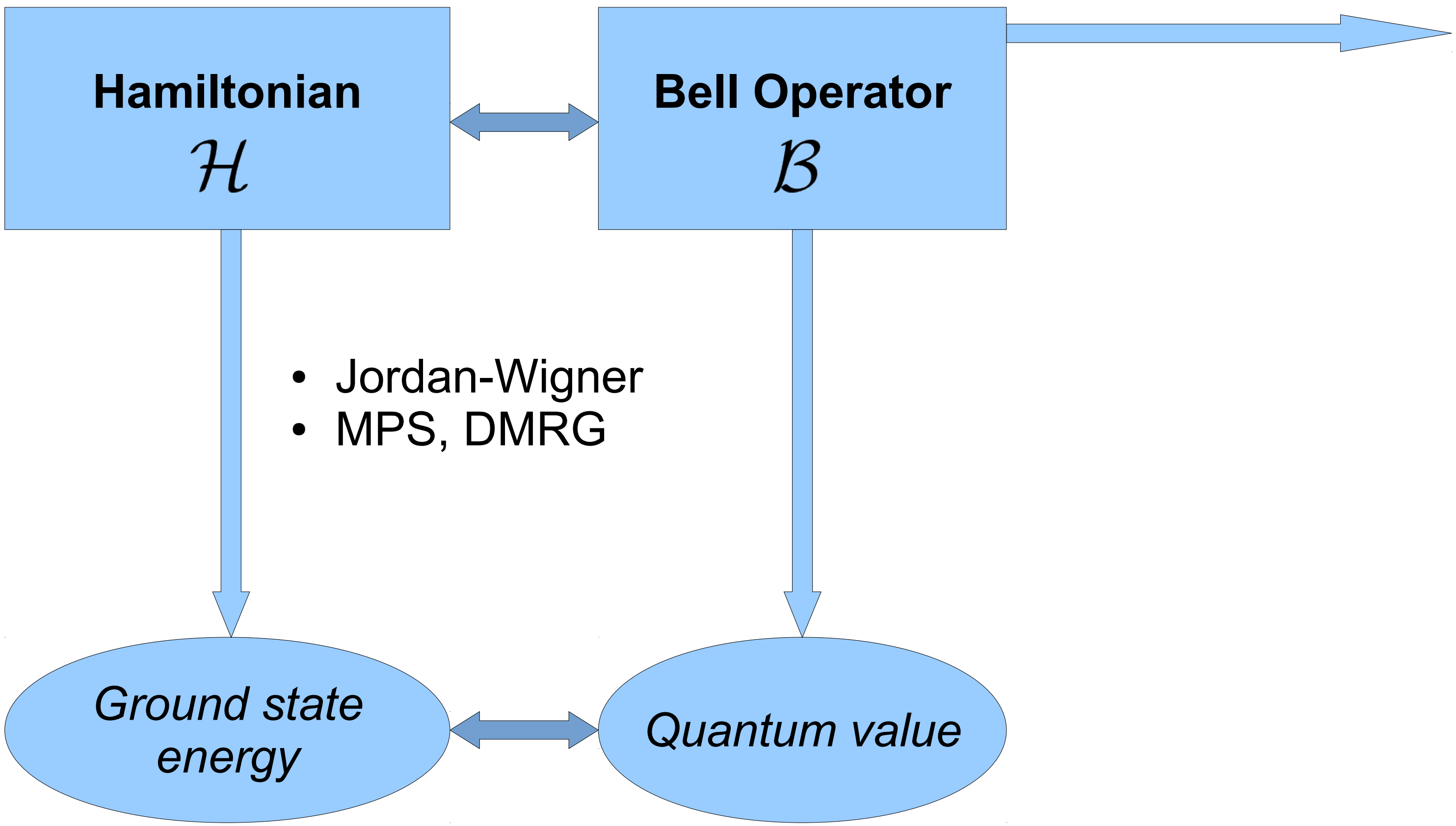
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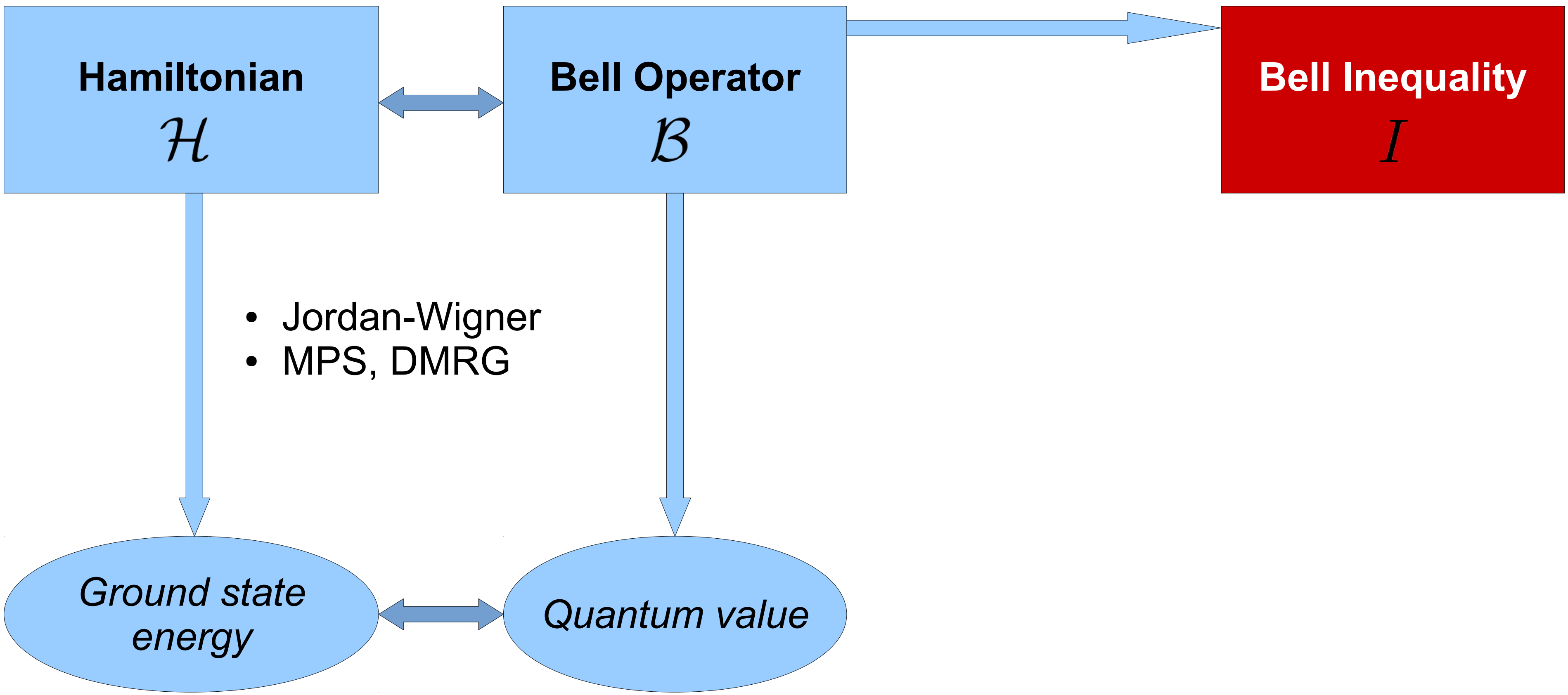
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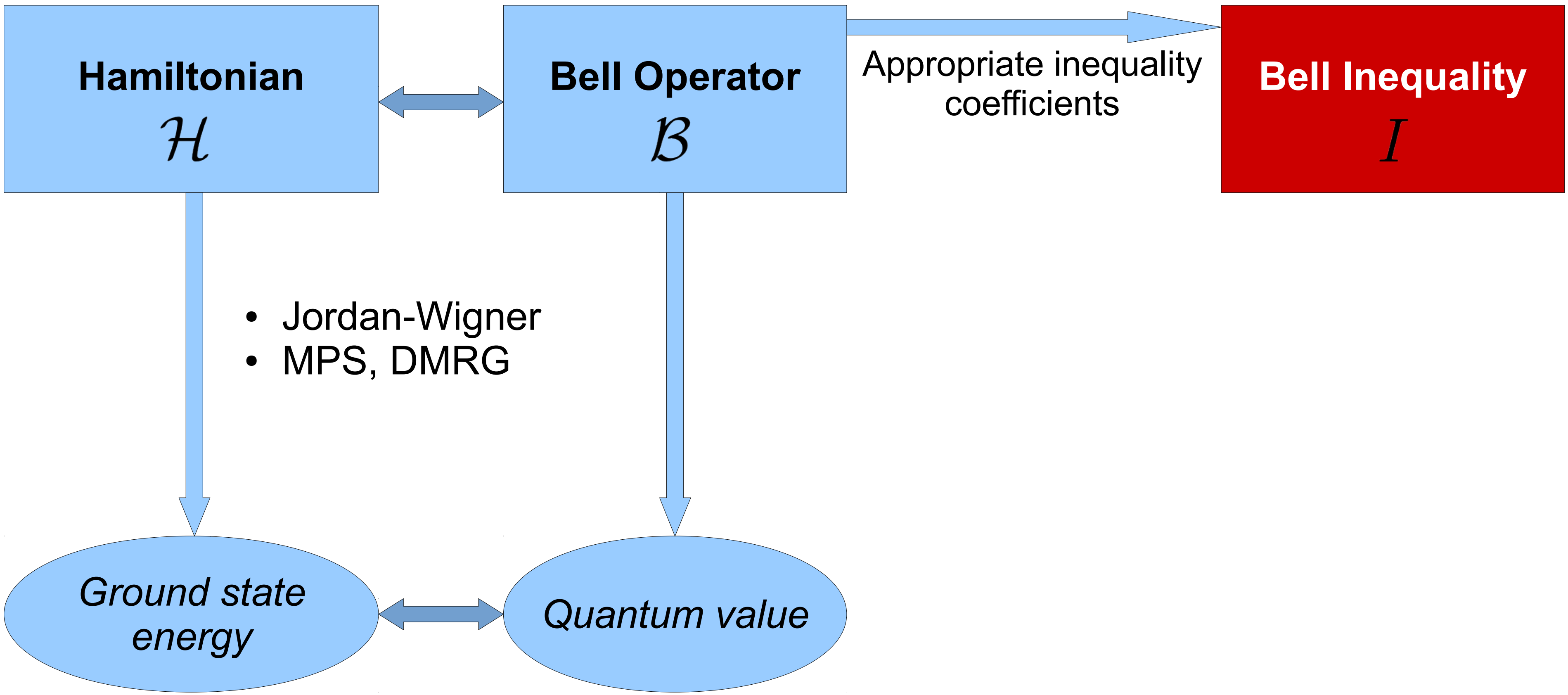
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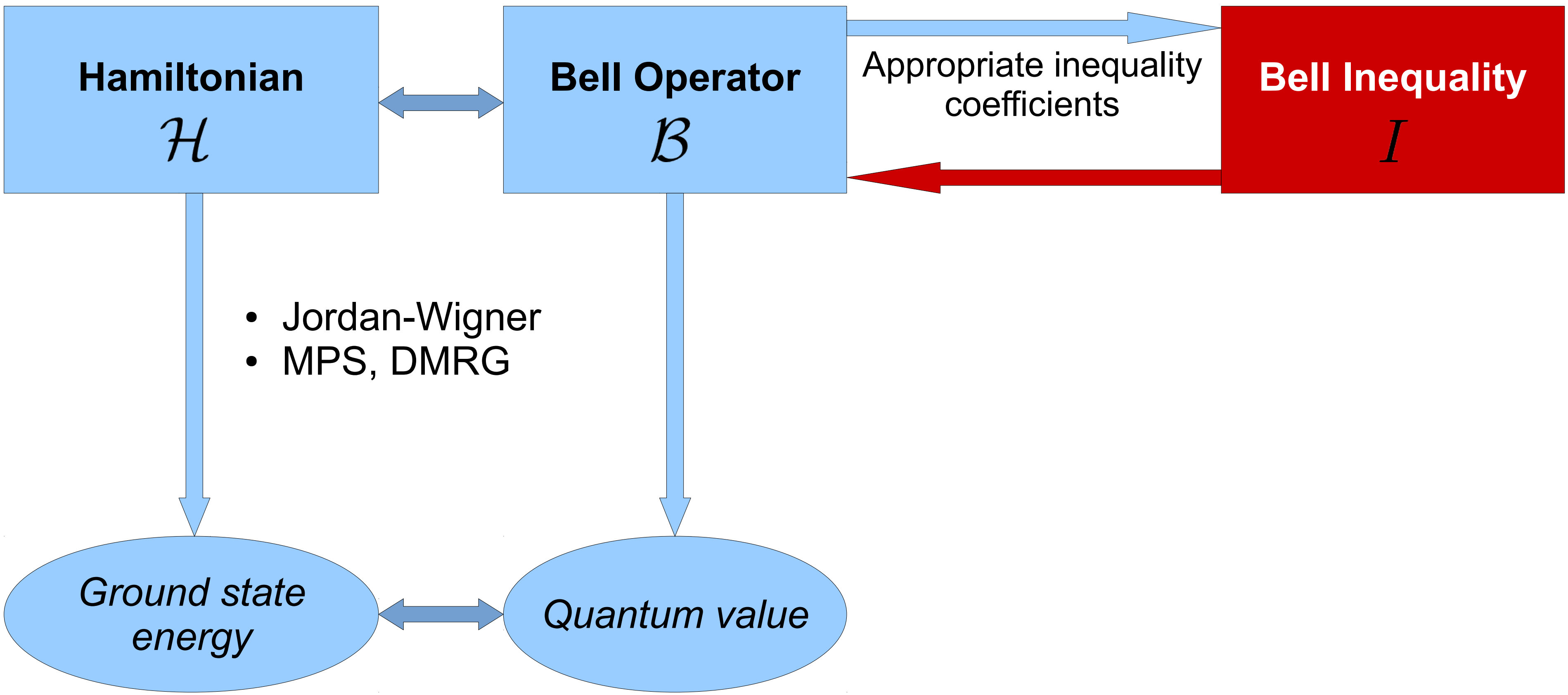
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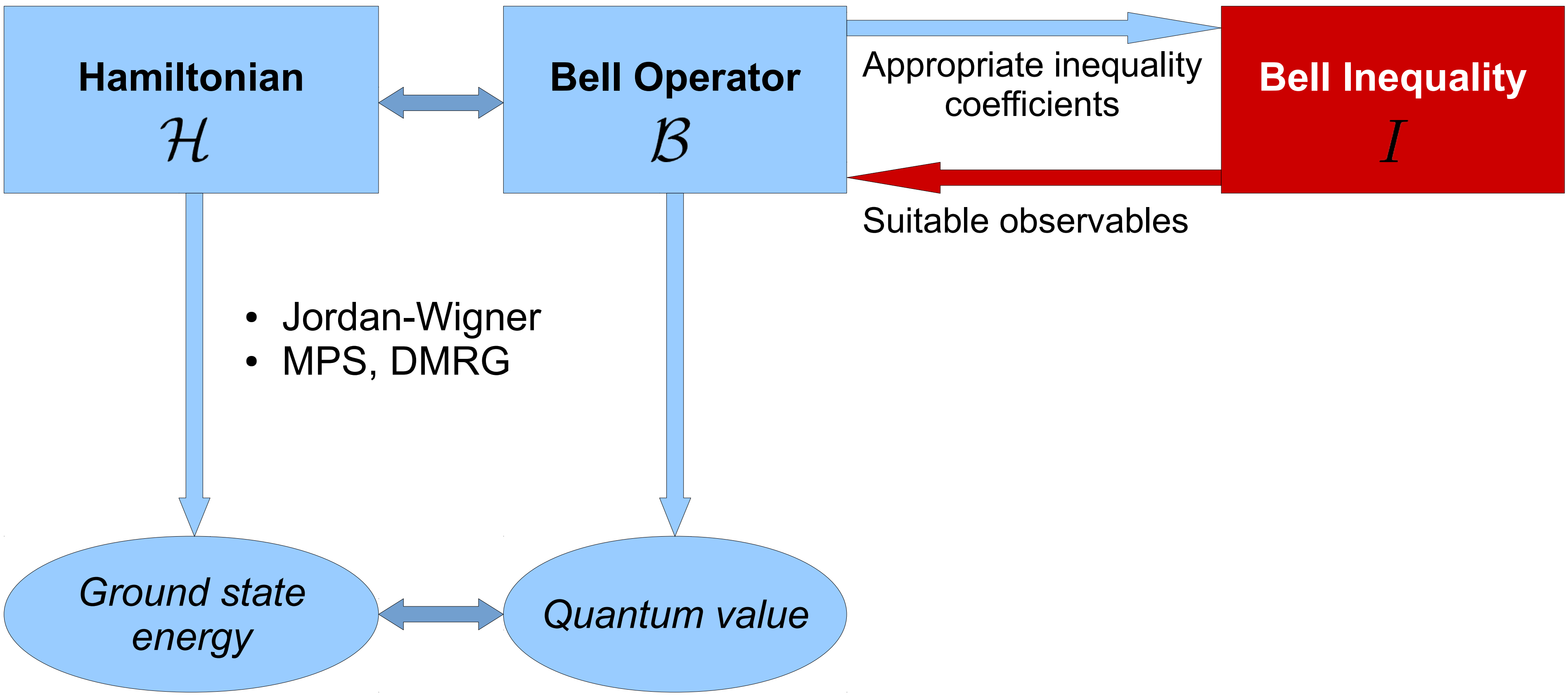

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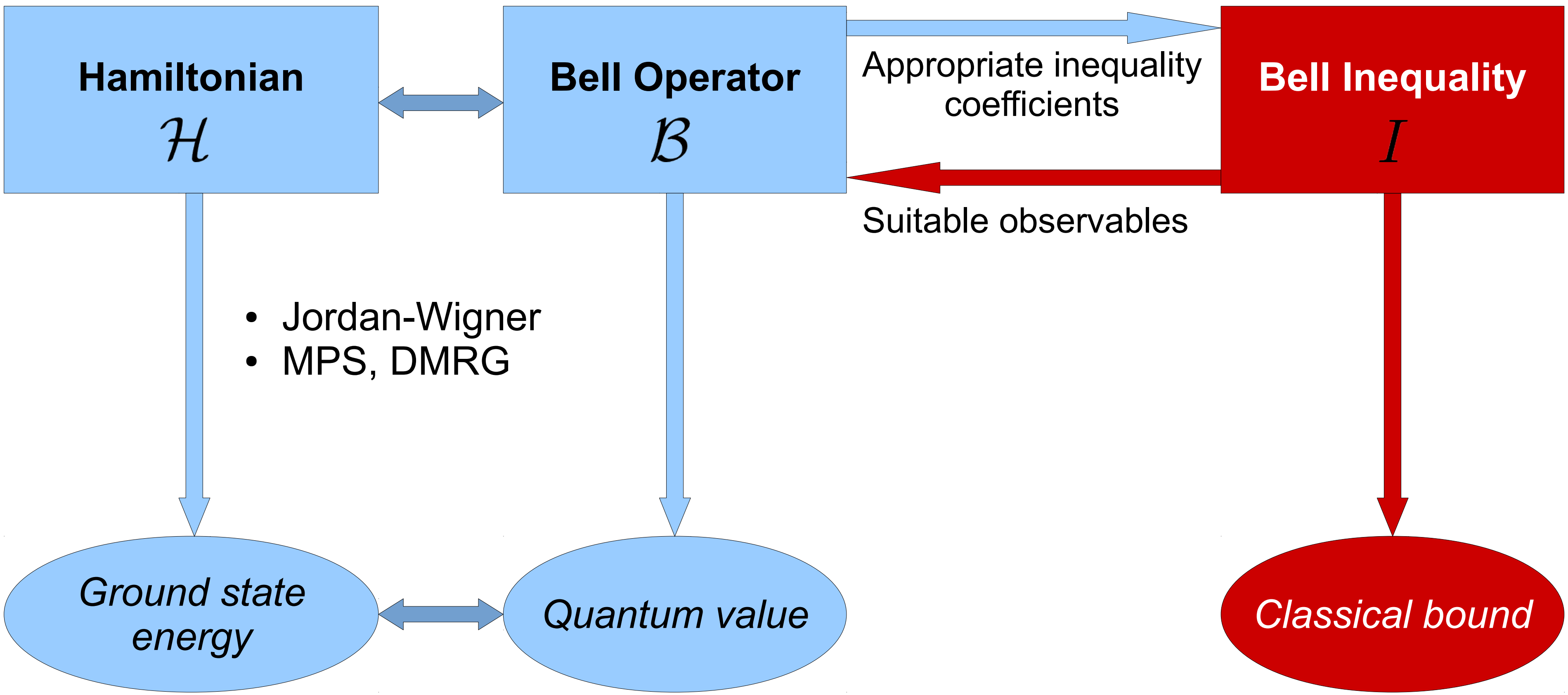
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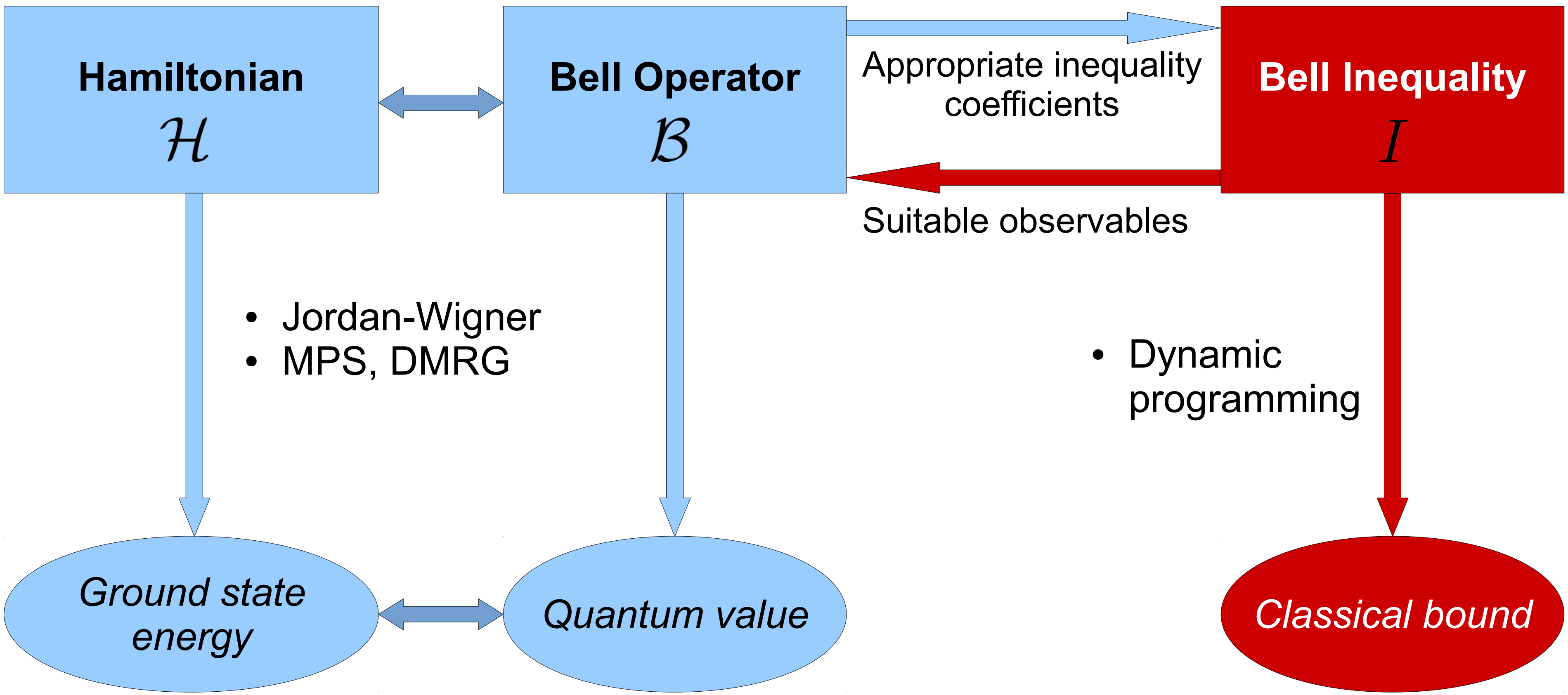
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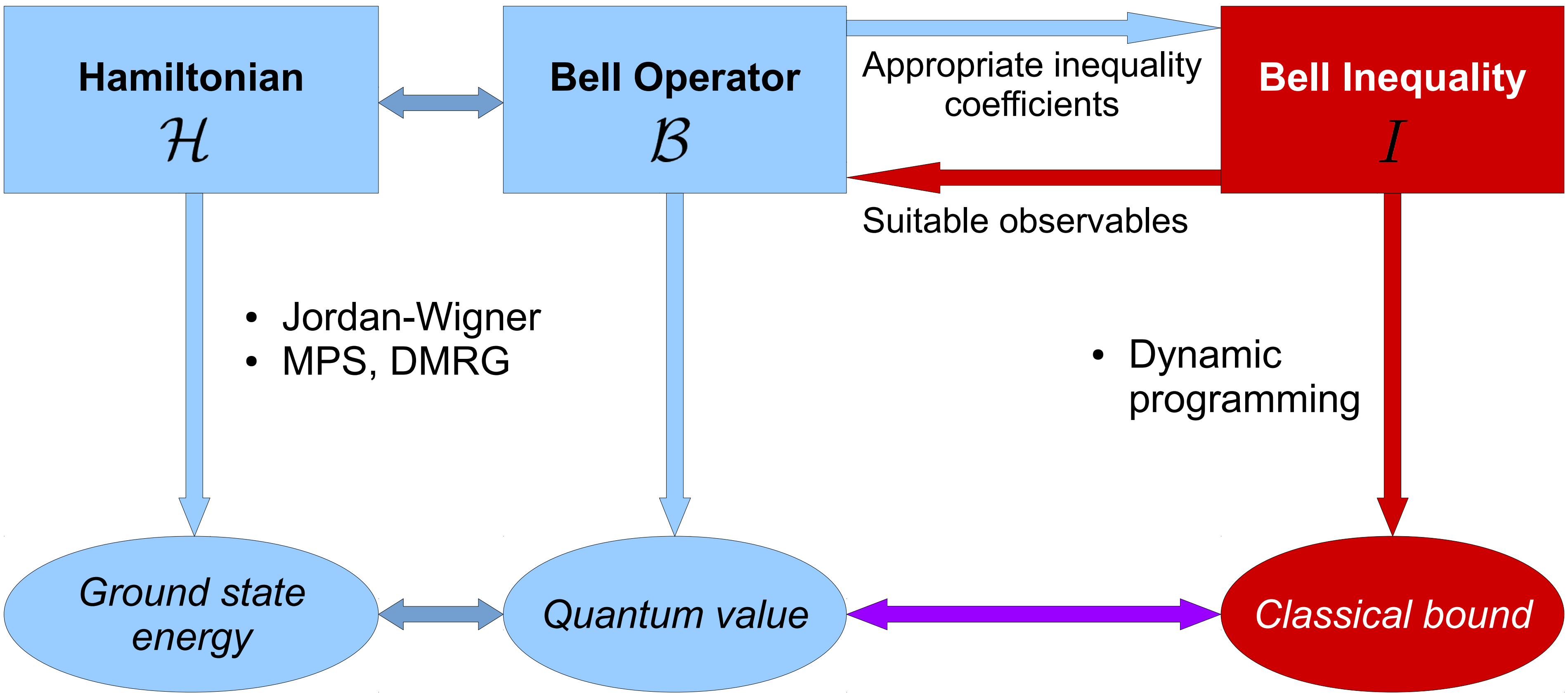
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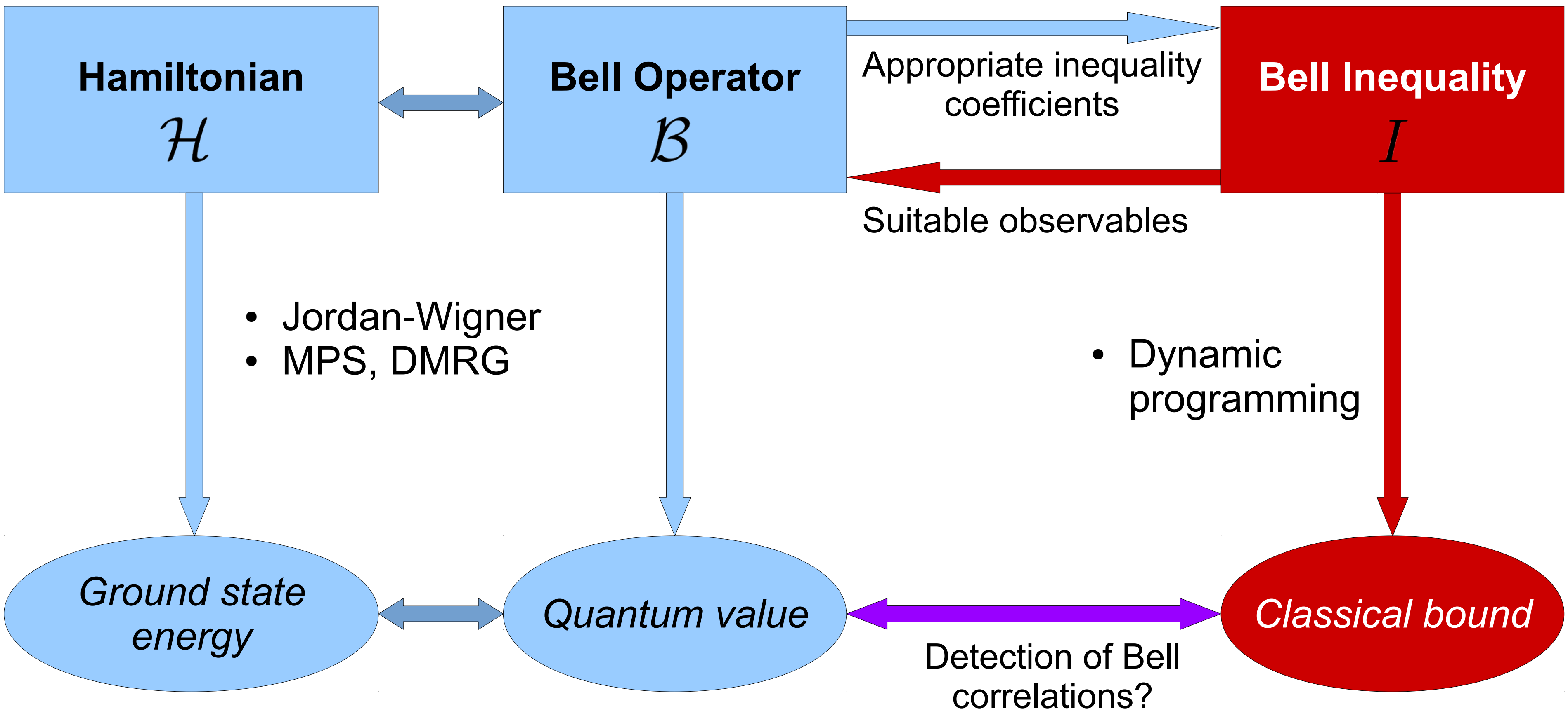
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Detection of Bell correlations?

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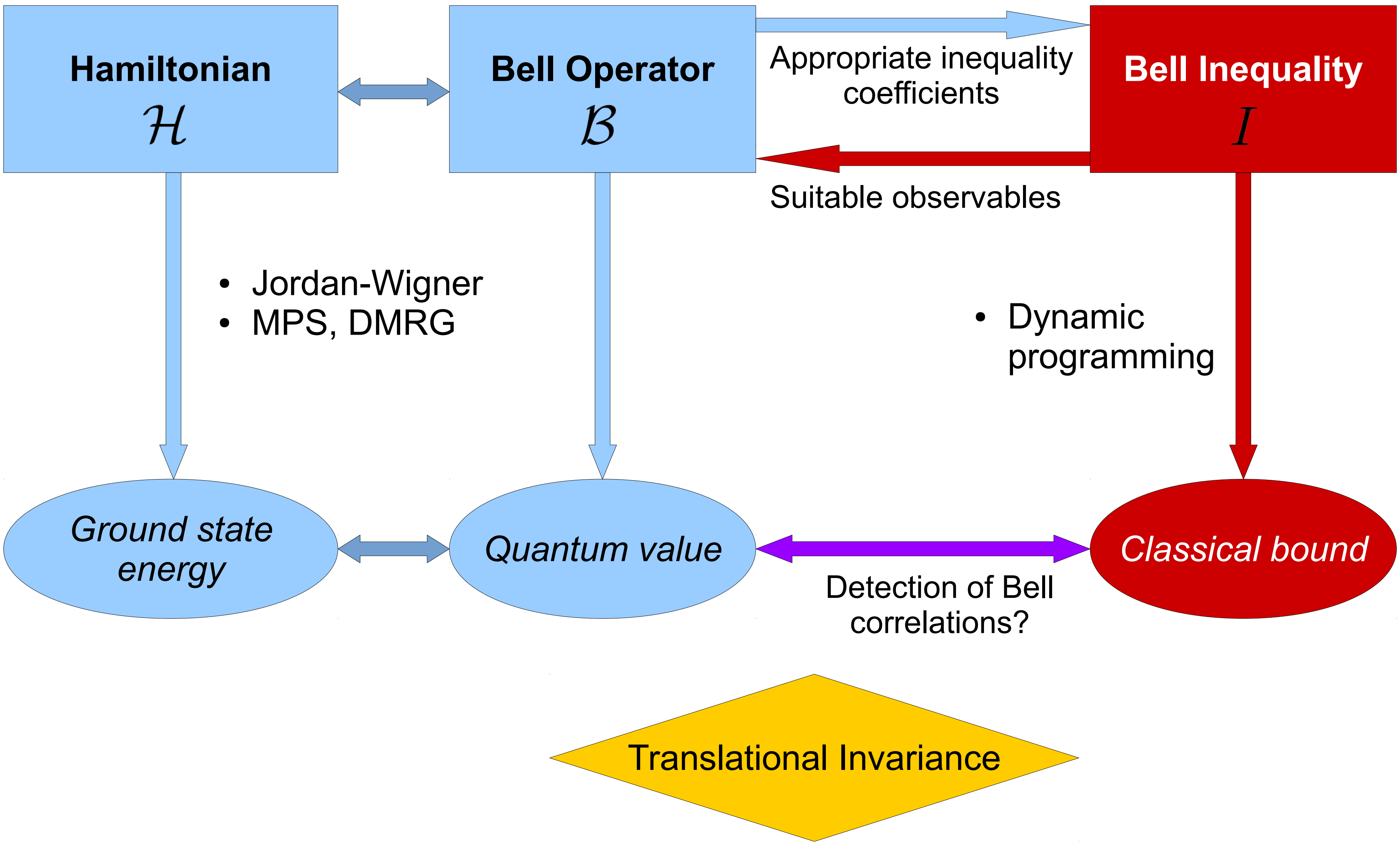
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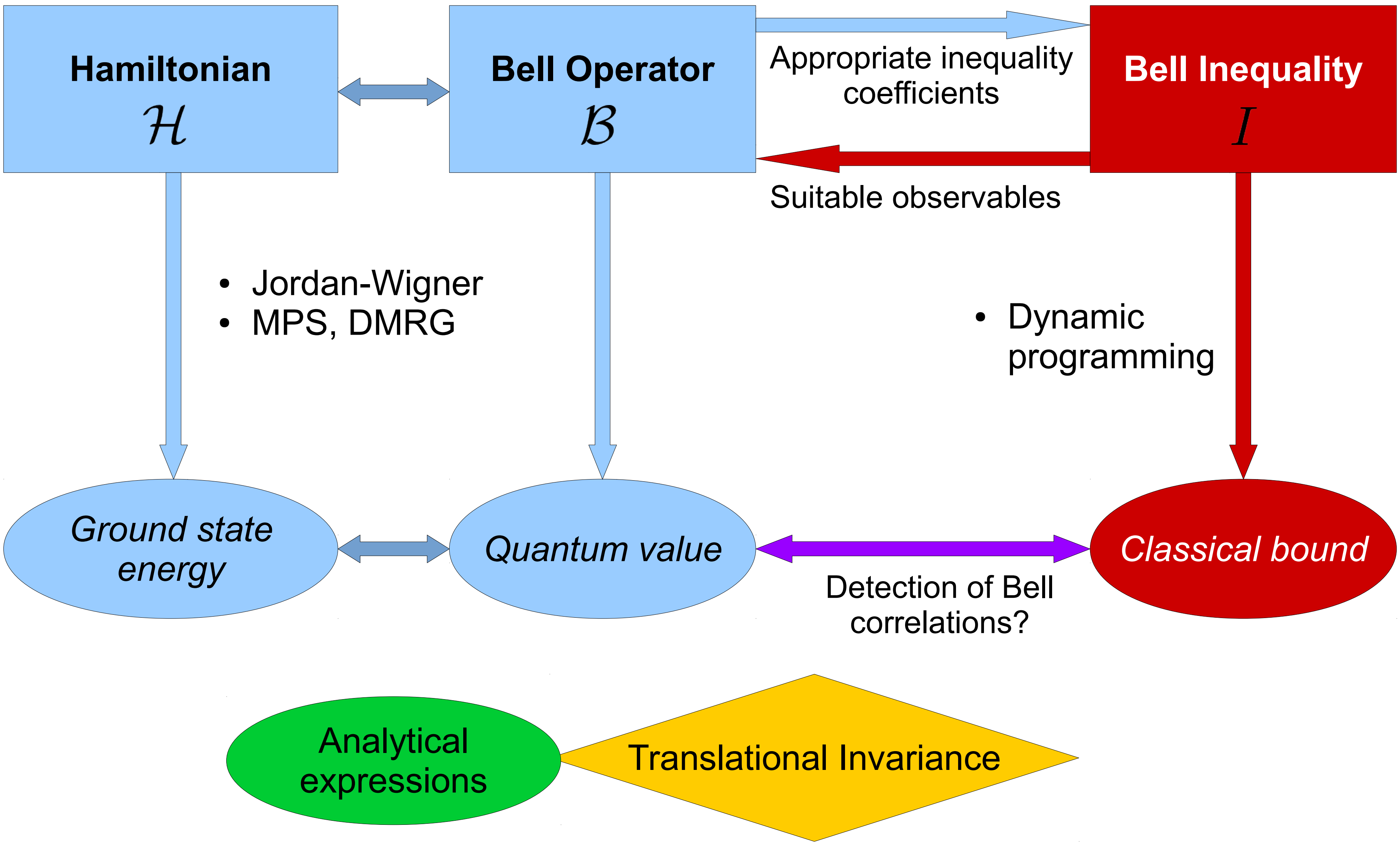
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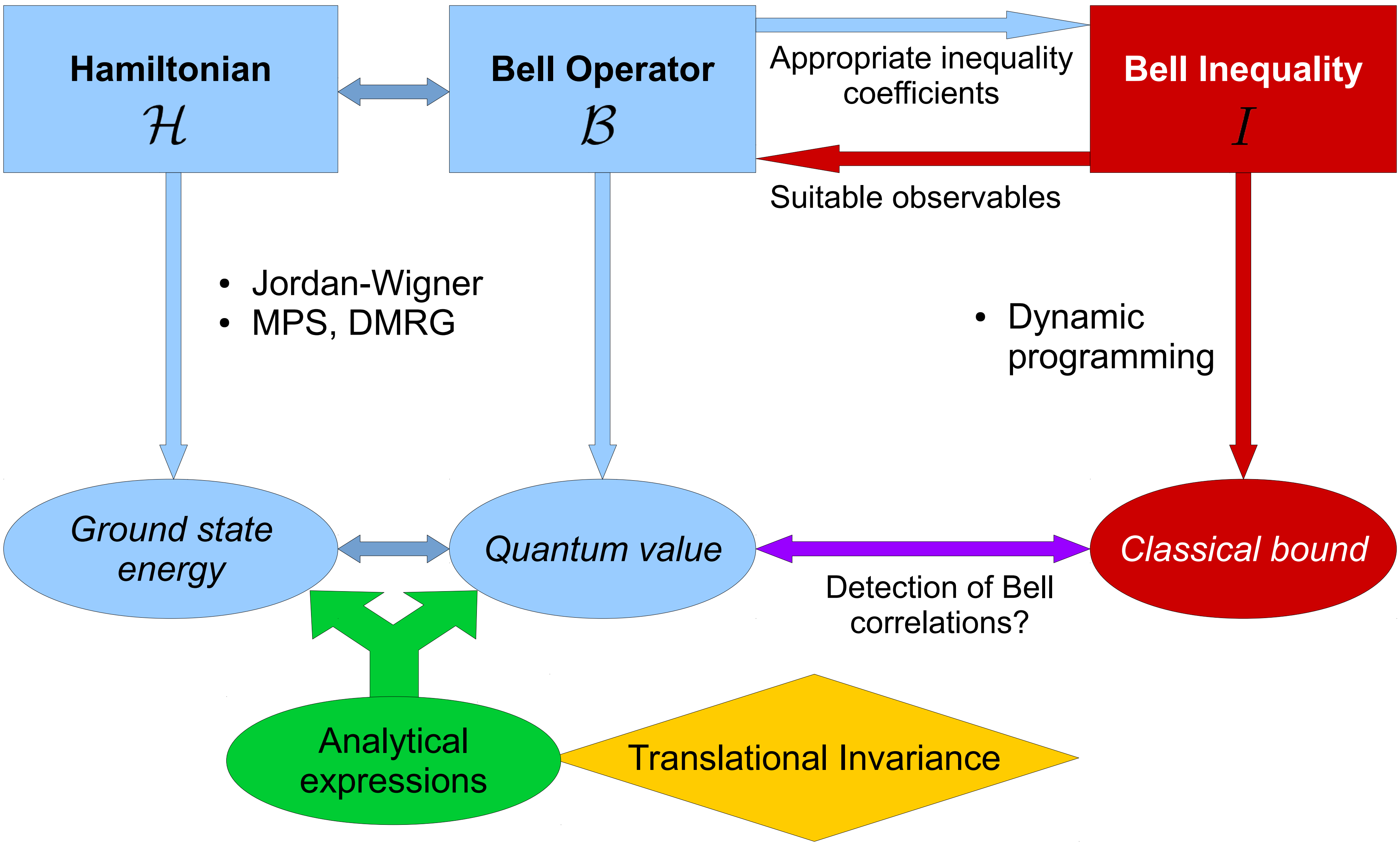

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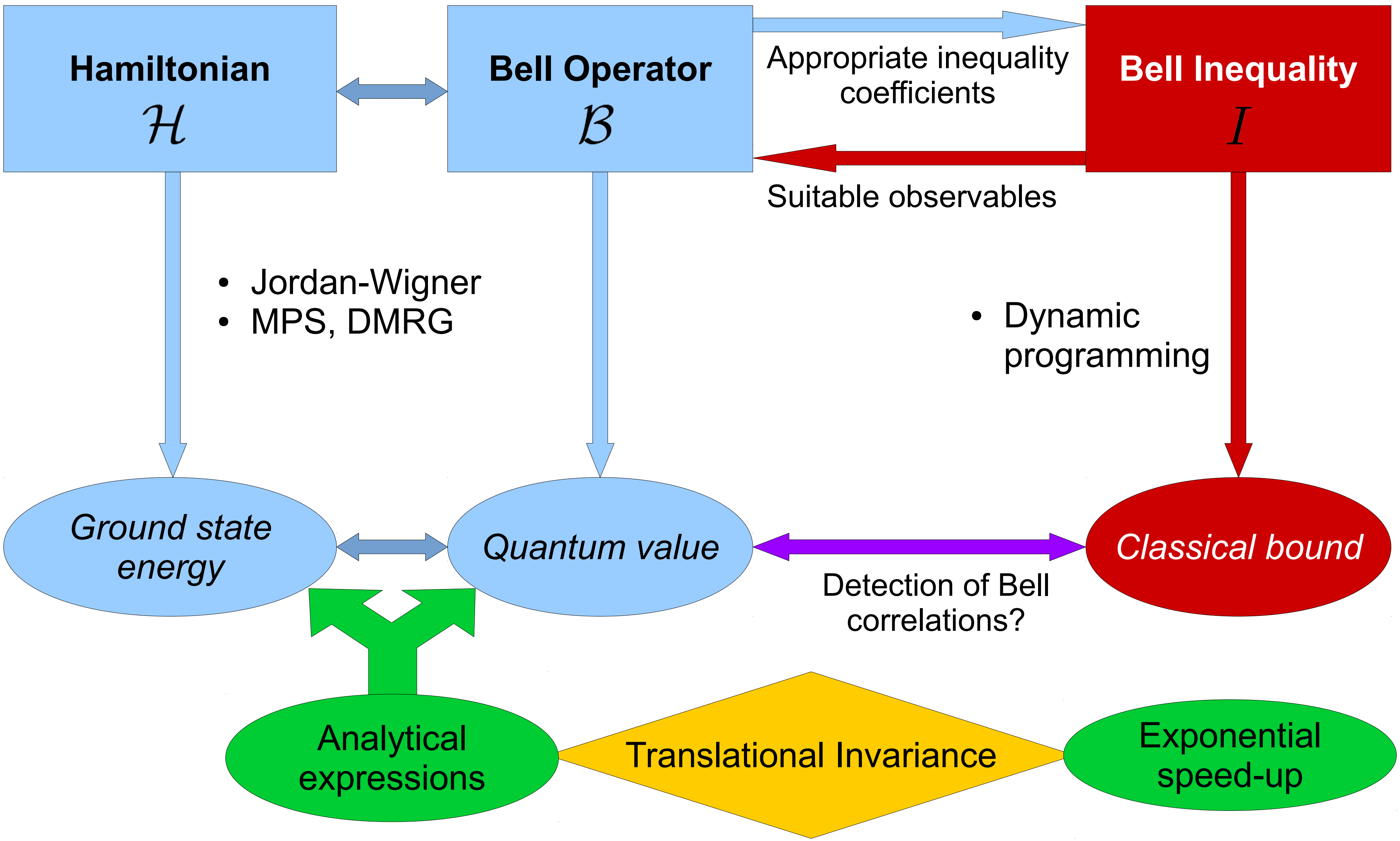
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Exponential speed-up

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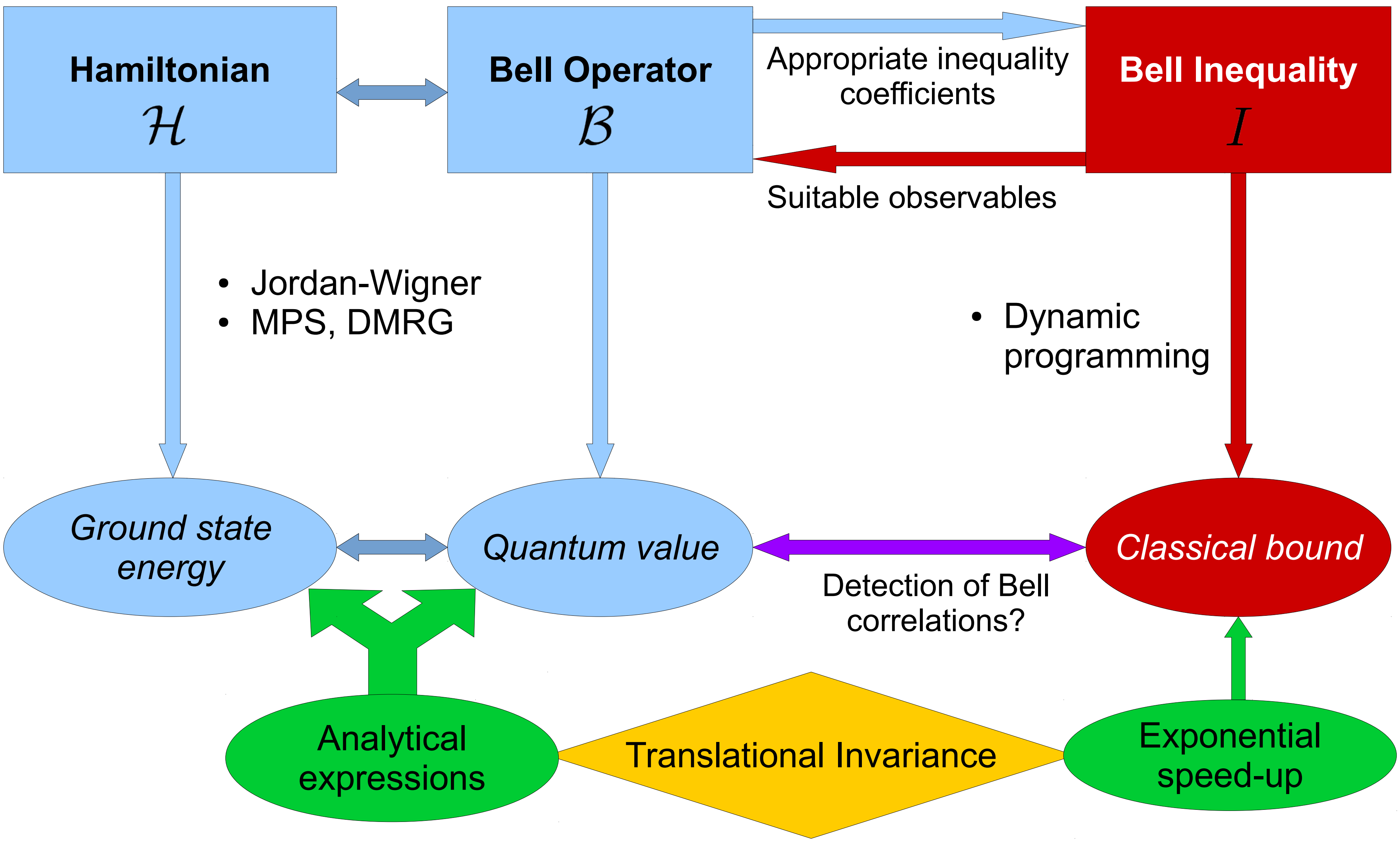
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The setting

The setting

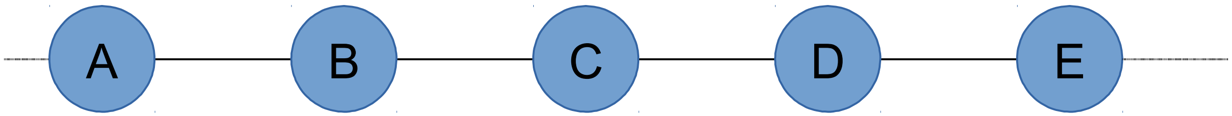
- Spin – 1/2 Hamiltonians

The setting

- Spin – 1/2 Hamiltonians
- n particles

The setting

- Spin – 1/2 Hamiltonians
- n particles
- One spatial dimension



The setting

- Spin – 1/2 Hamiltonians
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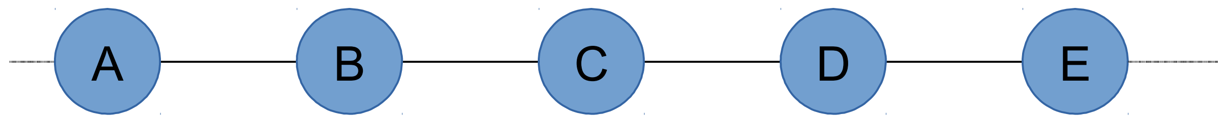

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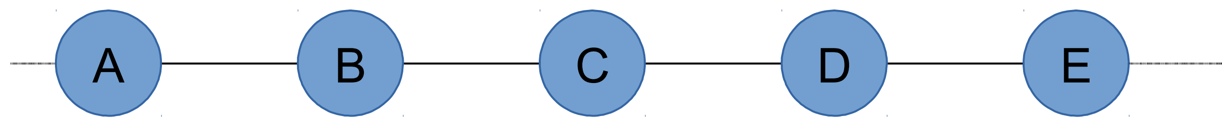

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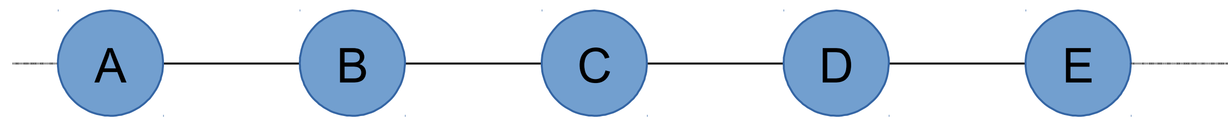

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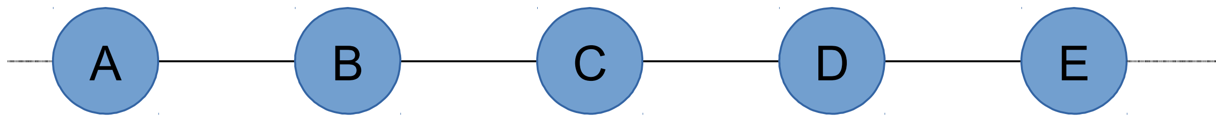

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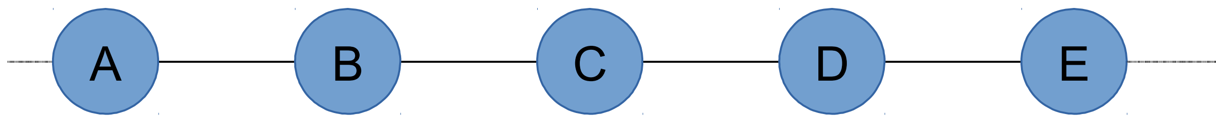

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e.g. XY-model in a transverse magnetic field

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  - Jordan – Wigner transformation: Spins to fermions

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Real, antisymmetric

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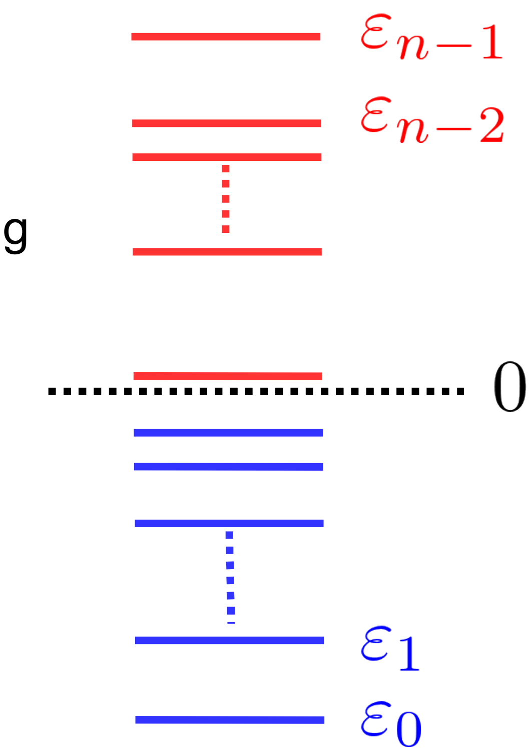

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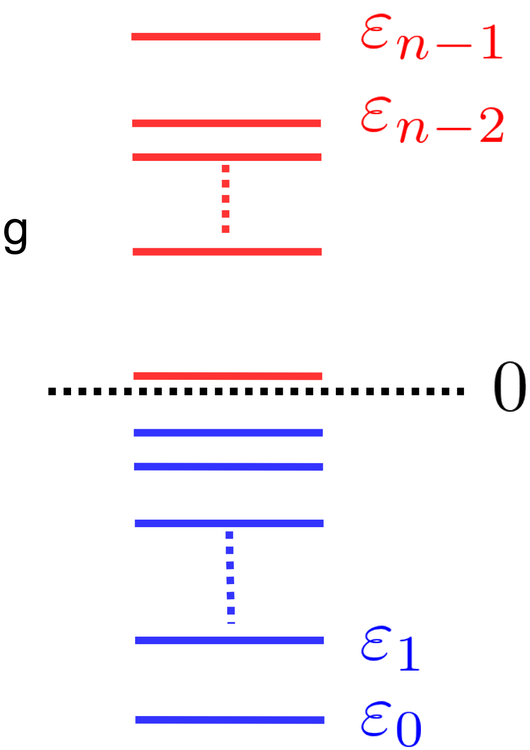

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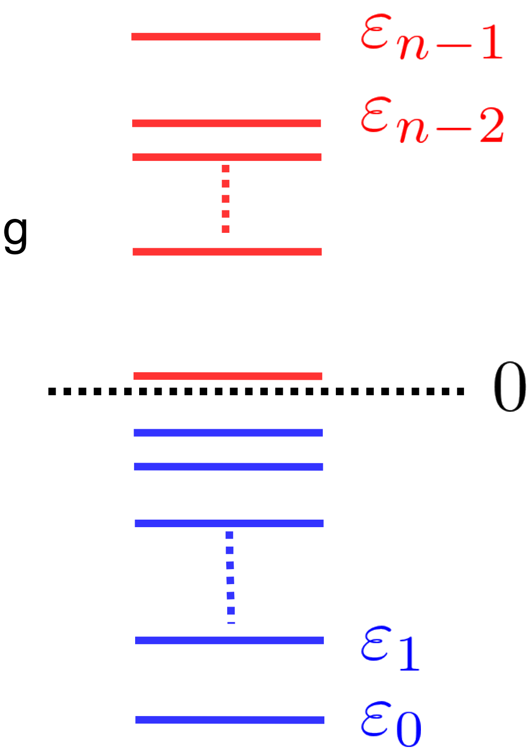

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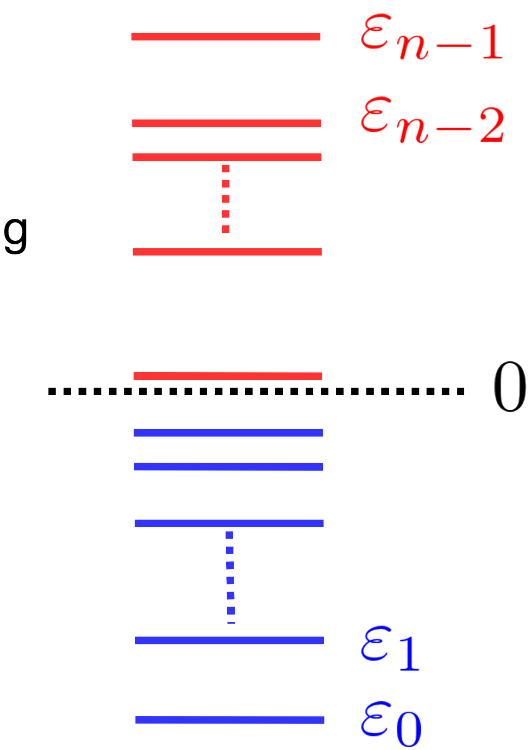

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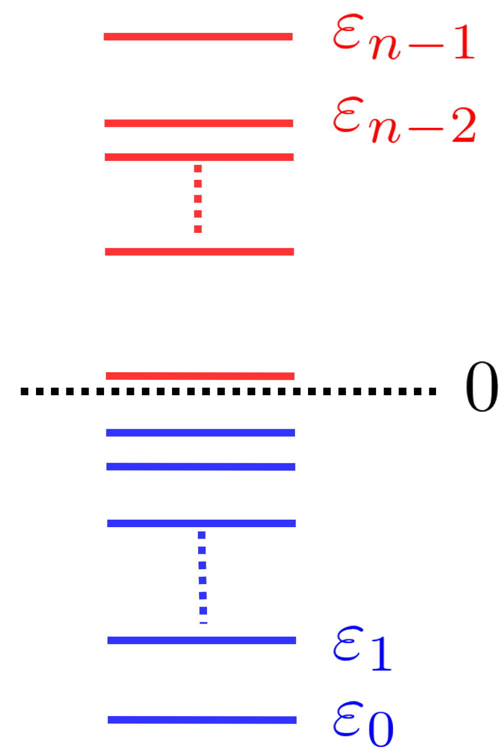

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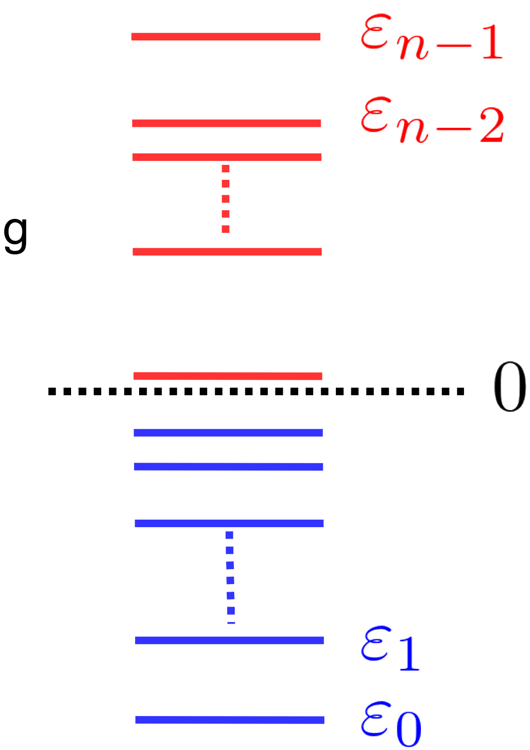

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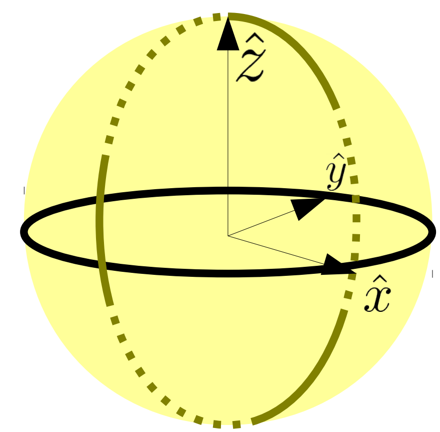

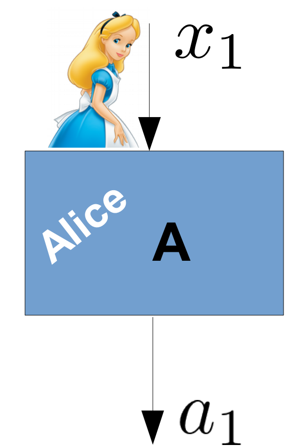

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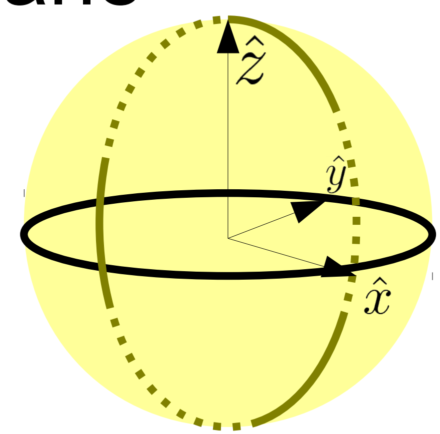

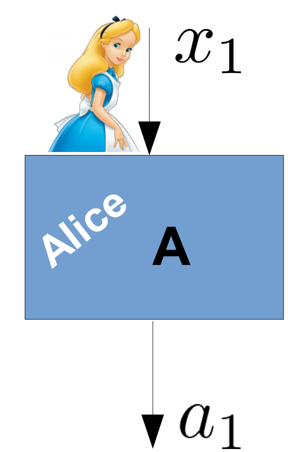

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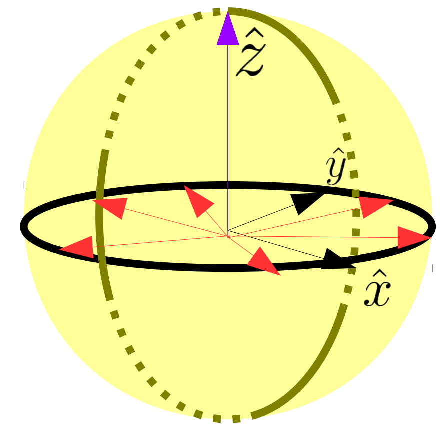

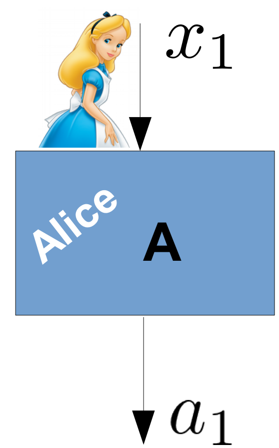

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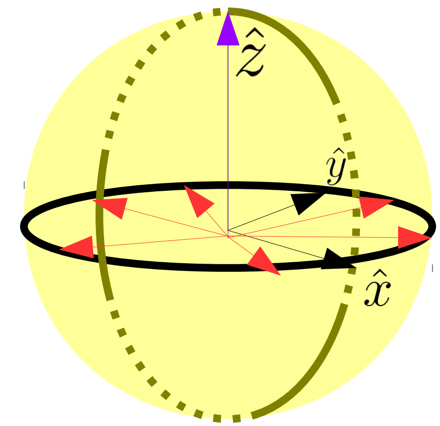

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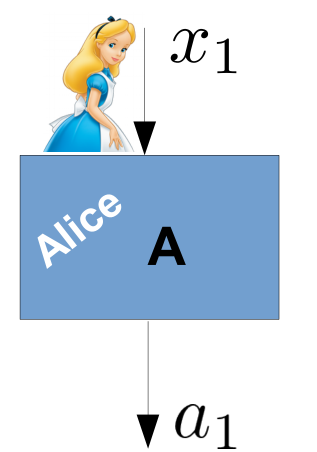

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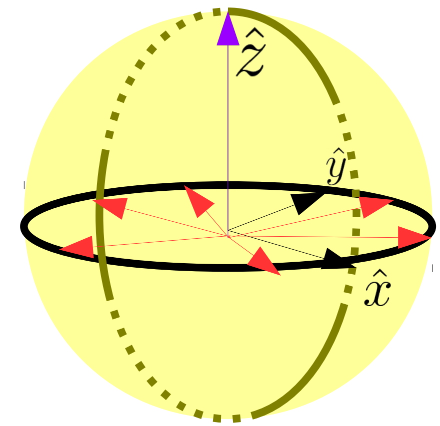

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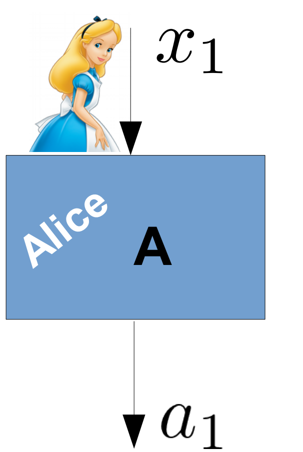

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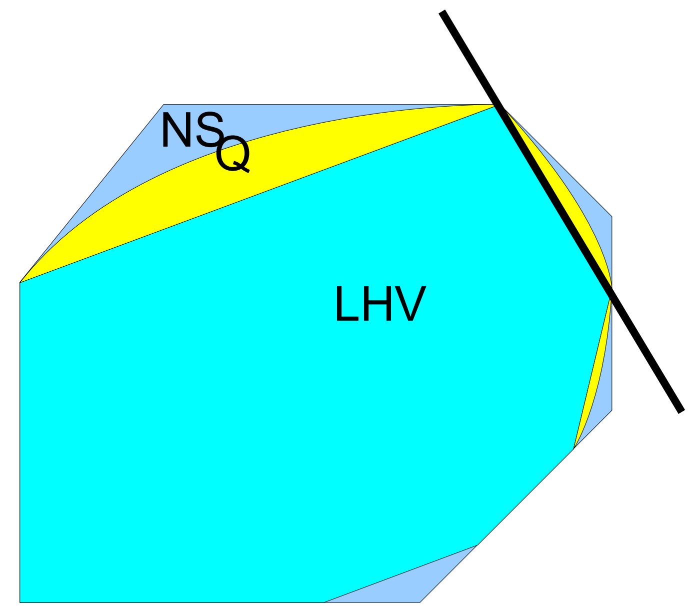

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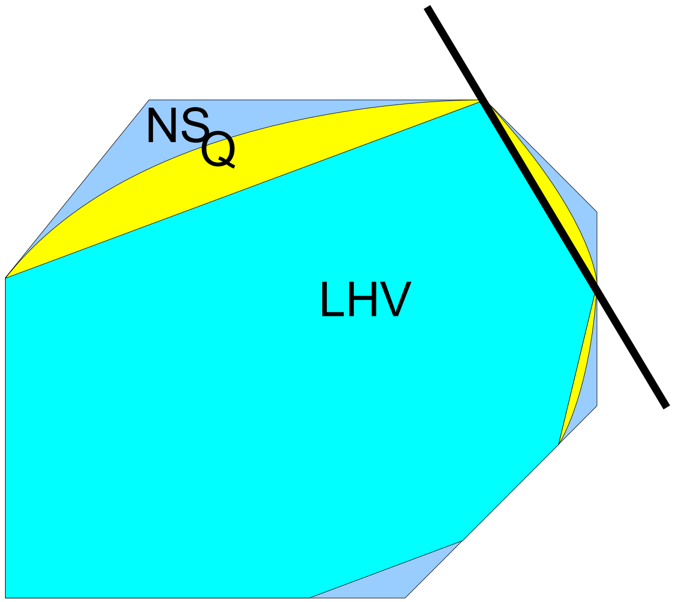

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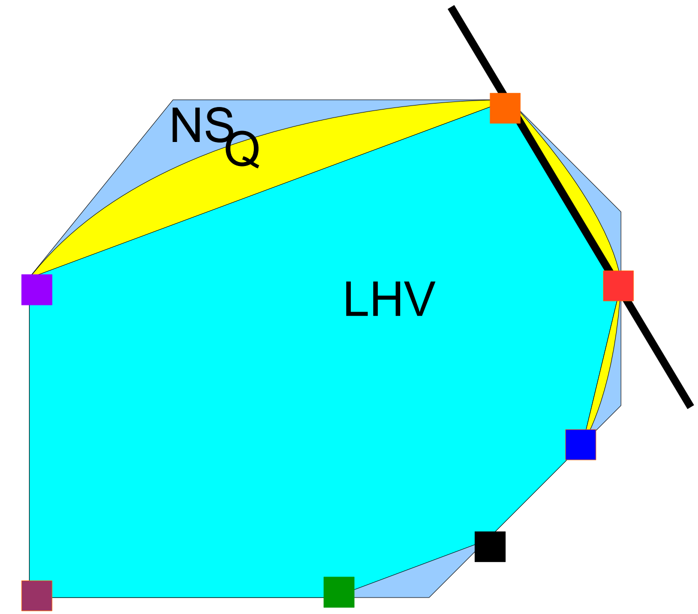

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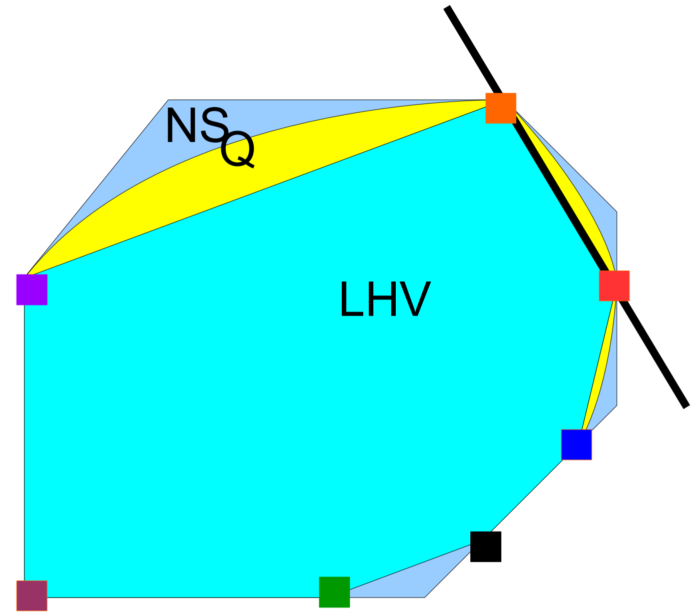

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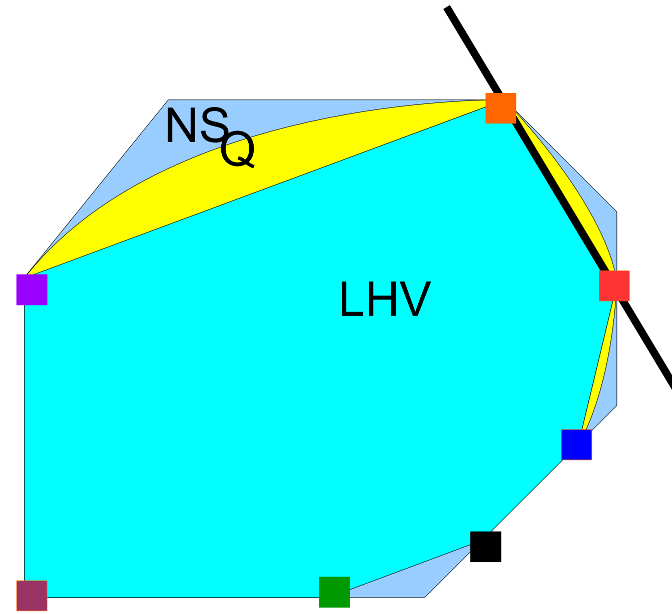

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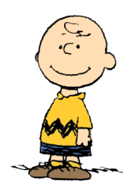
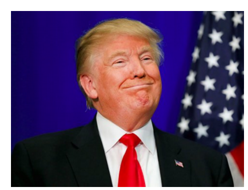

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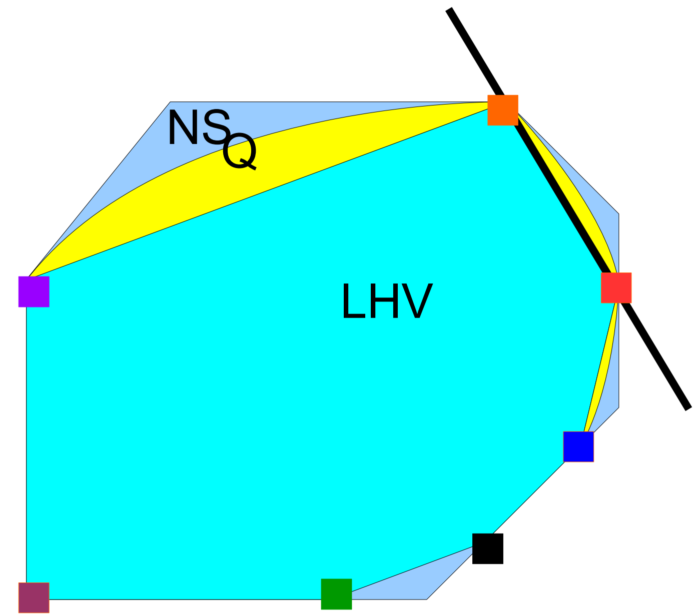

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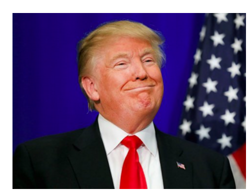


$M_0^{(i)}$

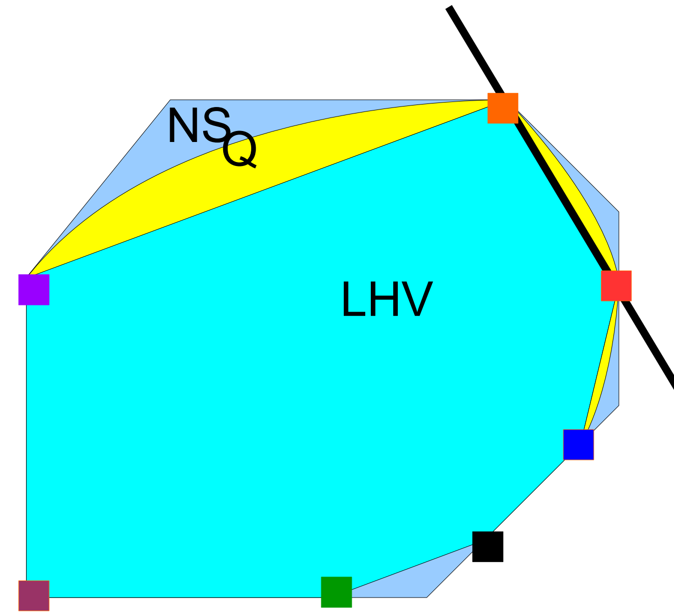
$M_1^{(i)}$

$M_2^{(i)}$

Finding the classical bound

- We want a Bell inequality of the form $I + \beta_C \geq 0$

$$\beta_C = - \min_{\text{LHVM}} I$$



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   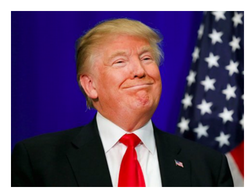

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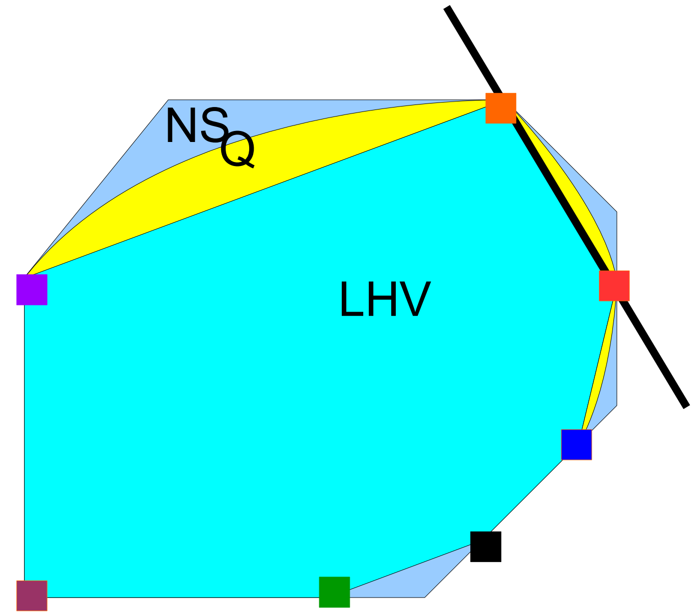

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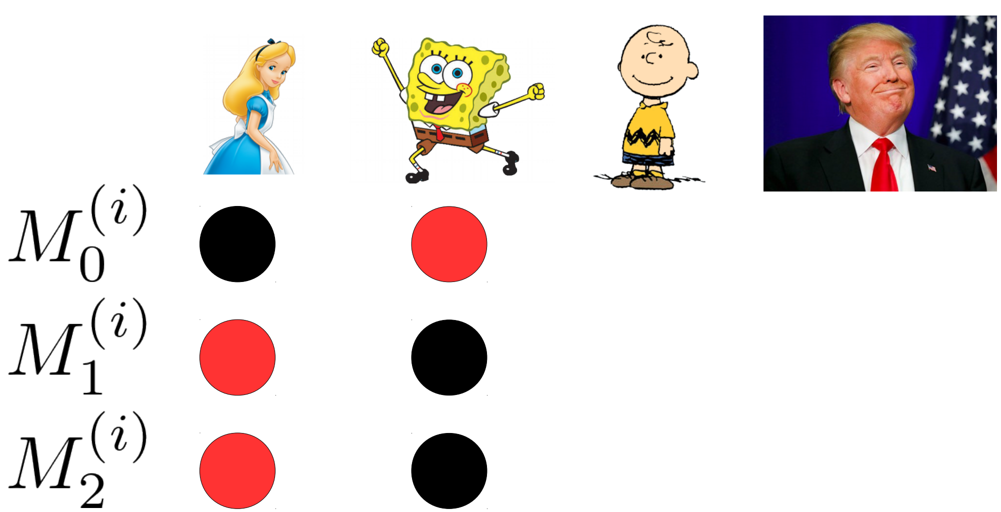

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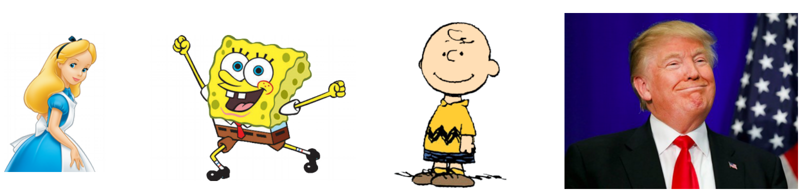

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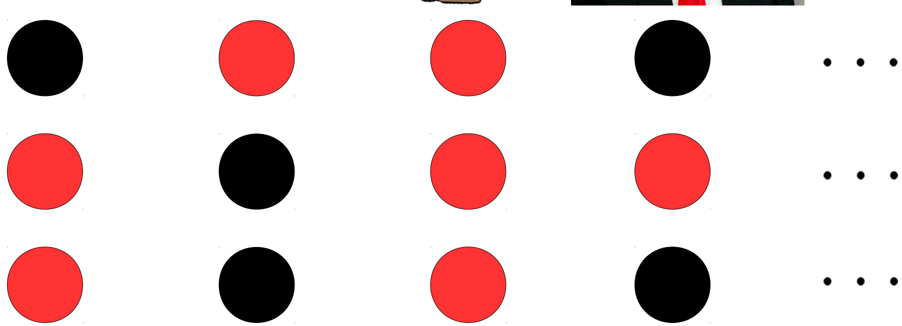

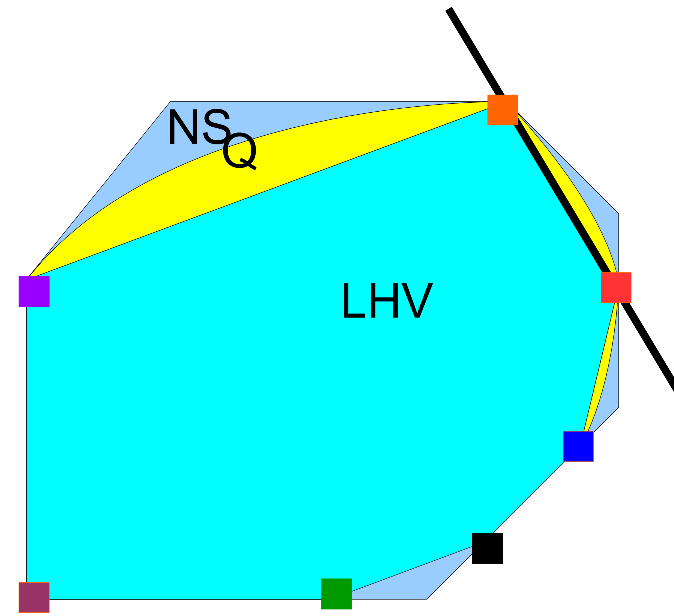

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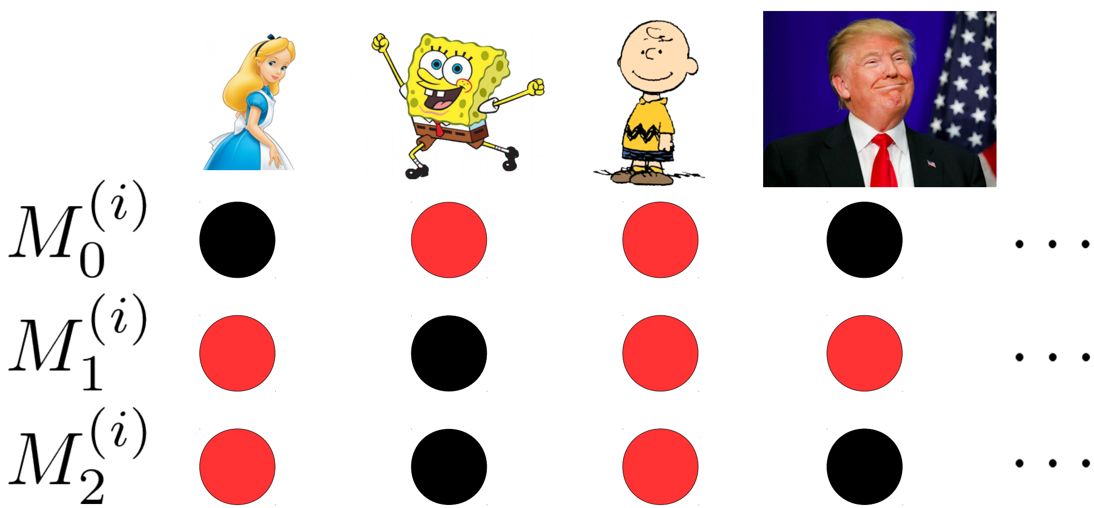

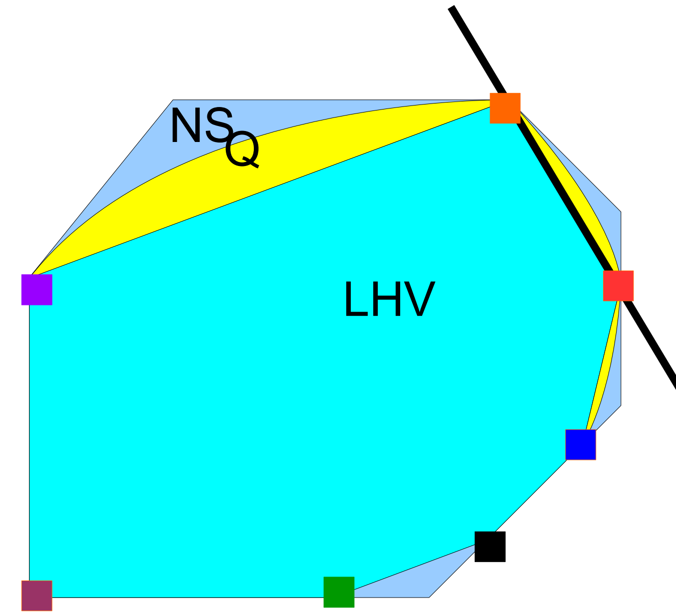

Problem

Finding the classical bound

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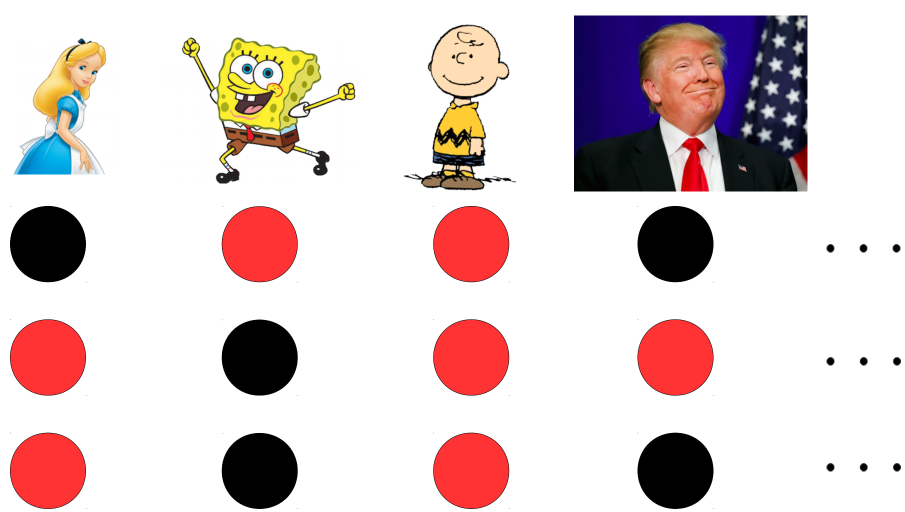

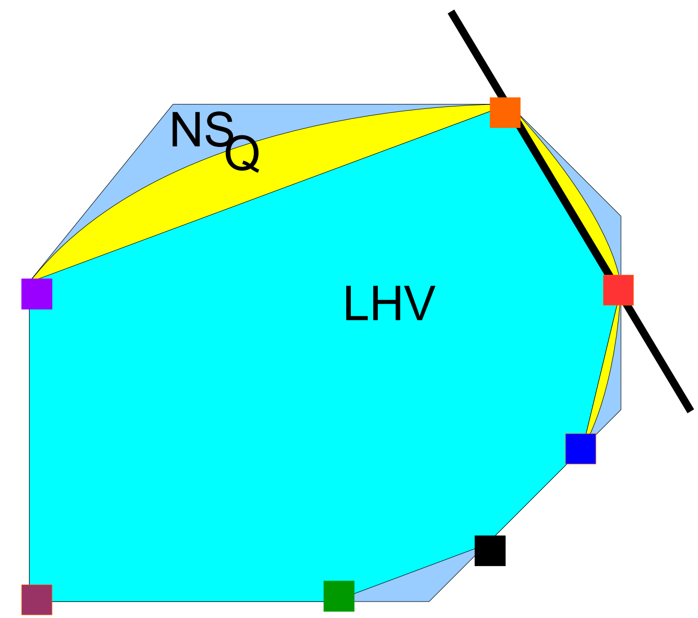

Problem

2^{mn} vertices

Finding the classical bound

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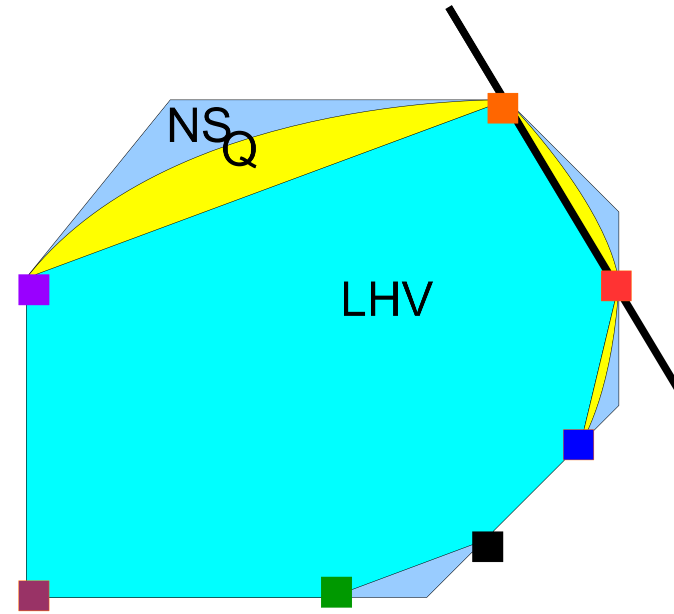

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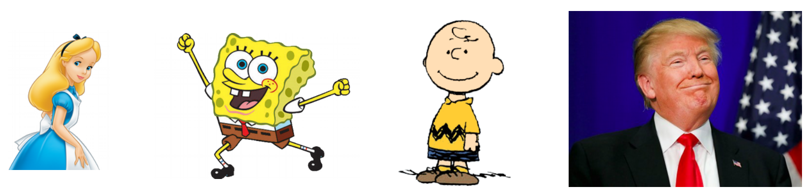

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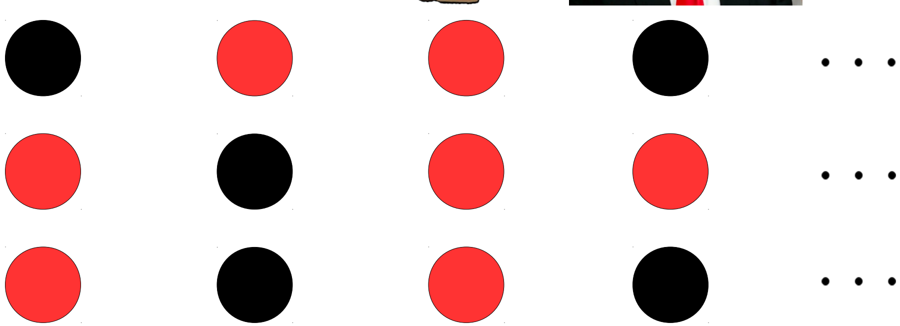


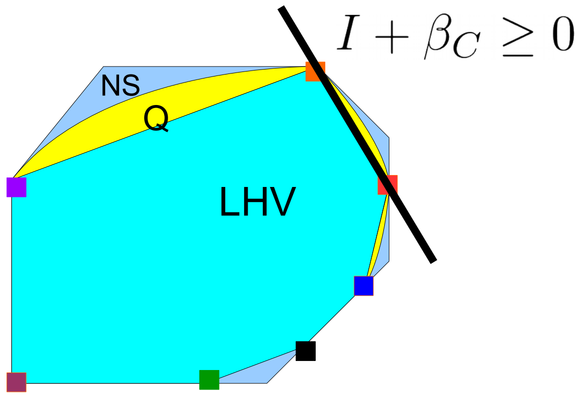
Problem

2^{mn} vertices

For our Bell inequalities

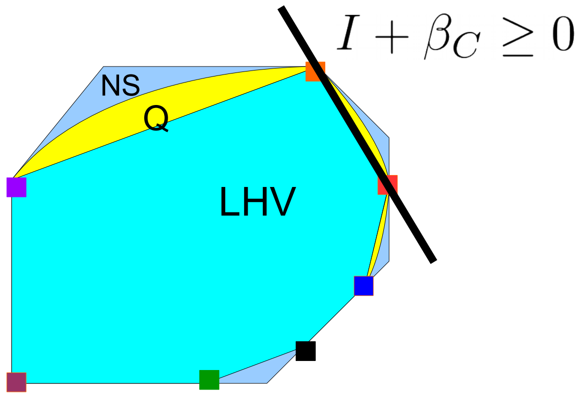
Dynamic programming

Finding the classical bound



Finding the classical bound

- Optimization over all LHV models



Finding the classical bound

- Optimization over all LHV models
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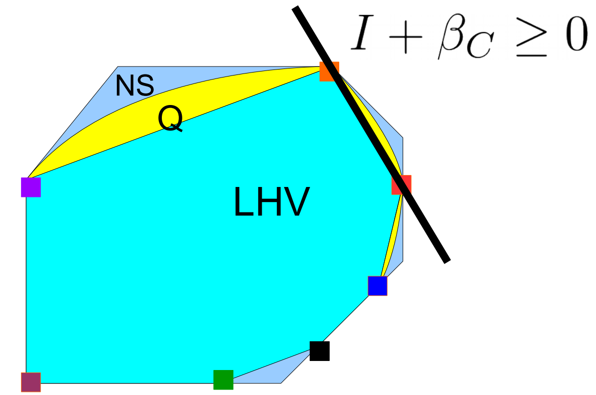

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- Optimization over all LHV models
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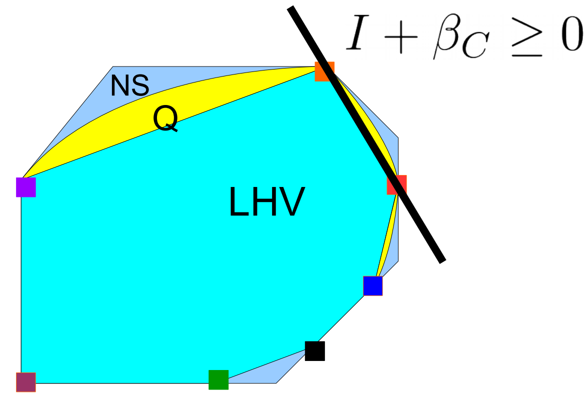

Finding the classical bound

- Optimization over all LHV models
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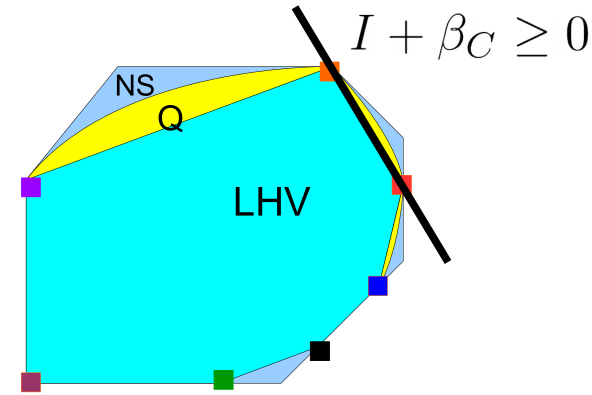
[N. Schuch, J. I. Cirac, Phys. Rev. A. **82**, 012314 (2010)]



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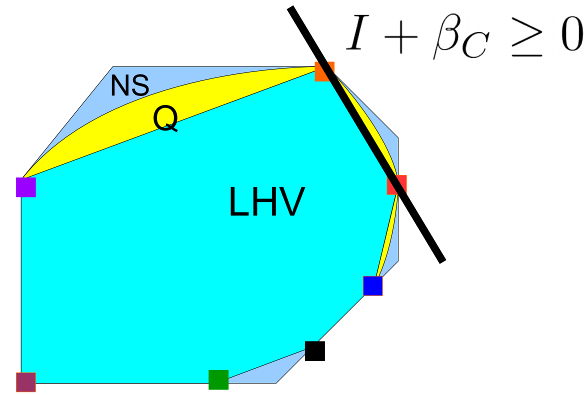

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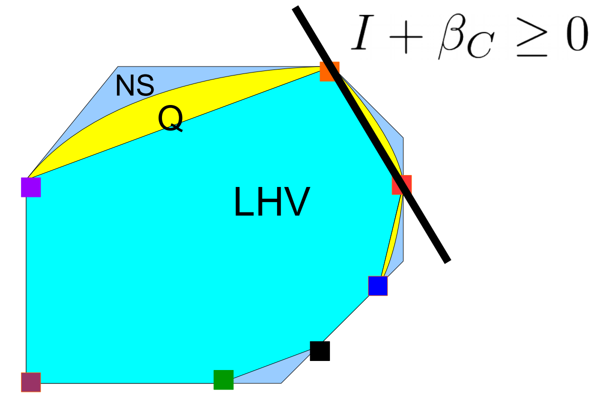

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Ingredients

•Recurrence relation

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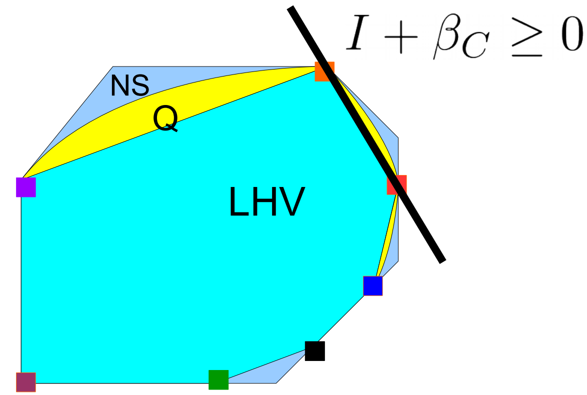

Ingredients
•Recurrence relation
•Compute & store intermediate sub-solutions

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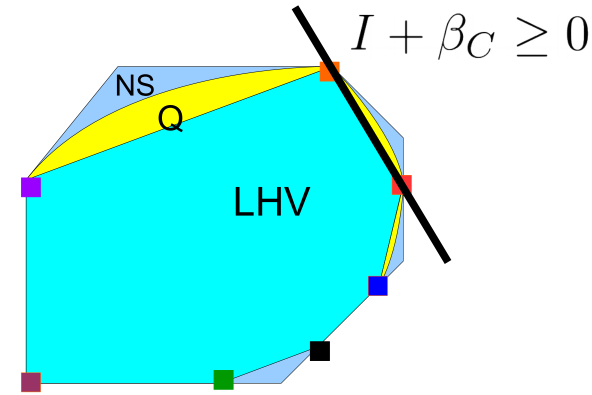

Ingredients
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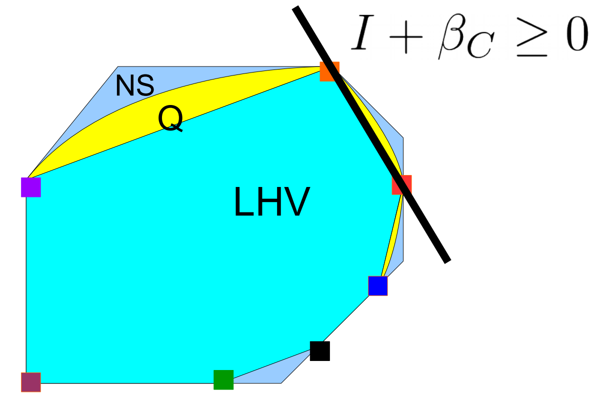

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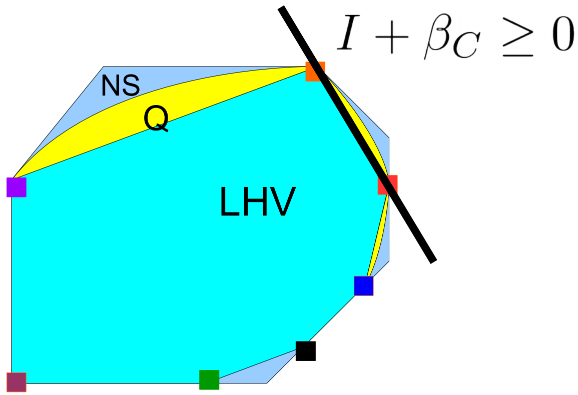

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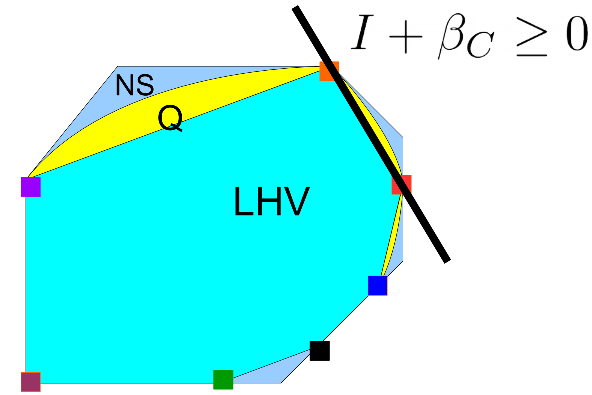

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Ingredients
•Recurrence relation
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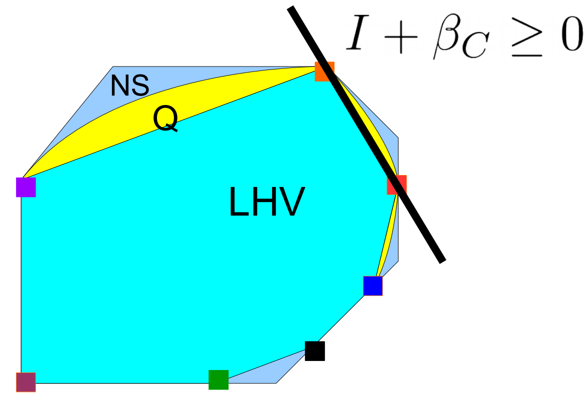
•Ordering of sub-solutions

Result
•Polynomial scaling

Finding the classical bound



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Ingredients
•Recurrence relation
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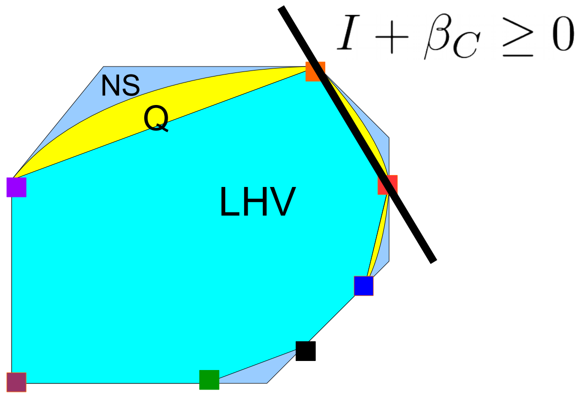
Result
•Polynomial scaling
•Constructive method of 1 optimal solution

Finding the classical bound



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Ingredients
•Recurrence relation
•Compute & store intermediate sub-solutions
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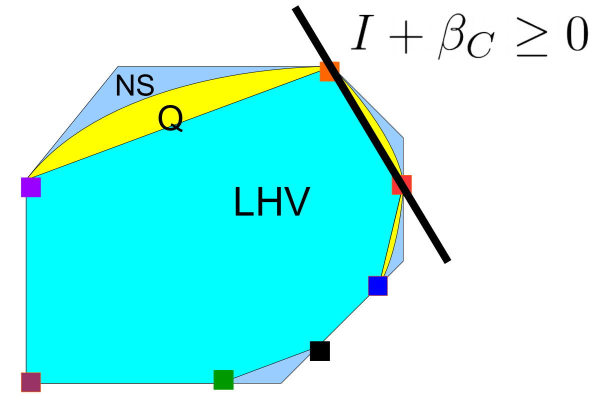
Result
•Polynomial scaling
•Constructive method of 1 optimal solution
•Much better than backtracking/brute force

Finding the classical bound



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# of problems by category at [TopCoder]

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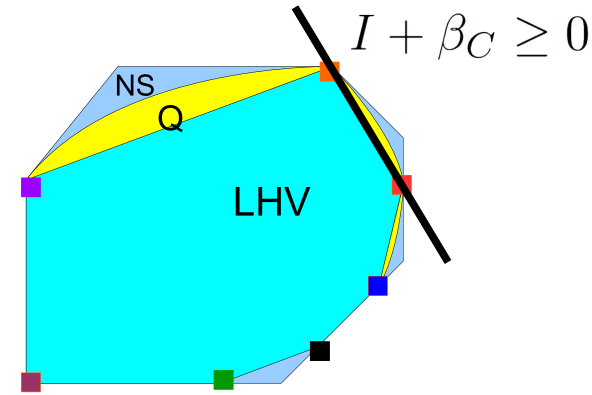

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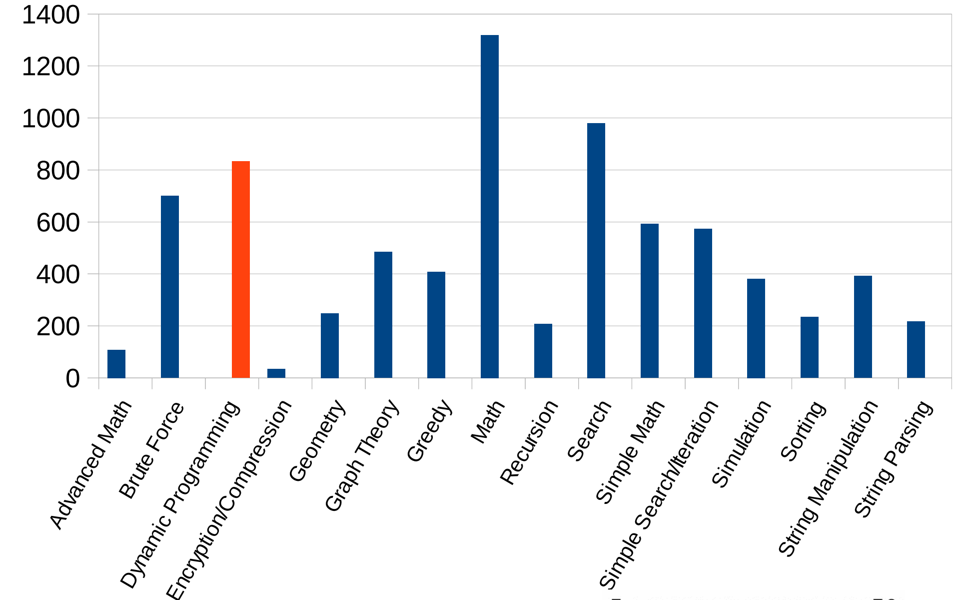
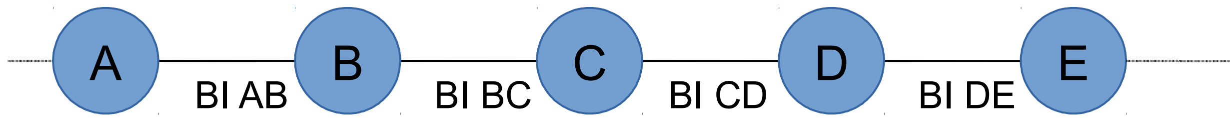


# of problems by category at [TopCoder]®

Dynamic programming

Dynamic programming

- The Bell Inequality as a sum of smaller BI



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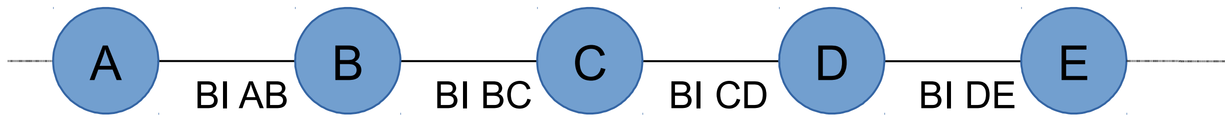

- The optimization

Dynamic programming

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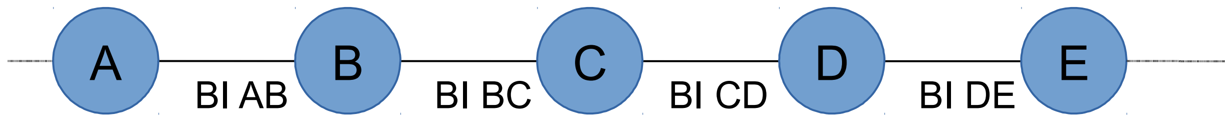

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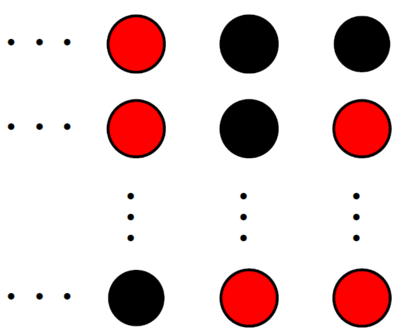


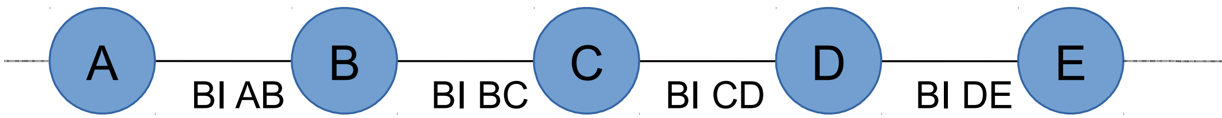
0

1

$m-1$

Dynamic programming

- The Bell Inequality as a sum of smaller BI



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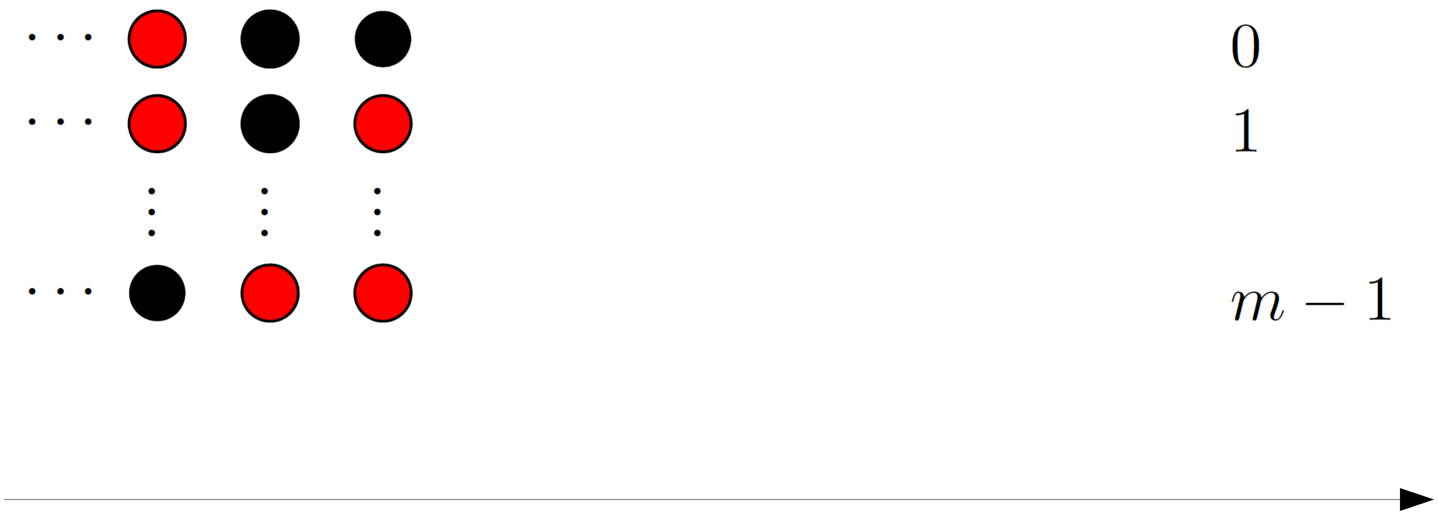

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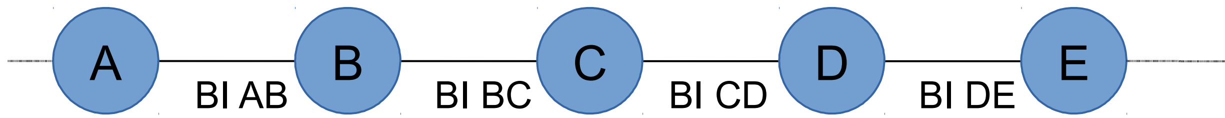

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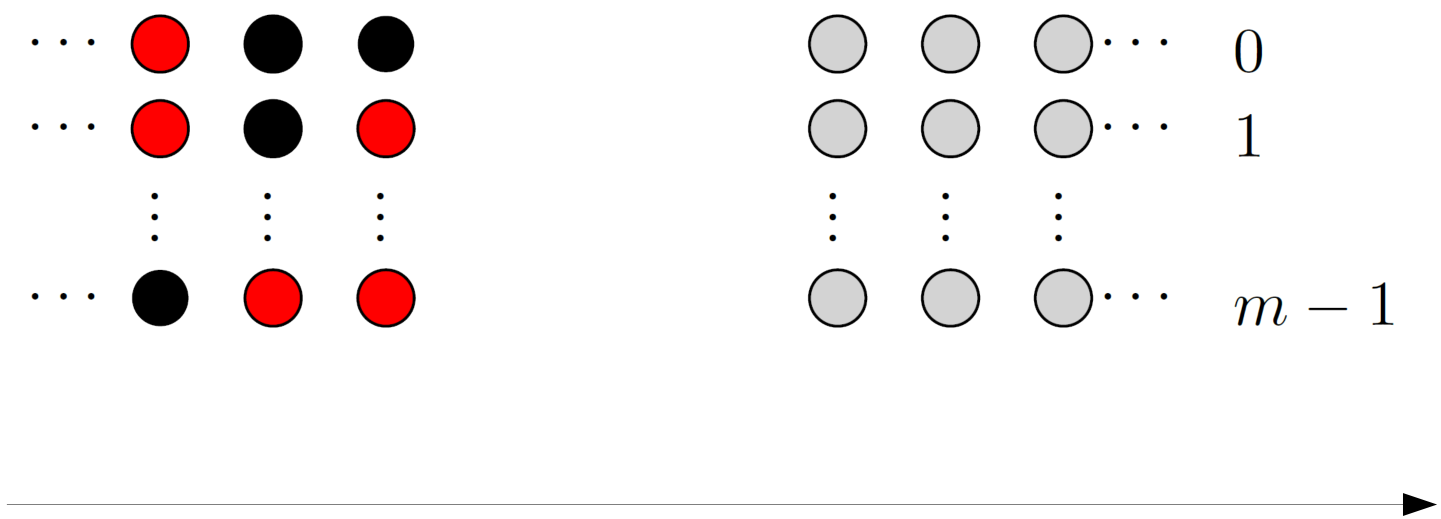

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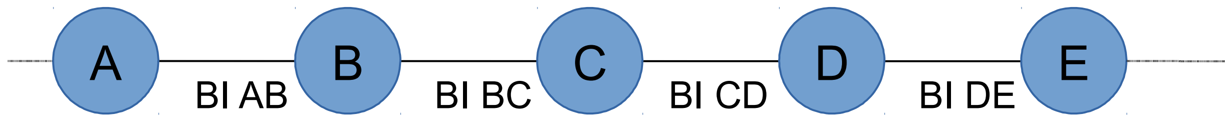

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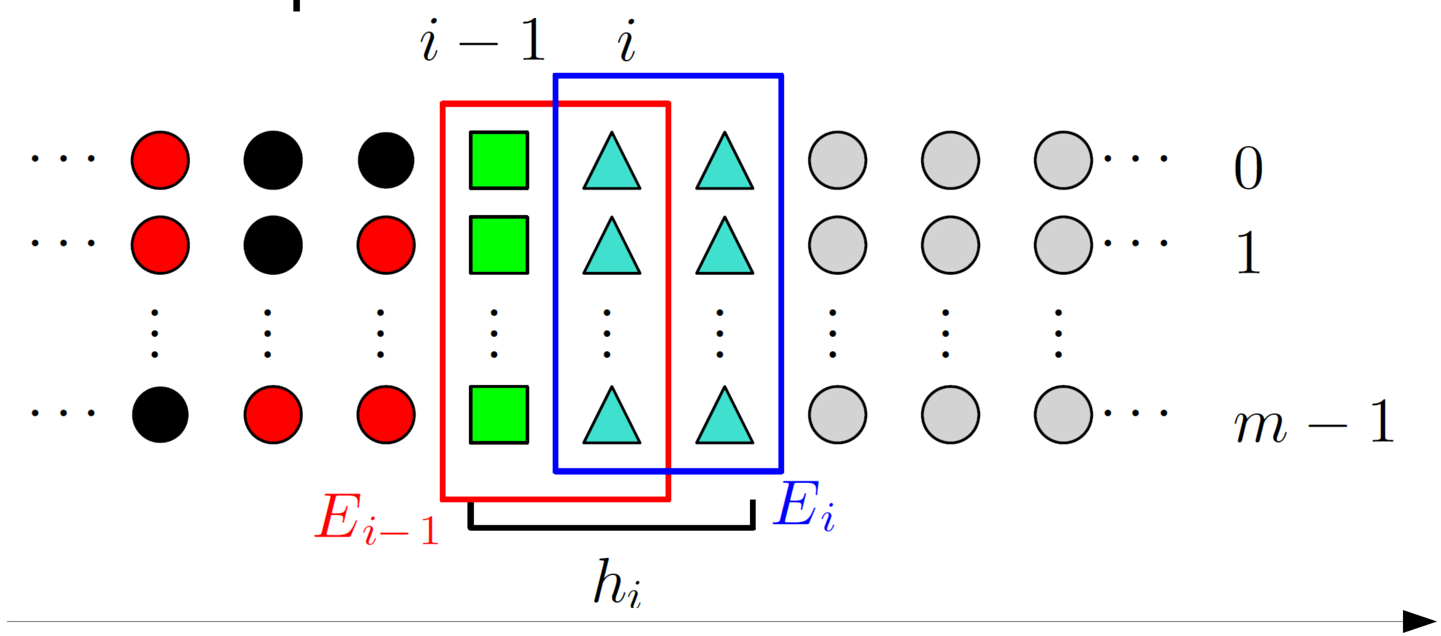

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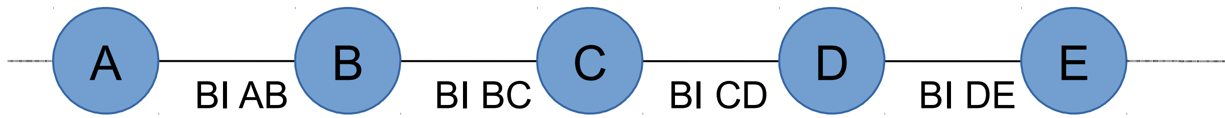

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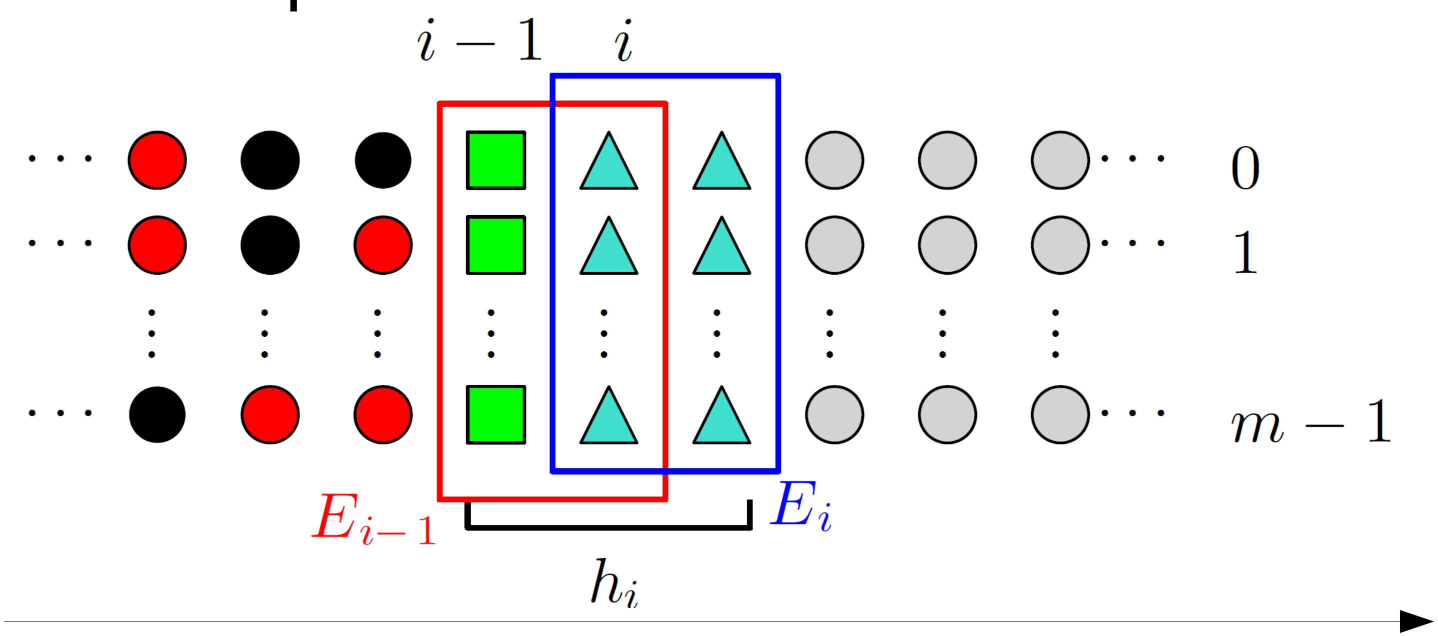
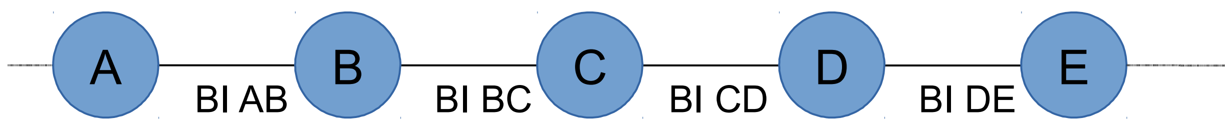


$$E_i(\triangle,\triangle) = \min_{\square} E_{i-1}(\square,\triangle) + h_i(\square,\triangle,\triangle)$$

Dynamic programming

- The Bell Inequality as a sum of smaller BI



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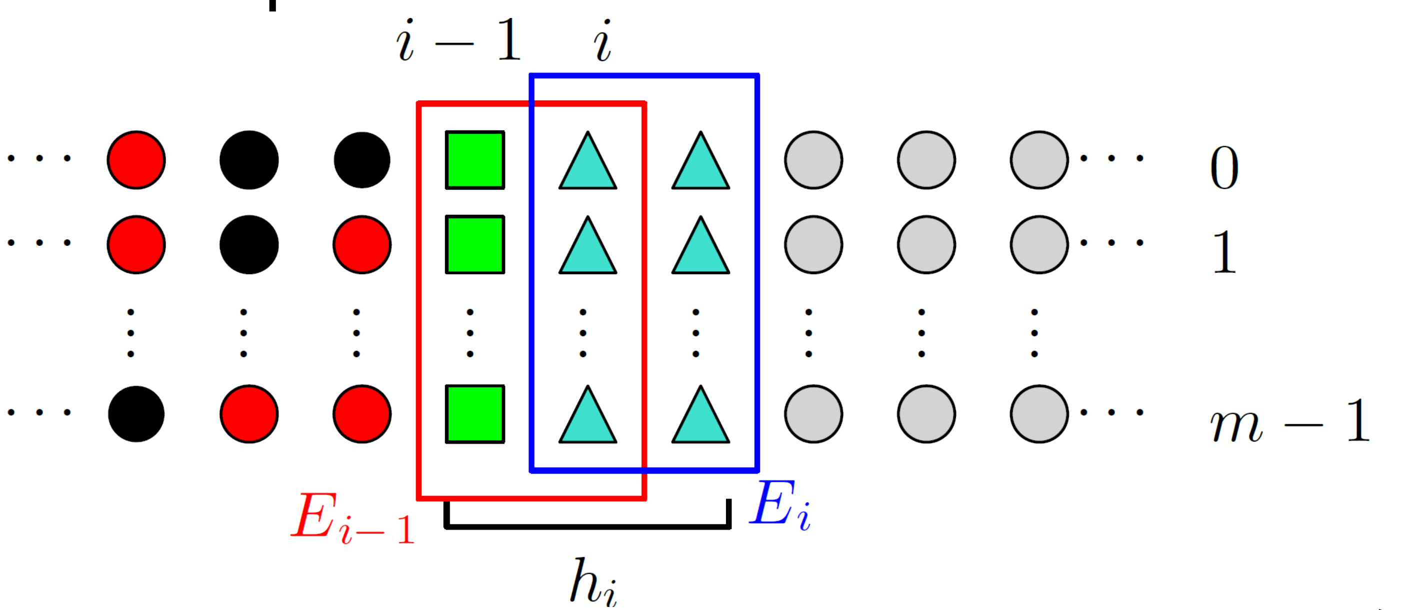


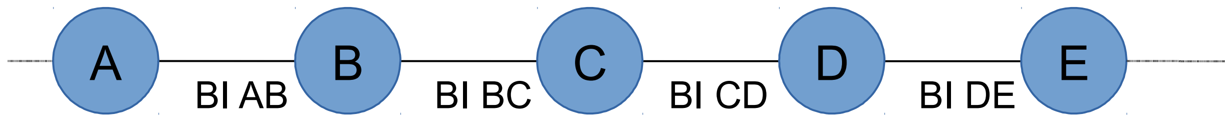
$$E_i(\triangle,\triangle) = \min_{\blacksquare} E_{i-1}(\blacksquare,\triangle) + h_i(\blacksquare,\triangle,\triangle)$$

Classical bound at

$$\beta_C := E_n$$

Dynamic programming

- The Bell Inequality as a sum of smaller BI



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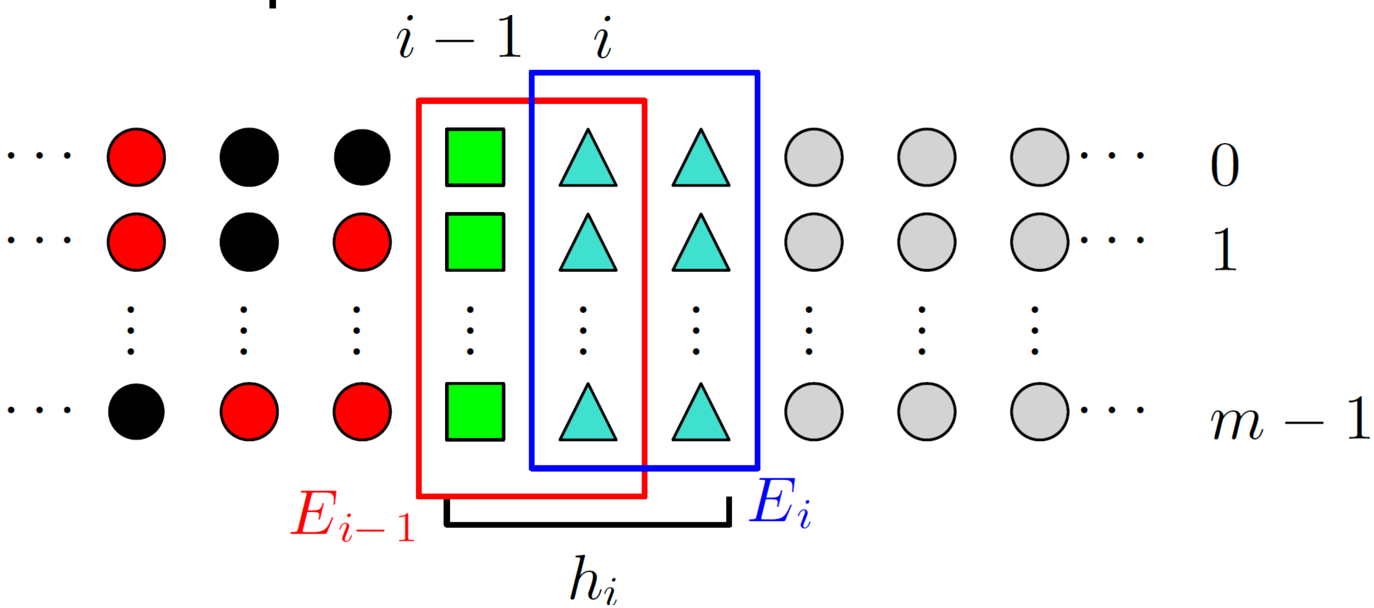

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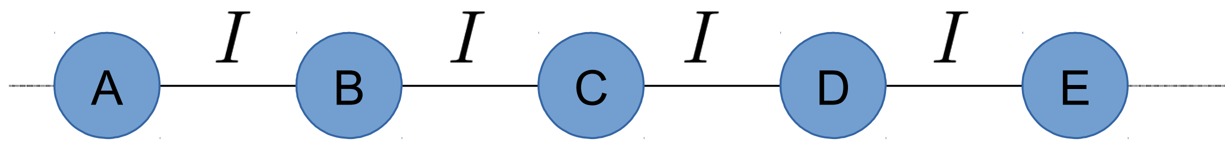
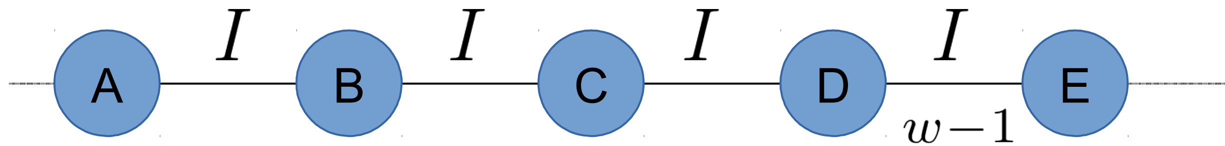
Overall complexity $O(n)$

Translationally invariant BI

Translationally invariant BI

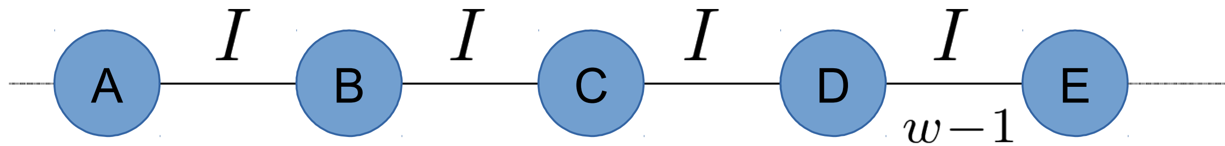


Translationally invariant BI



- **Idea**: Minimize a function $$F = \min_{x_0,\ldots,x_w} \sum_{j=0}^{w-1} f^{(0)}(x_j, x_{j+1})$$

Translationally invariant BI

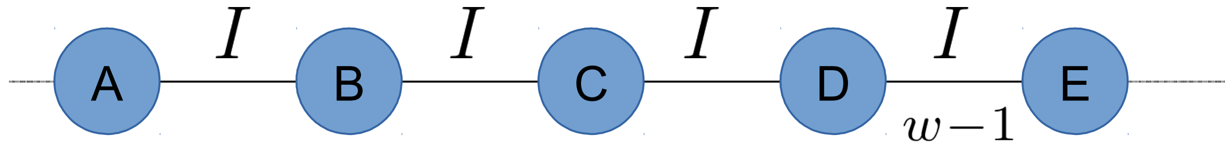


- **Idea**: Minimize a function $$F = \min_{x_0,\ldots,x_w} \sum_{j=0}^{w-1} f^{(0)}(x_j, x_{j+1})$$

  by eliminating half of the variables at each step

$$f^{(t+1)}(x,y) = \min_z (f^{(t)}(x,z) + f^{(t)}(z,y))$$

Translationally invariant BI



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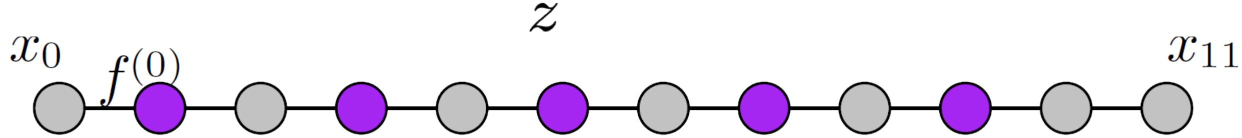

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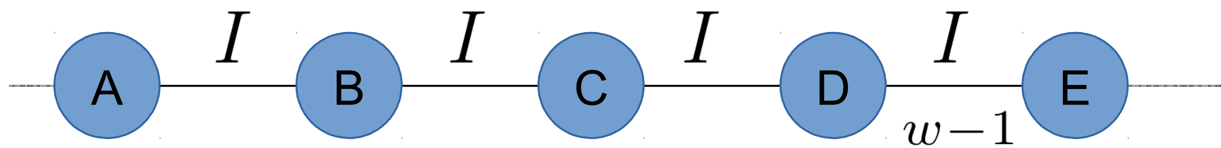

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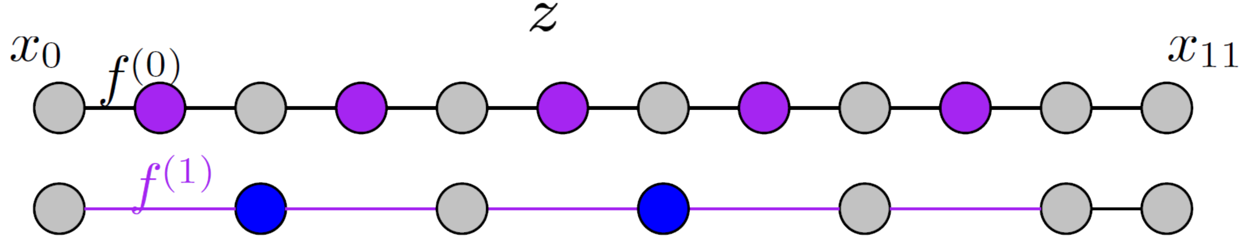

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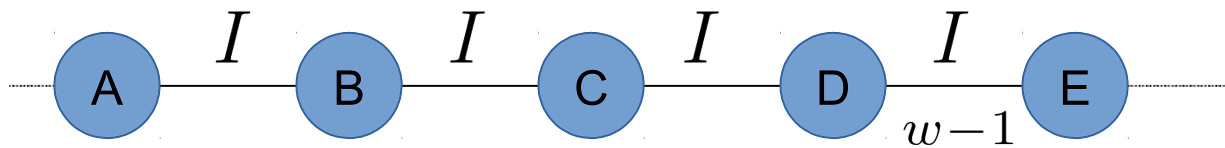

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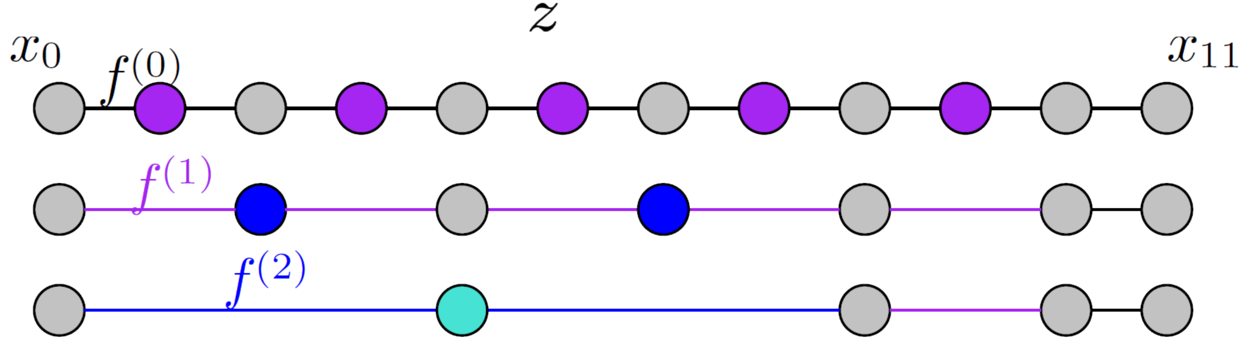

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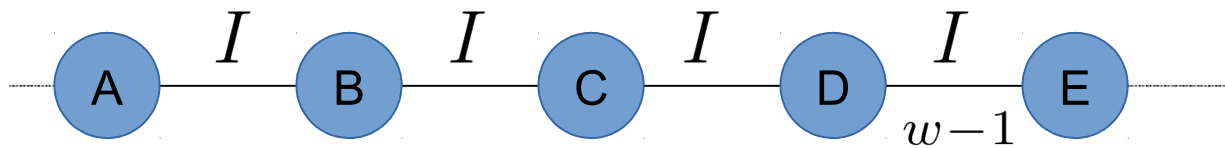

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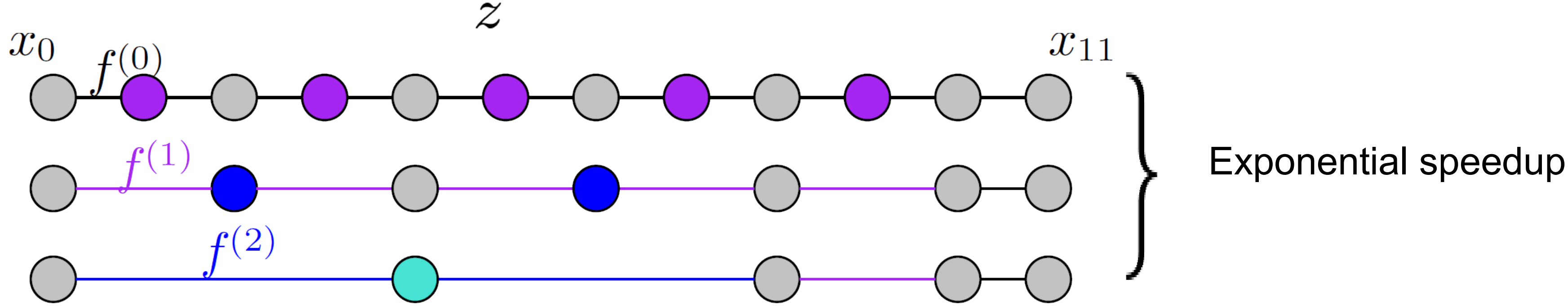

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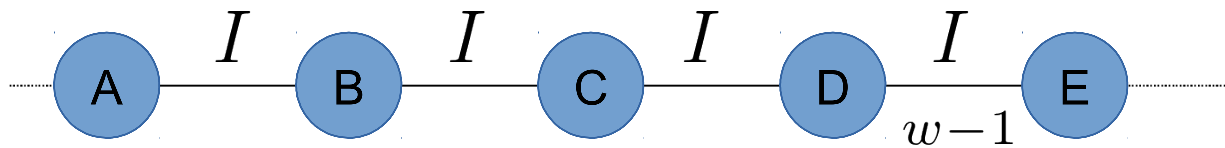

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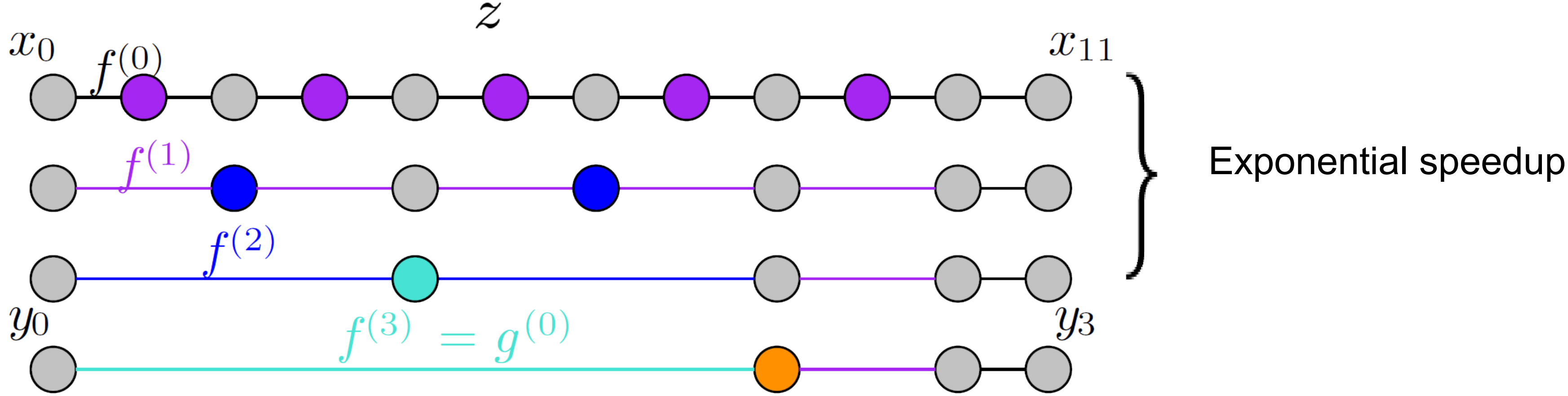

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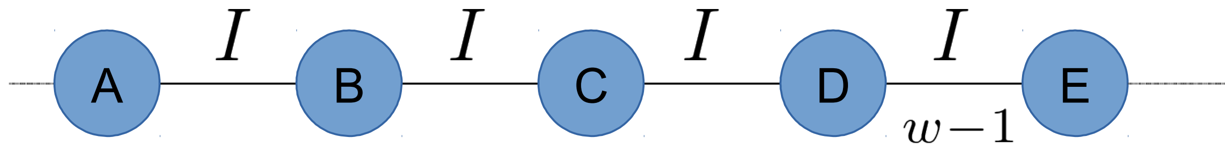

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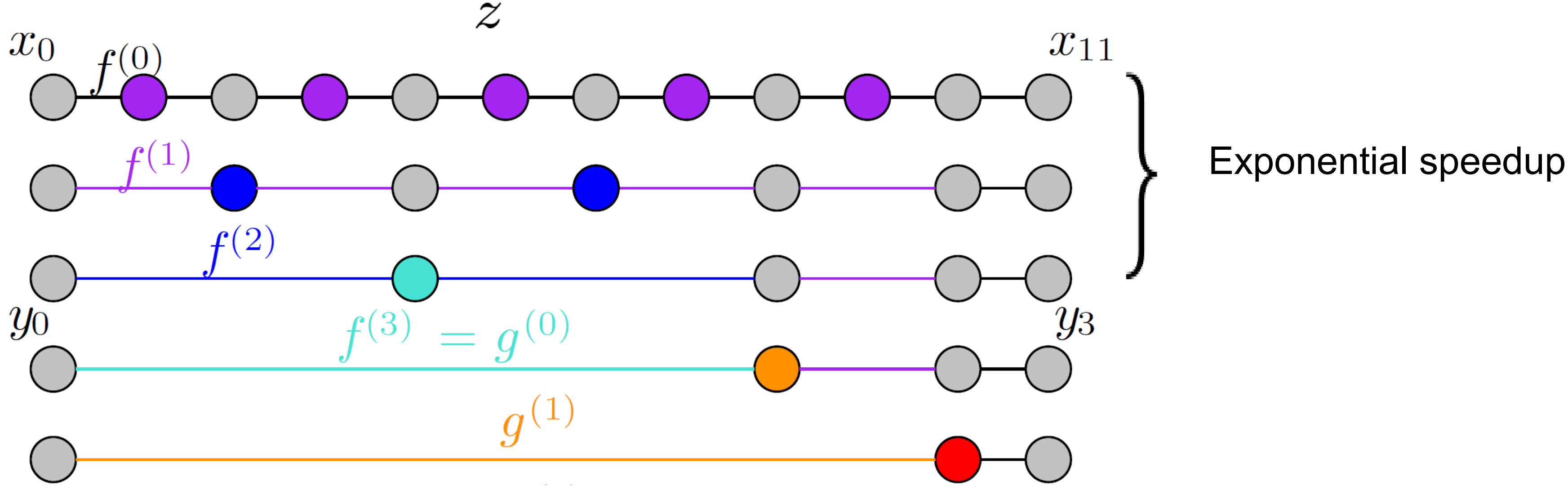

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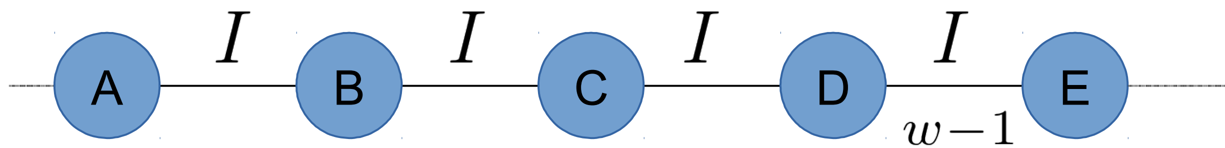

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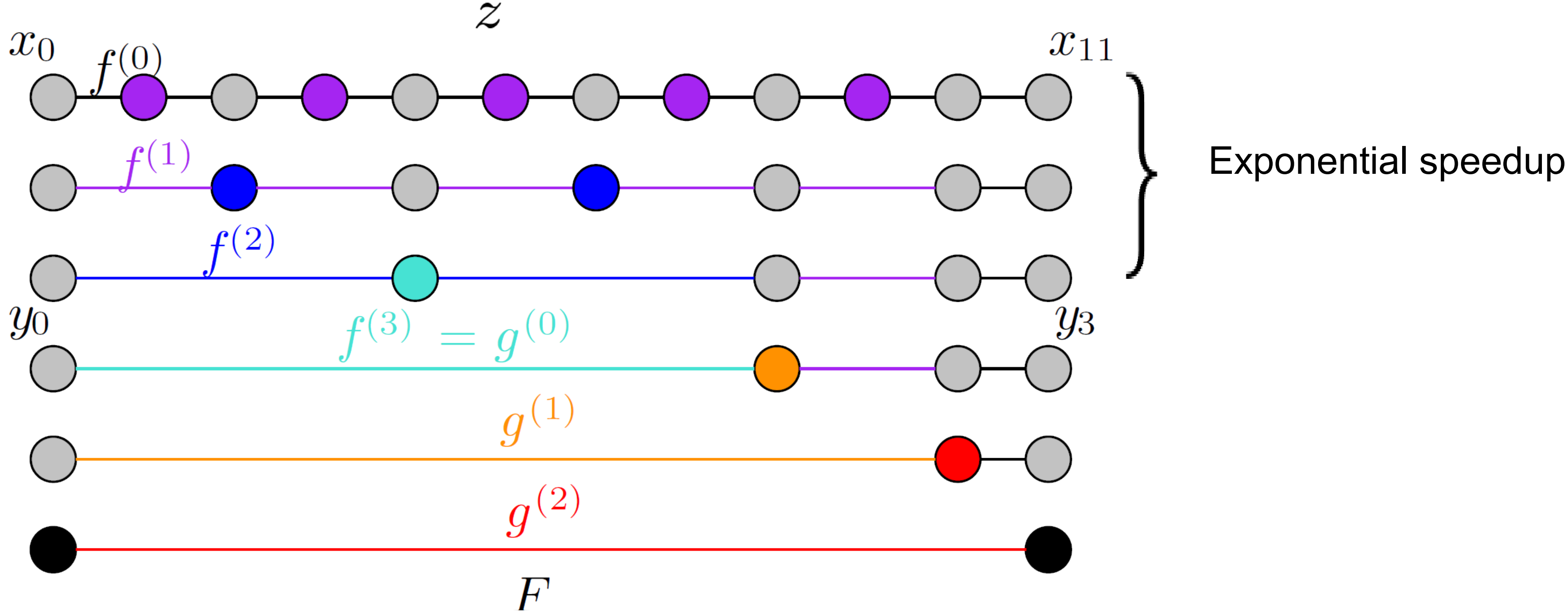

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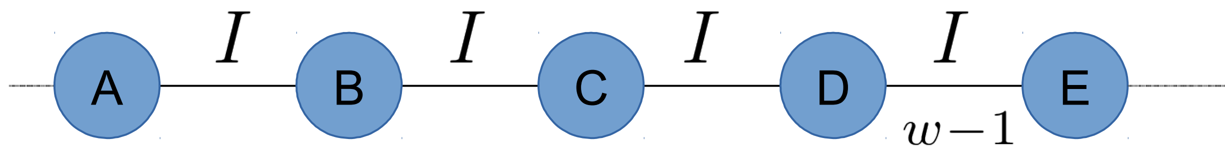

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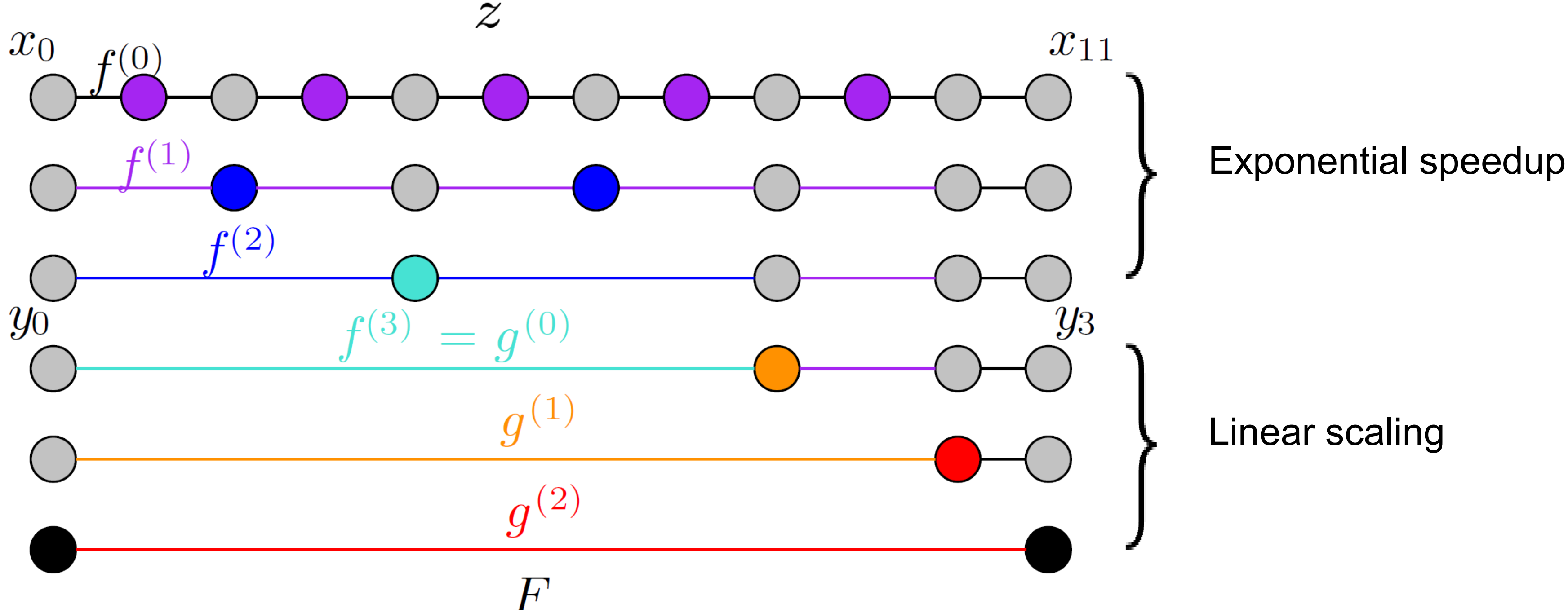

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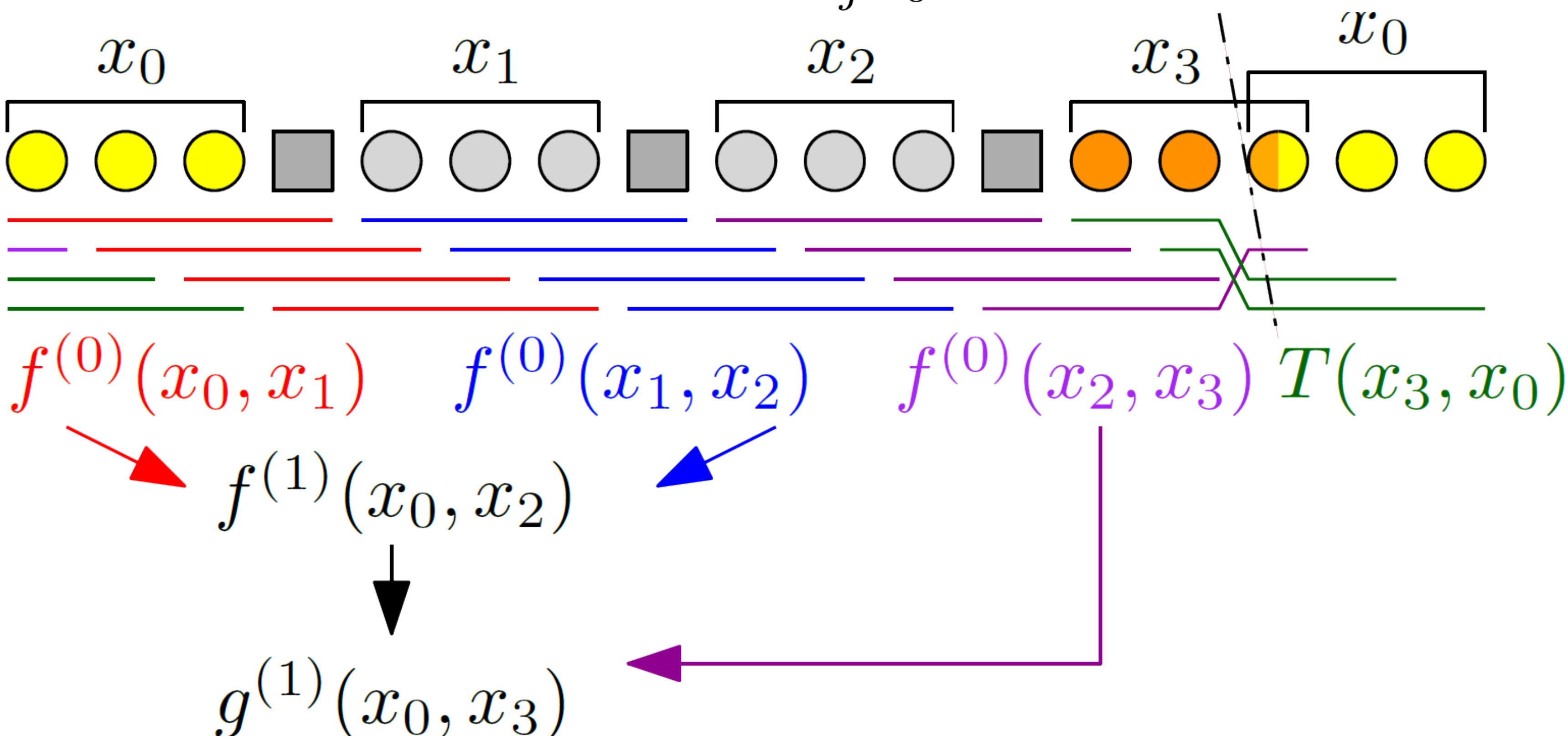

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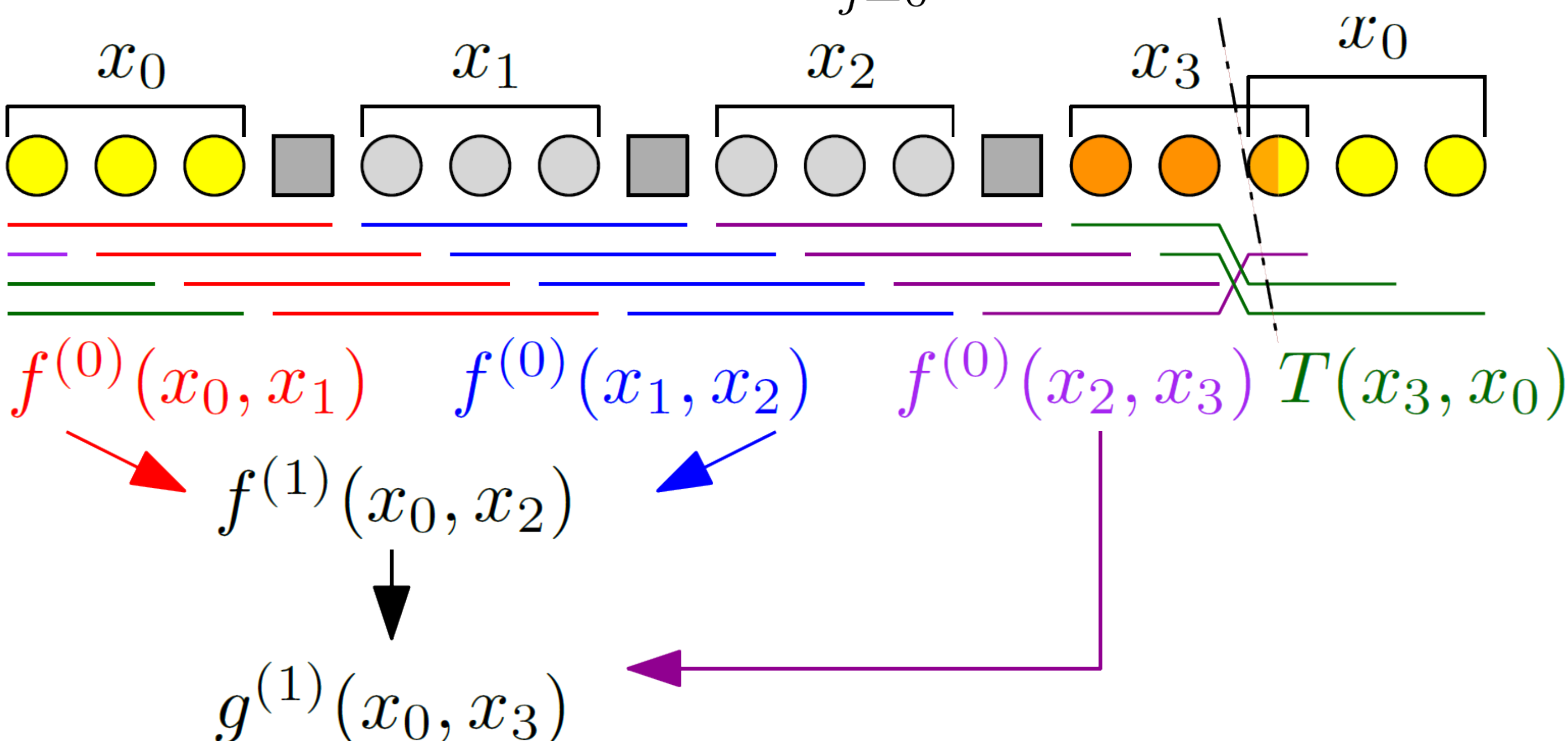

$O(\log n)$ overall complexity

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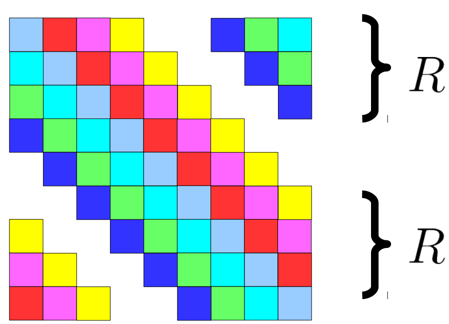

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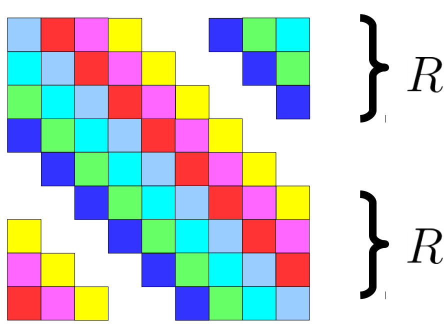

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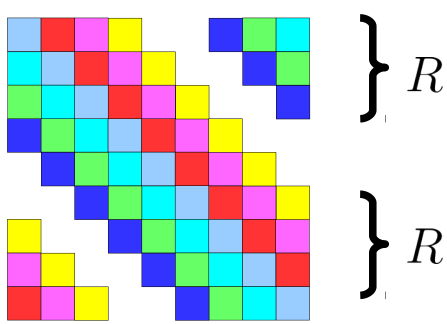

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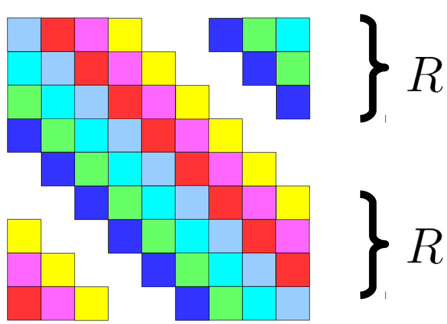

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$$(\mathcal{F}_n)_{kl} := \frac{1}{\sqrt{n}} \omega^{k \cdot l}, \qquad \omega^n = 1$$

Translationally invariant Hamiltonian (II)

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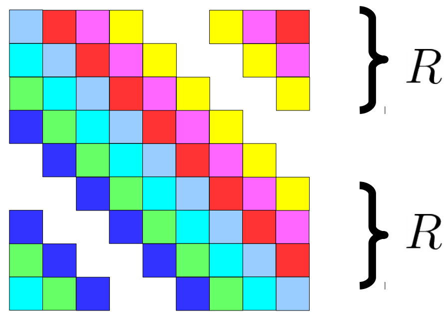

If the fermion system has parity 1
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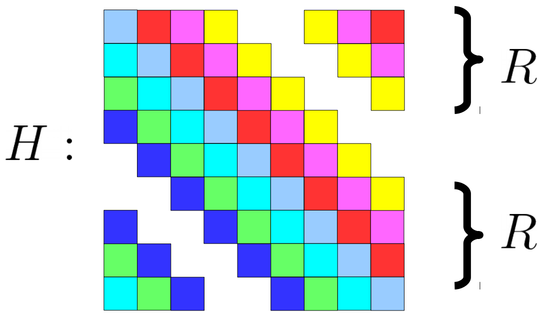

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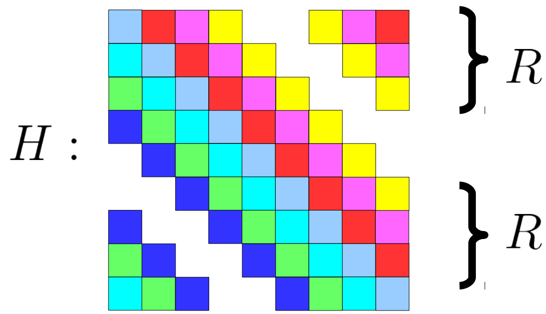

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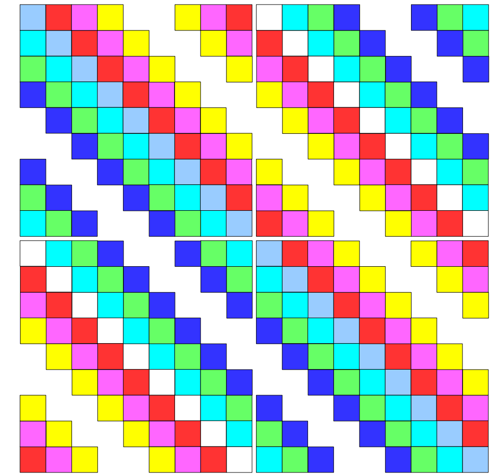

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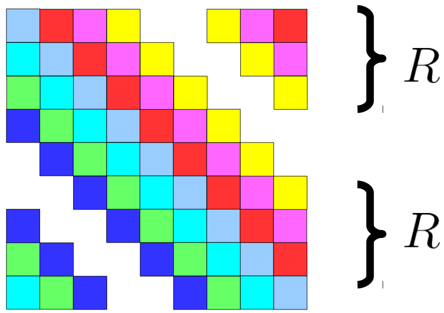

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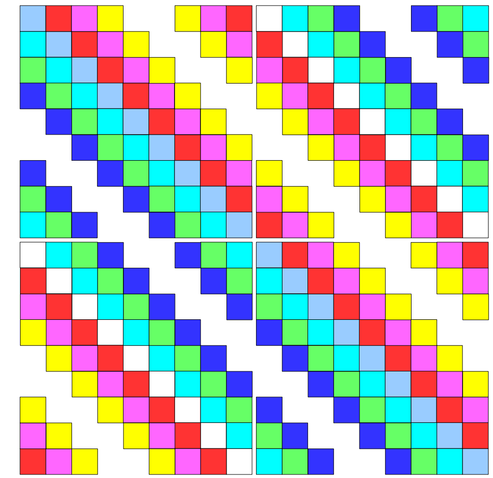

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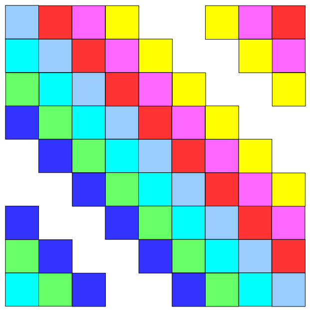

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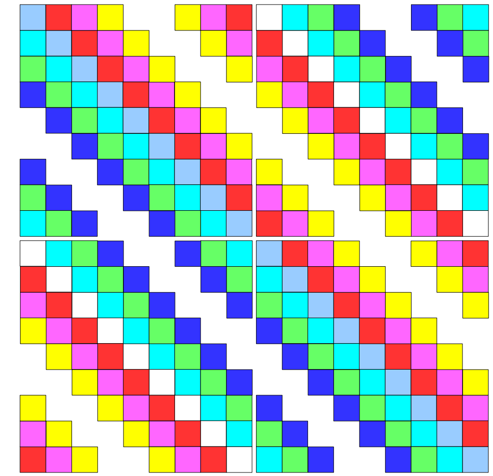

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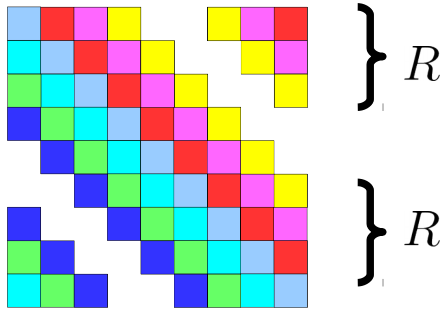

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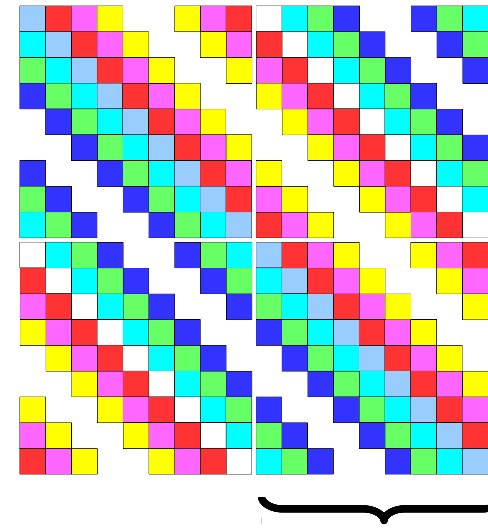

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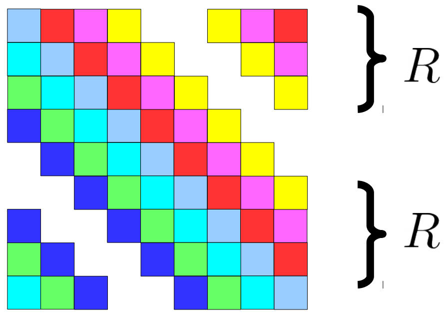

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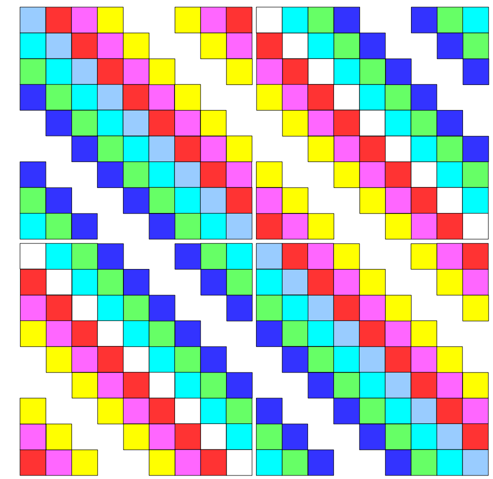

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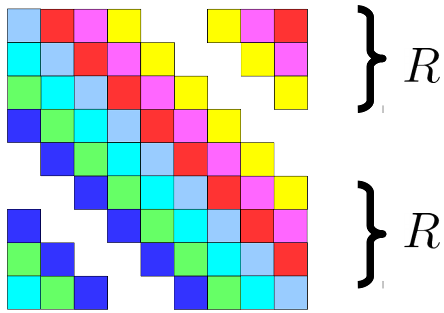

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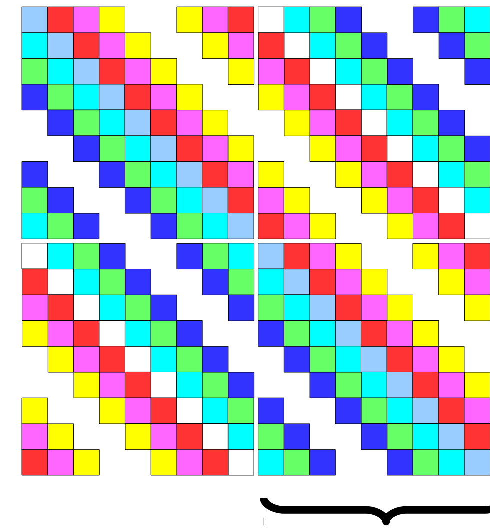

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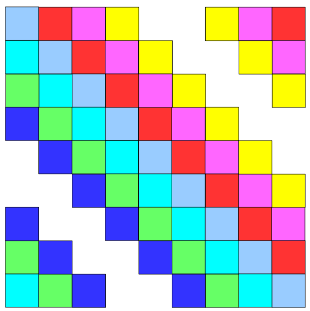
$H:$  $\} R$ $\} R$

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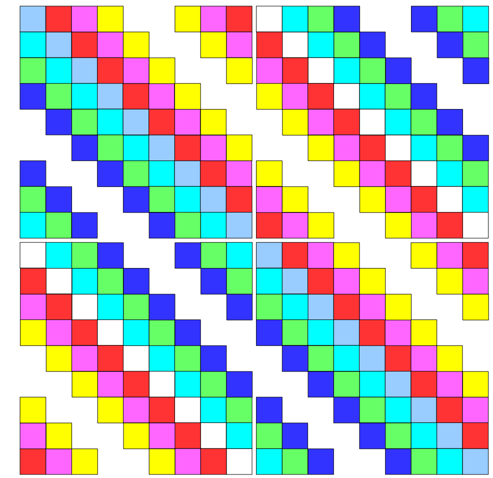

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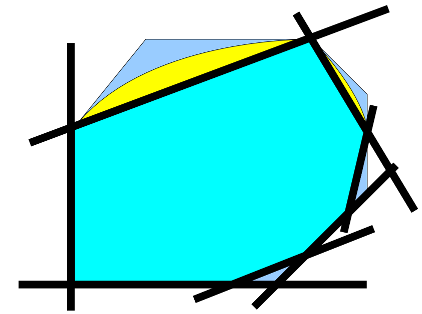

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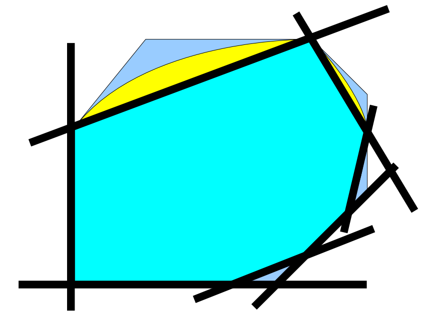
(n, m, d) scenario



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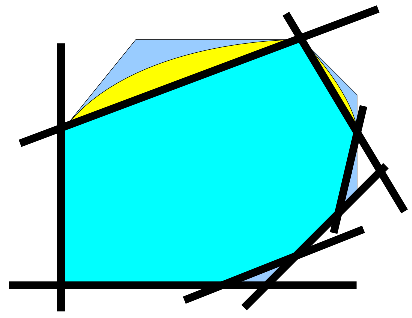
Dimension of the Local Polytope $D \approx (md)^n$



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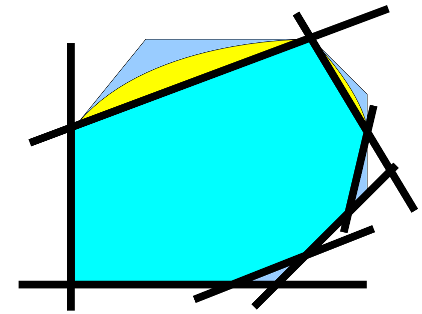

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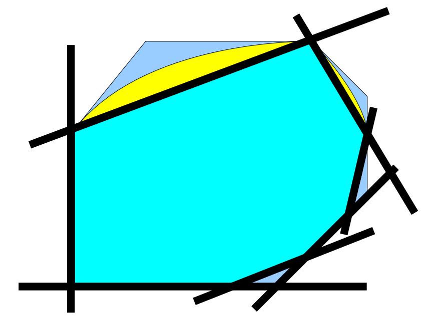

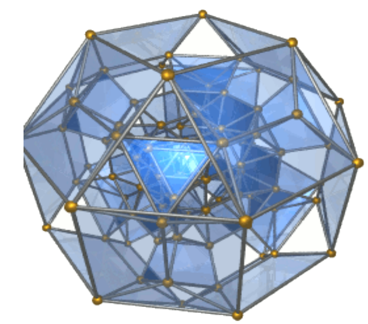

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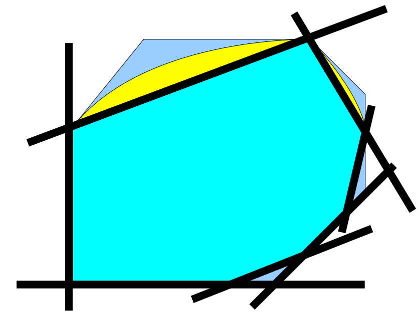

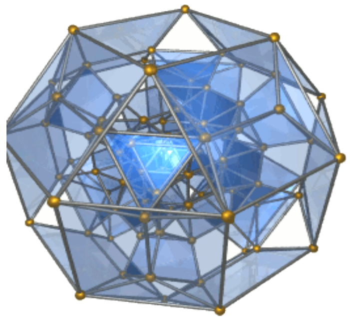

Dimension of the Local Polytope $D \approx (md)^n$

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Complexity of dual description: $O(v^{\lfloor D/2 \rfloor} + v \log v)$

[B. Chazelle, *An optimal convex hull algorithm in any fixed dimension, Discrete Comput. Geom.* **10** 377409 (1993)]

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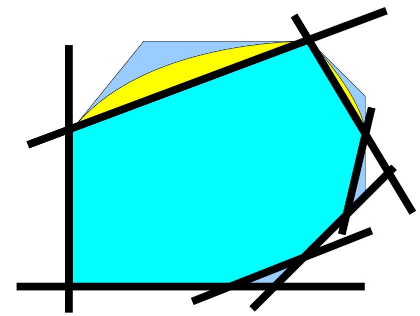

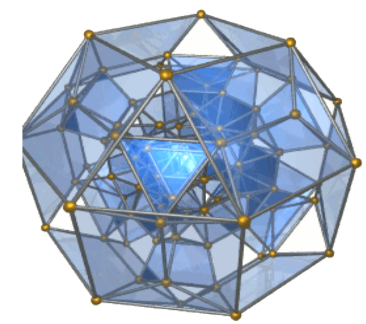

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Examples

[redacted]

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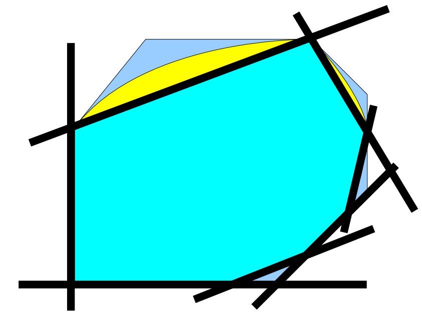

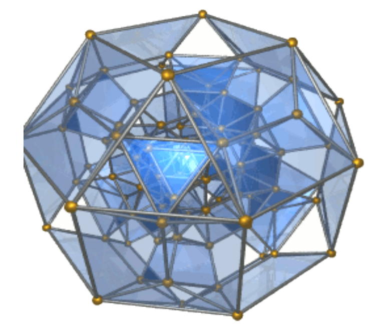

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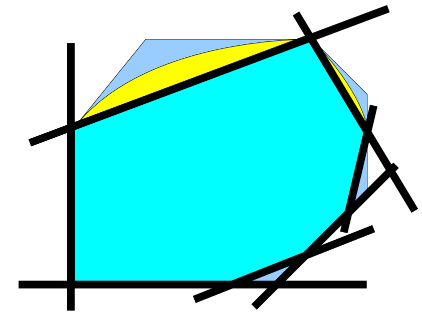

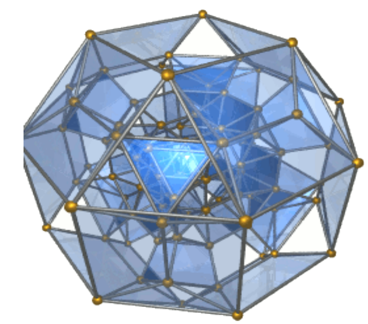

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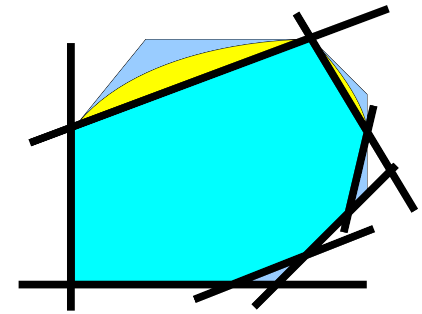

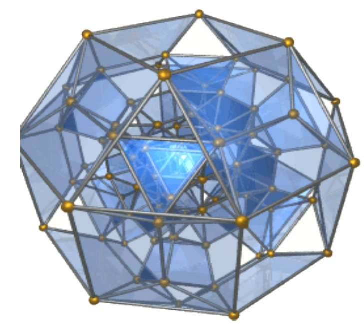

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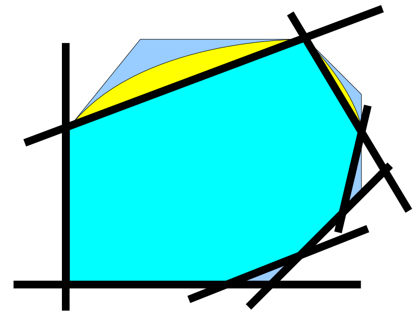

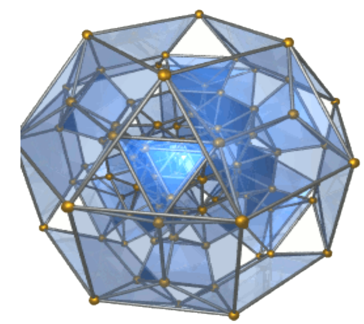

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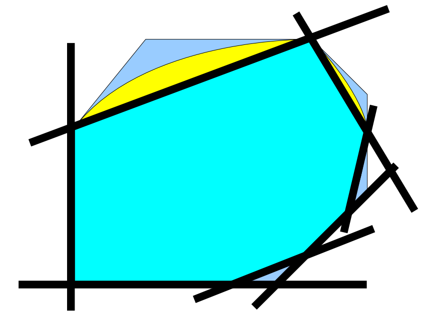

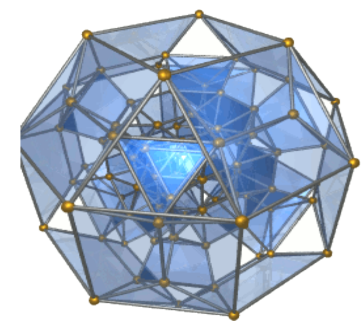

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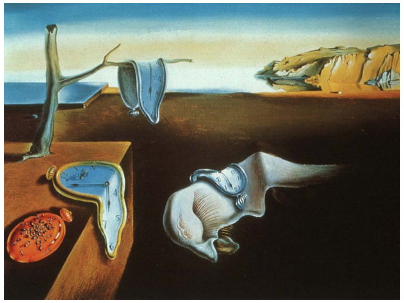
$(10^4, 2, 2) \longrightarrow 10^{10^{10^{4.67867\ldots}}}$ basically any timescale you want



[S. Dalí *The persistence of memory (1931)*]

Examples (Ib)

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- Projecting $\mathbb{P}_L$ to the space of few-body, TI BI

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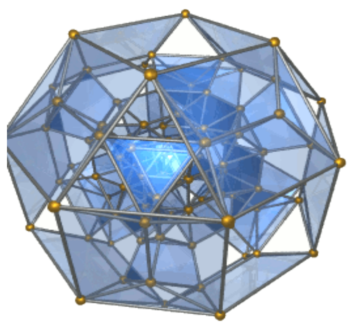

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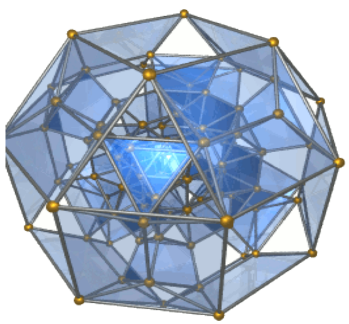

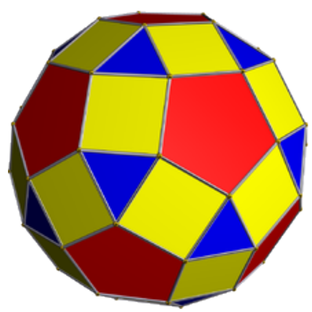

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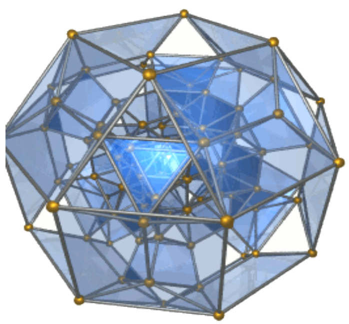
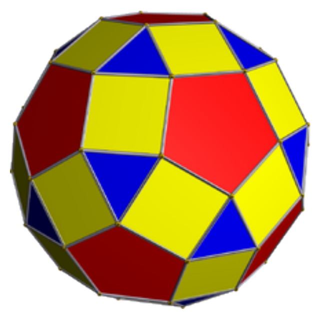
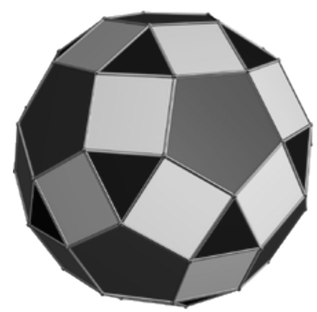

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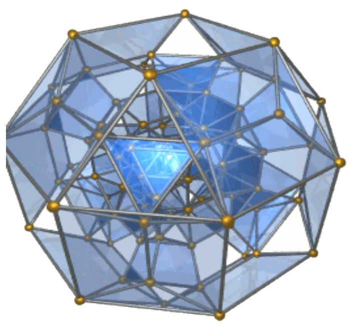
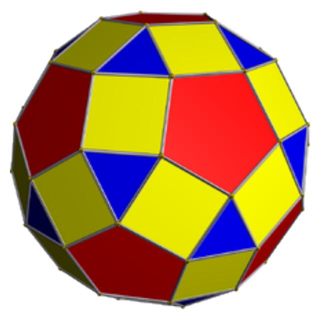
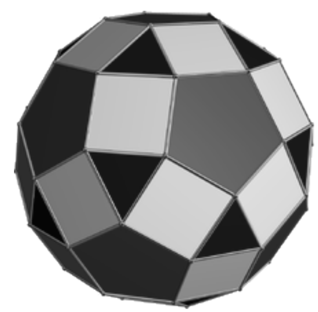
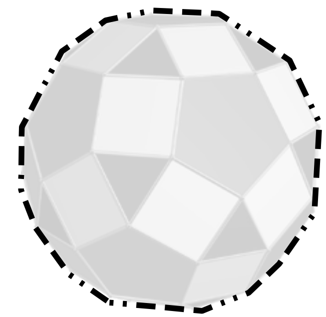

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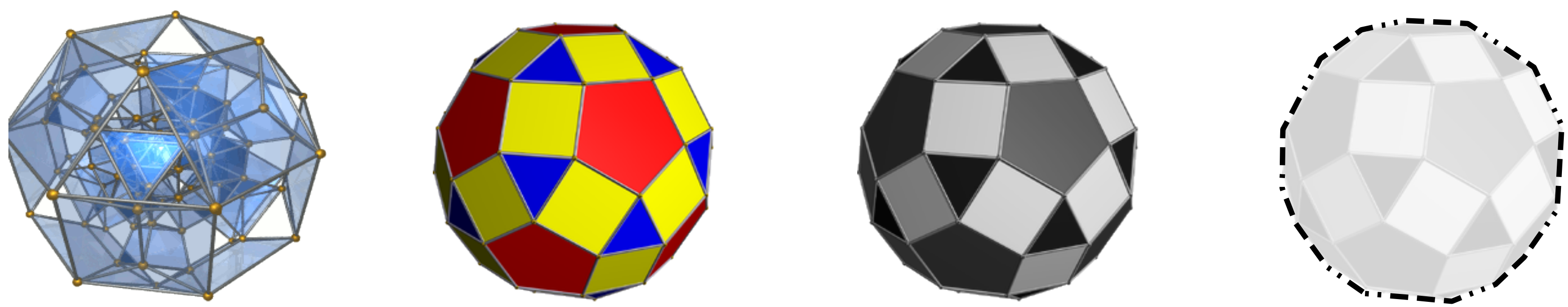

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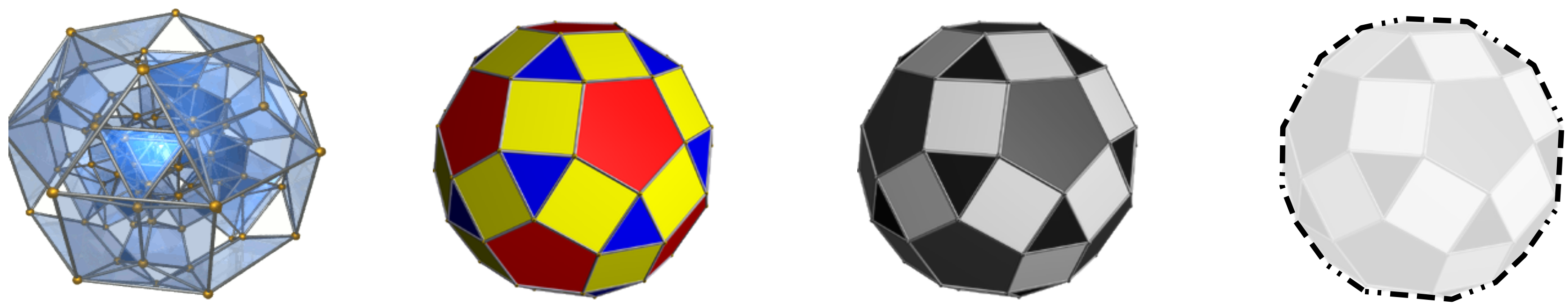

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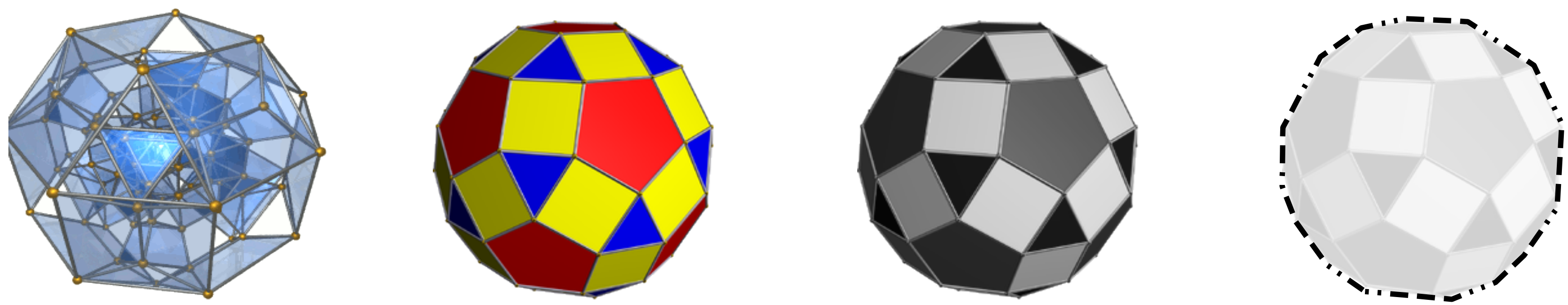

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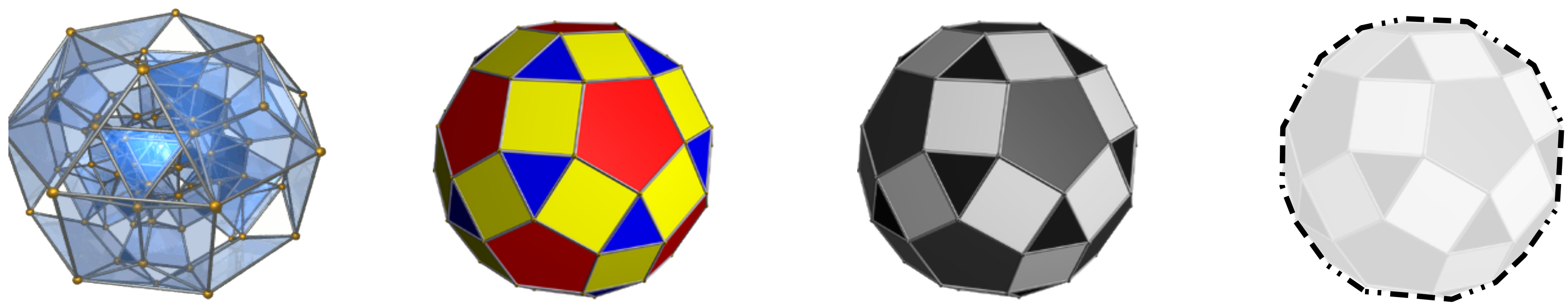

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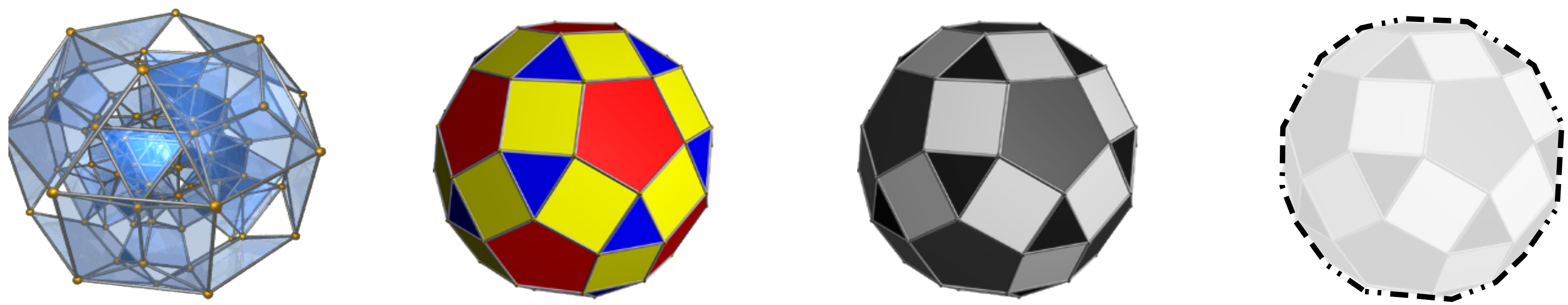

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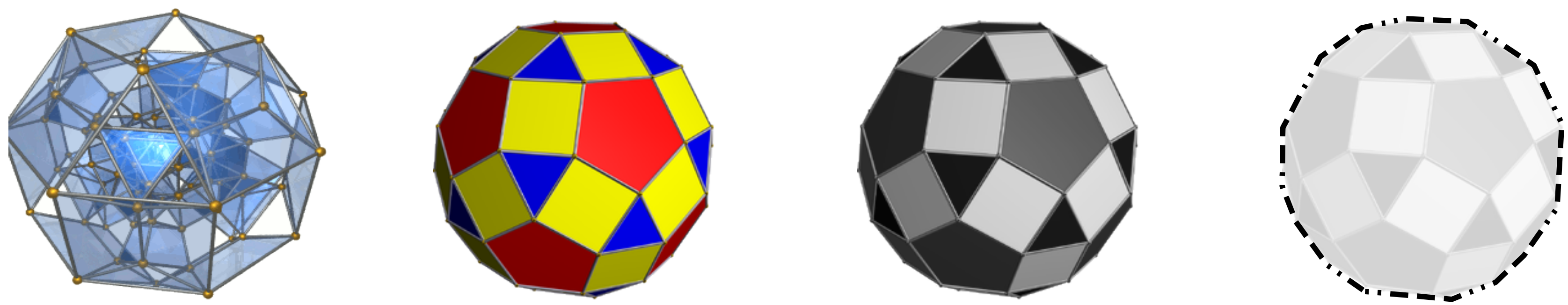

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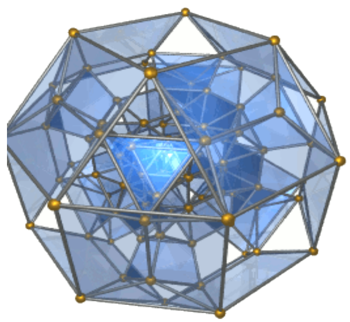
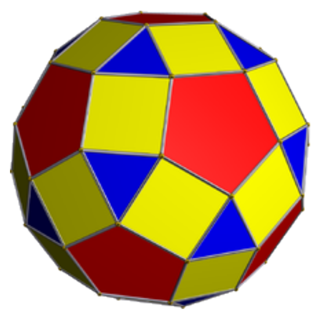
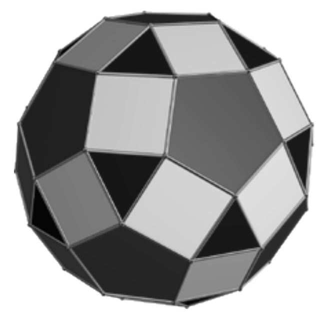
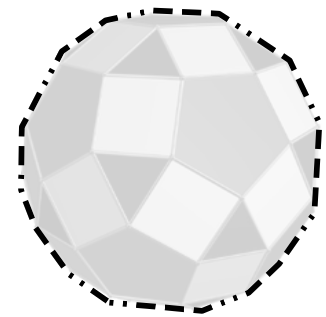

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which exceeds the classical bound, showing that the correlations are nonlocal

Examples (IIa)

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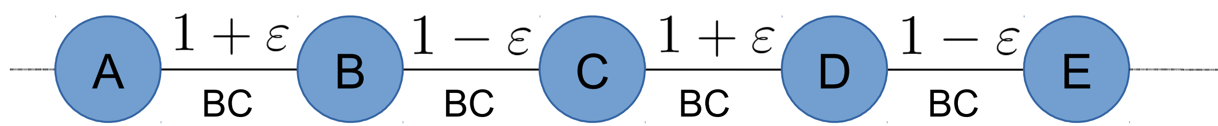

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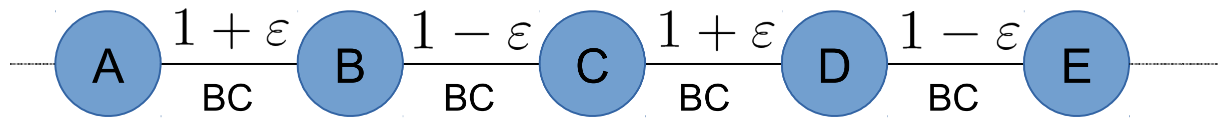

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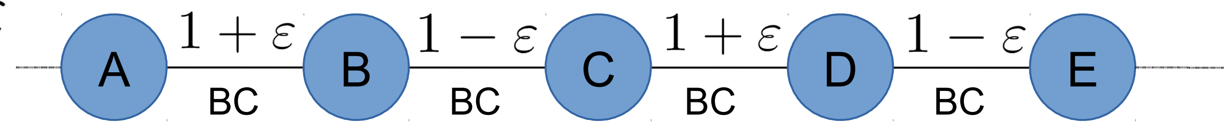

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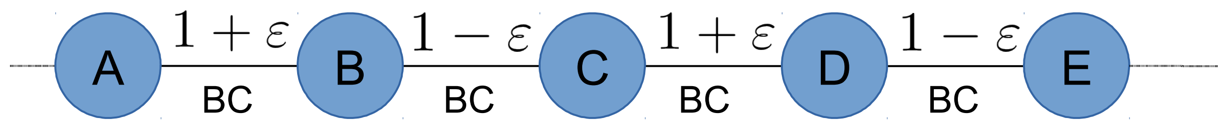

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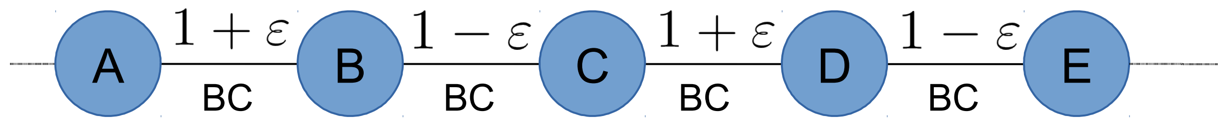

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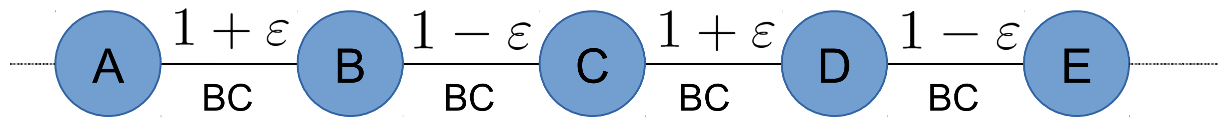

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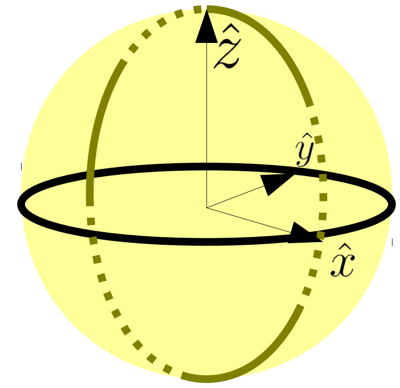

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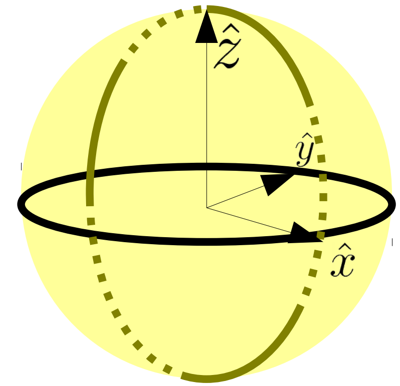

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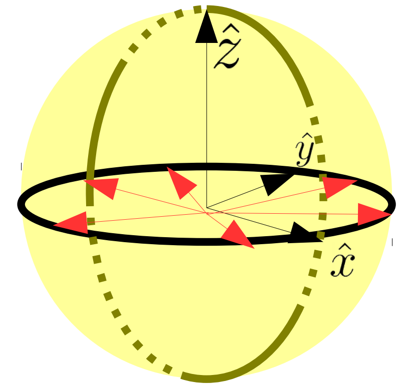

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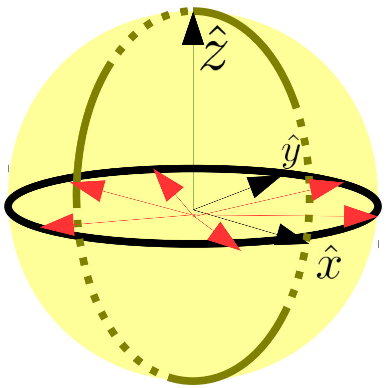

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Asymptotic contributions per particle
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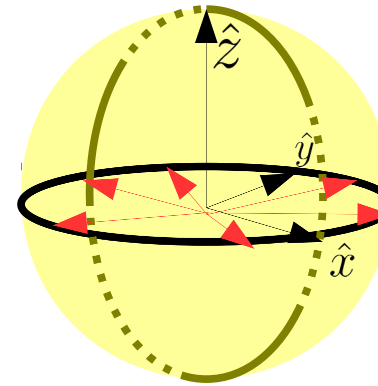

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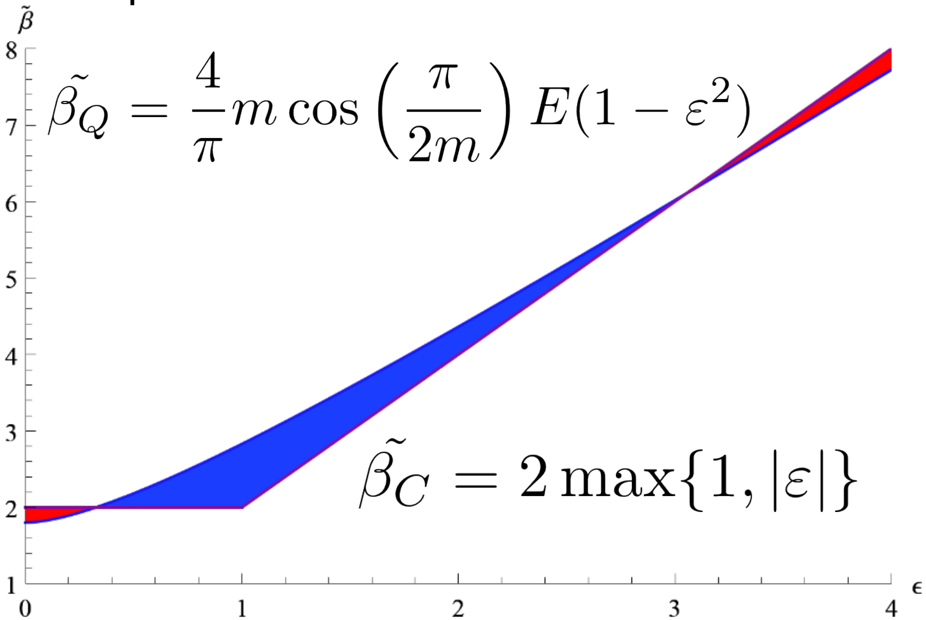

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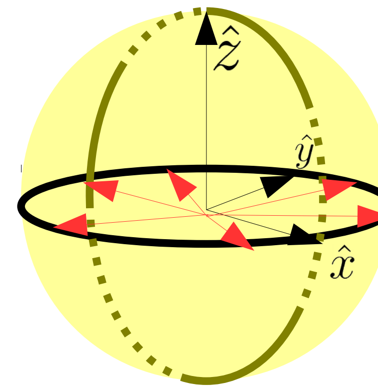

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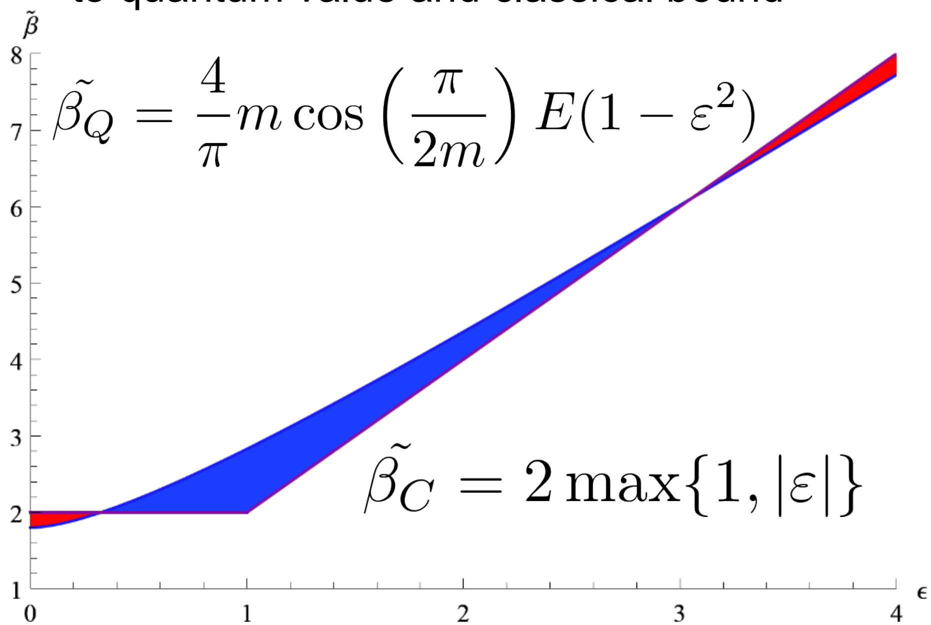

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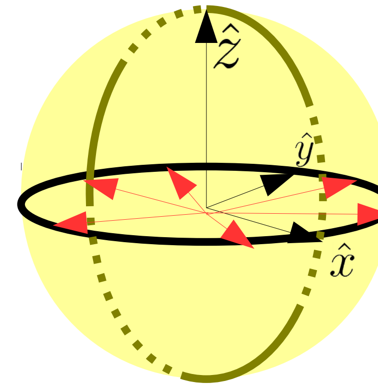

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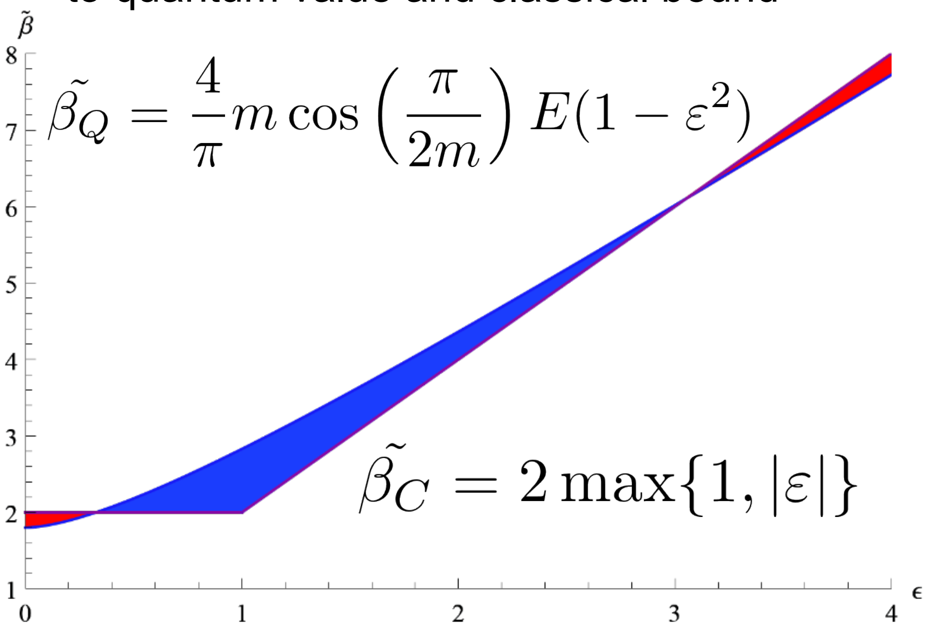
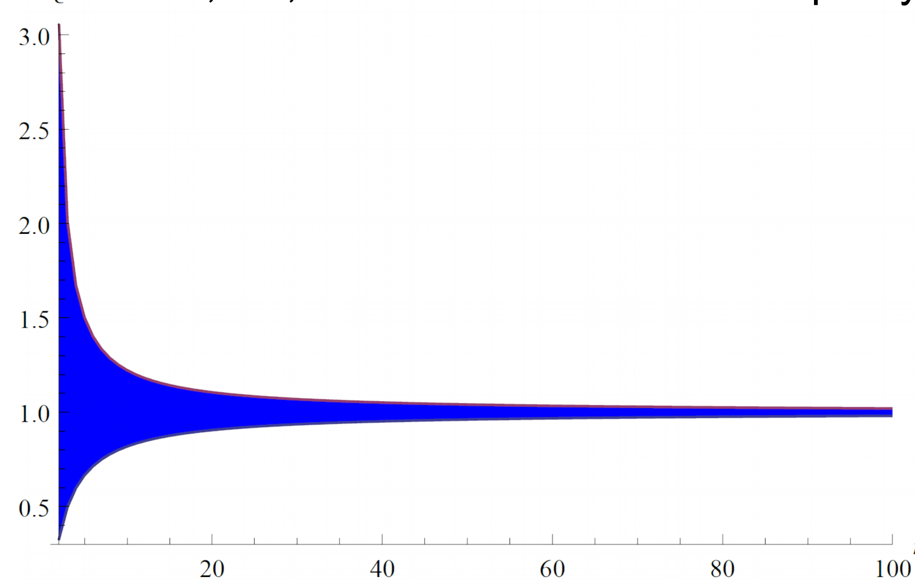


The optimal number of measurements is m=2, i.e., when BC is the CHSH inequality



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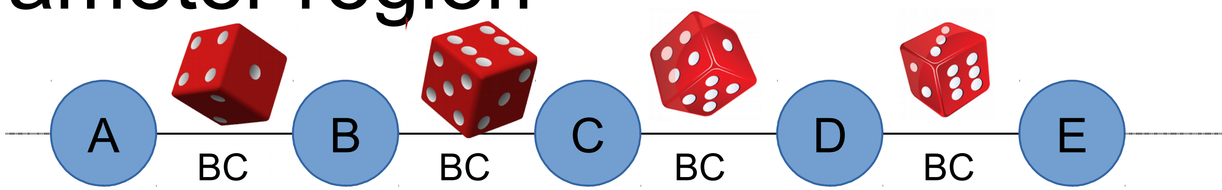

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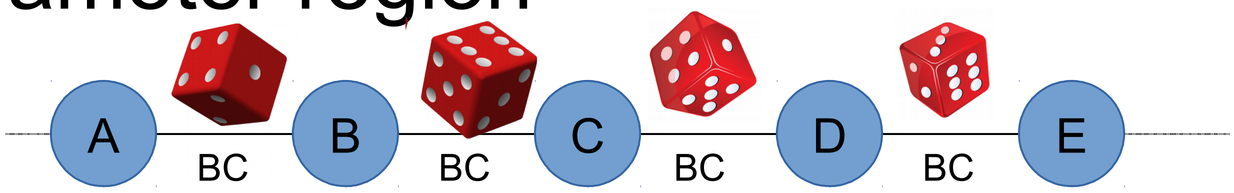

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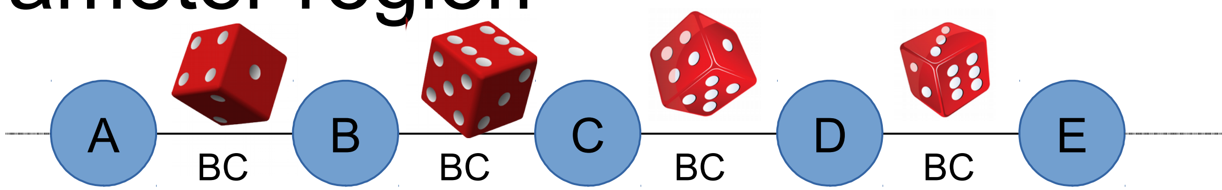

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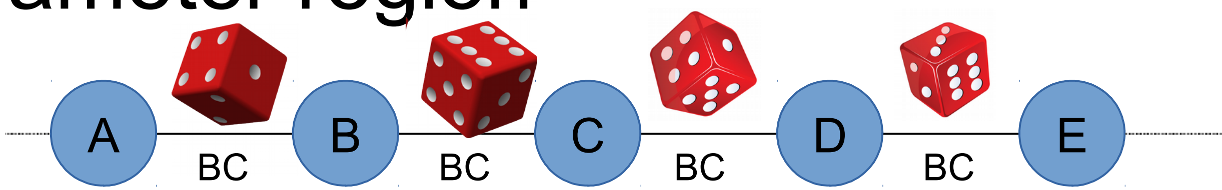

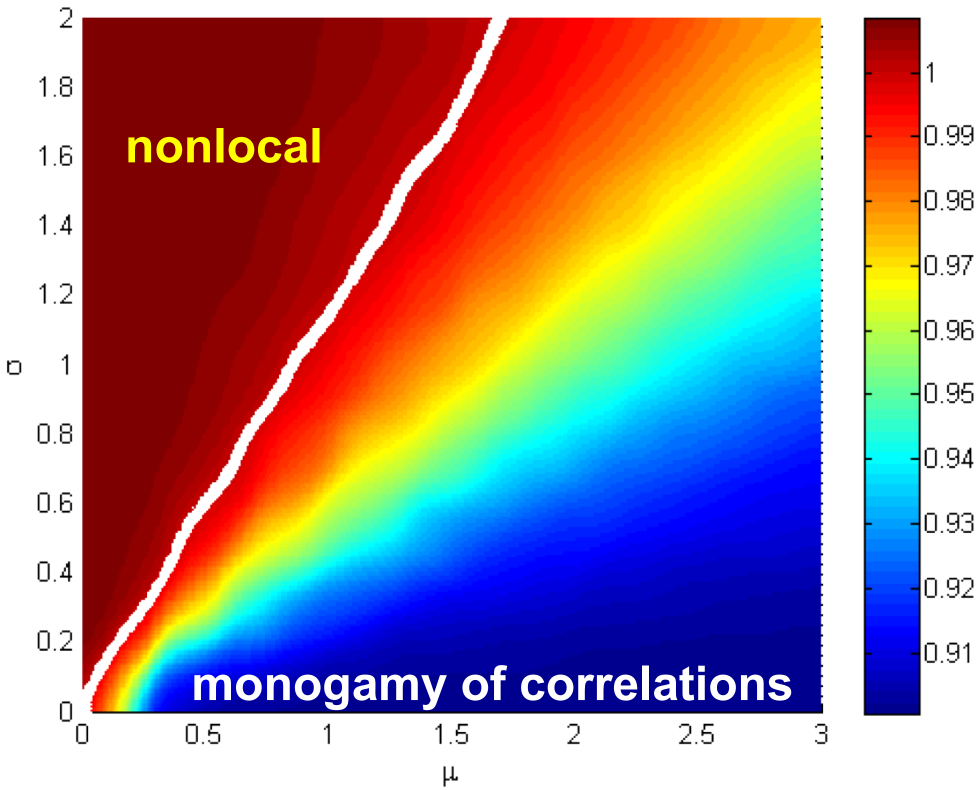

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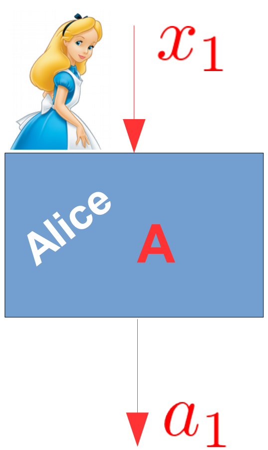

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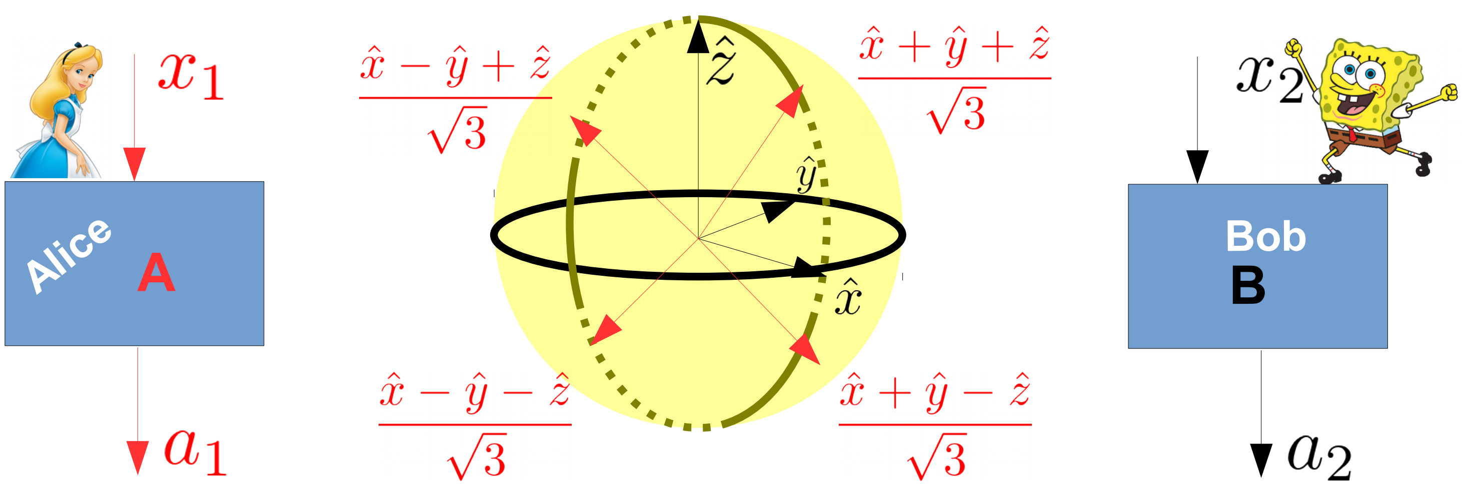

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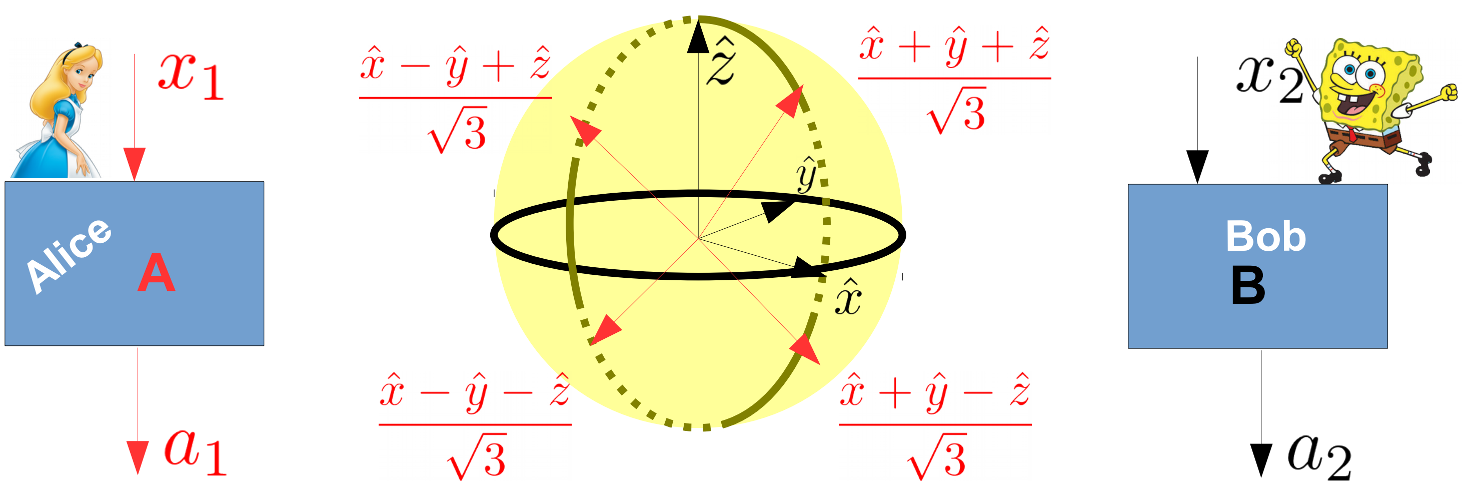

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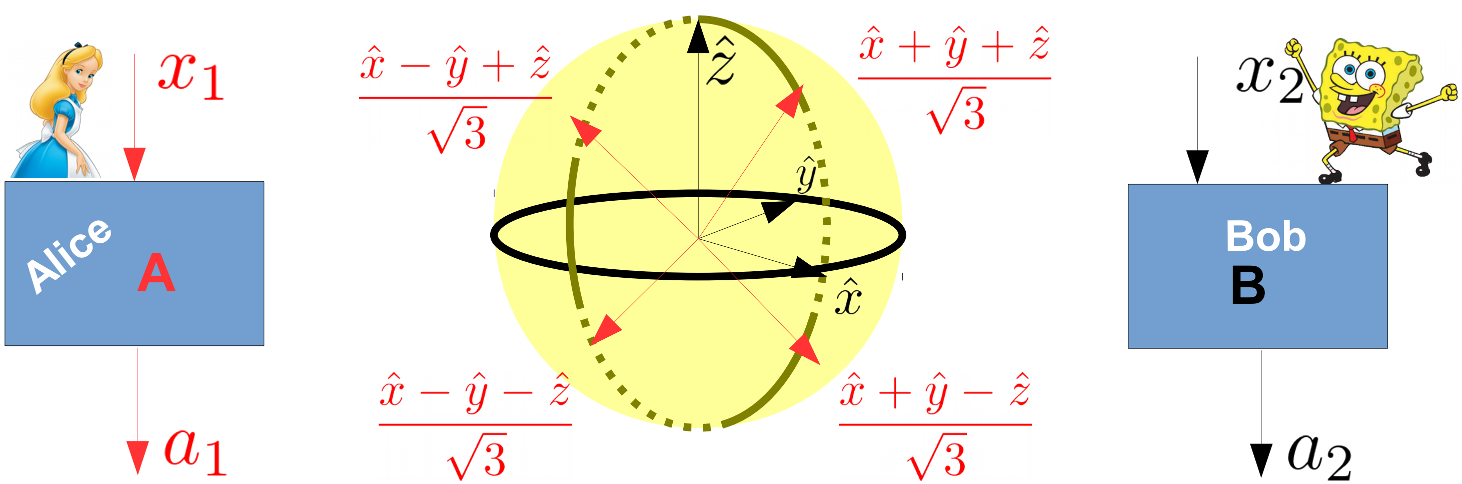

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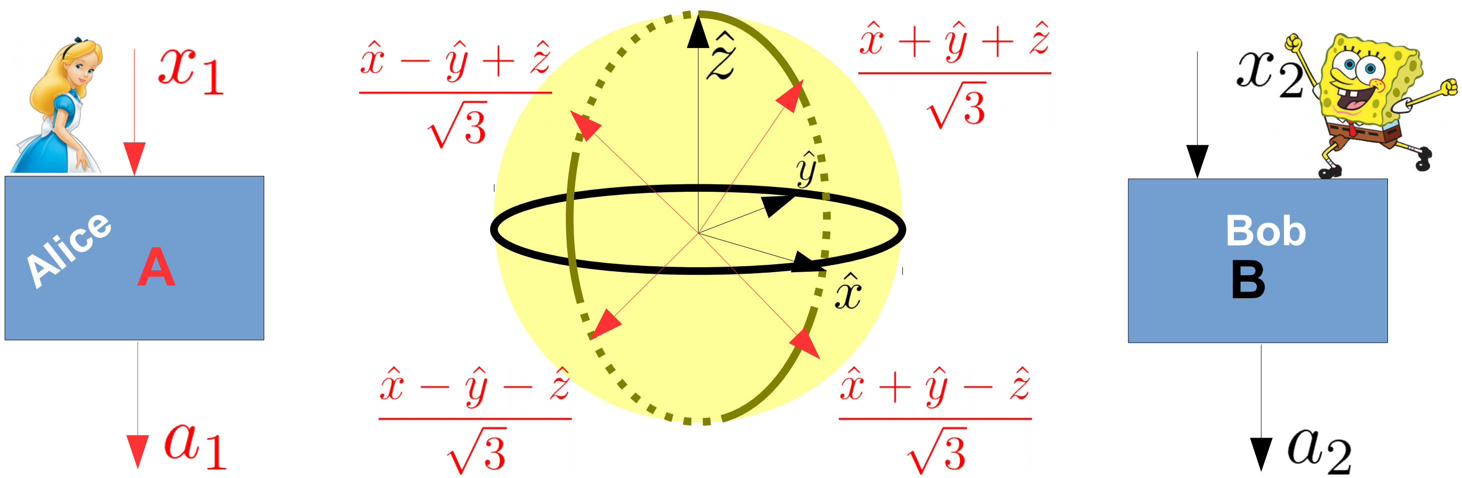

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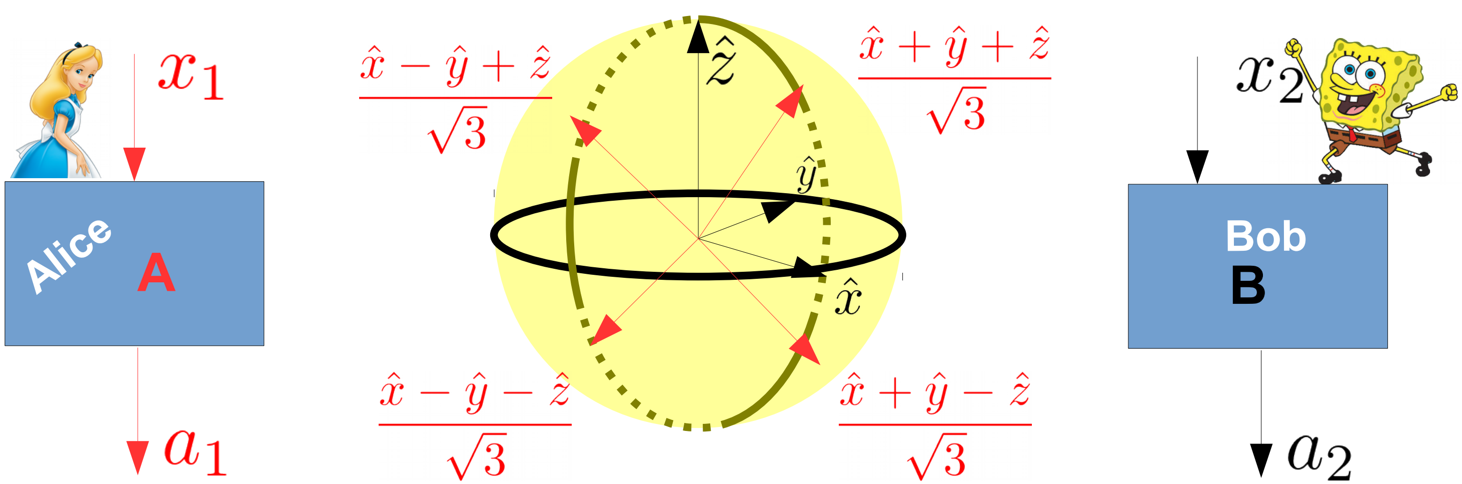

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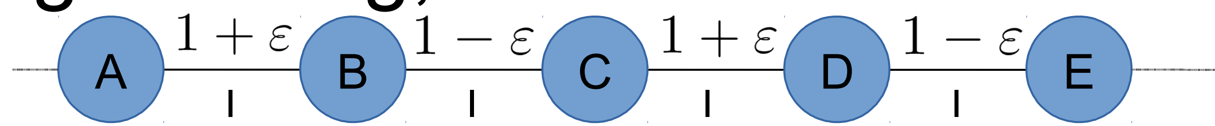
- Using Dynamic Programming, we find the classical bound of 

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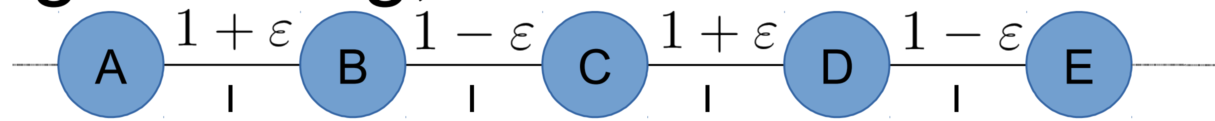

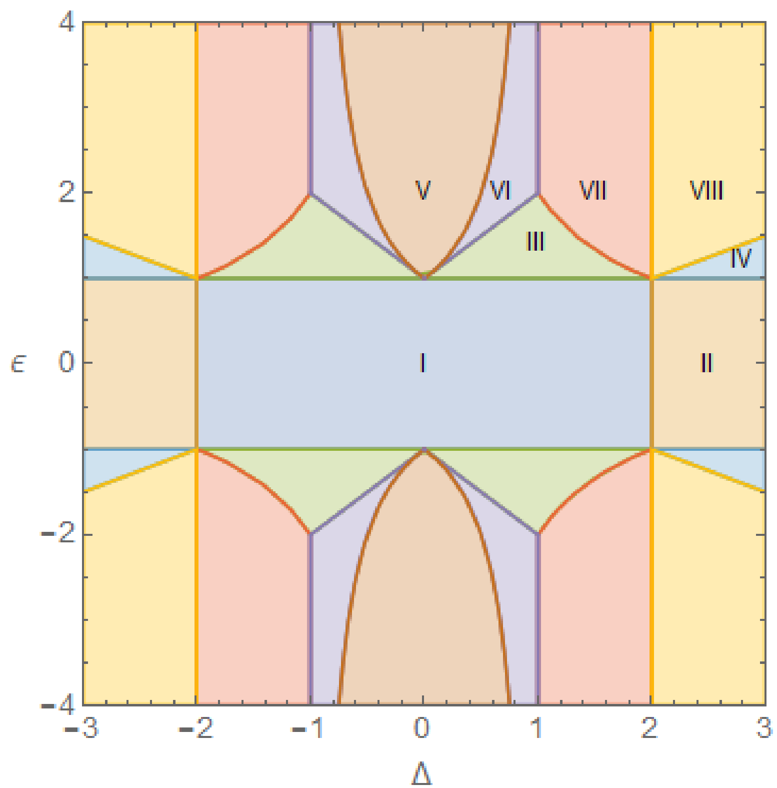

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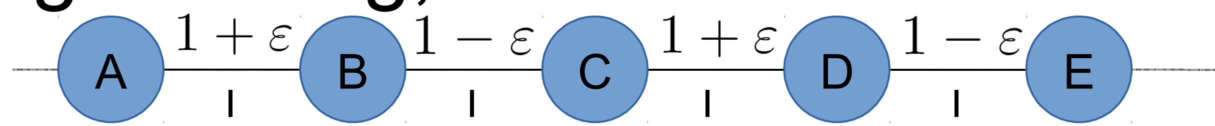

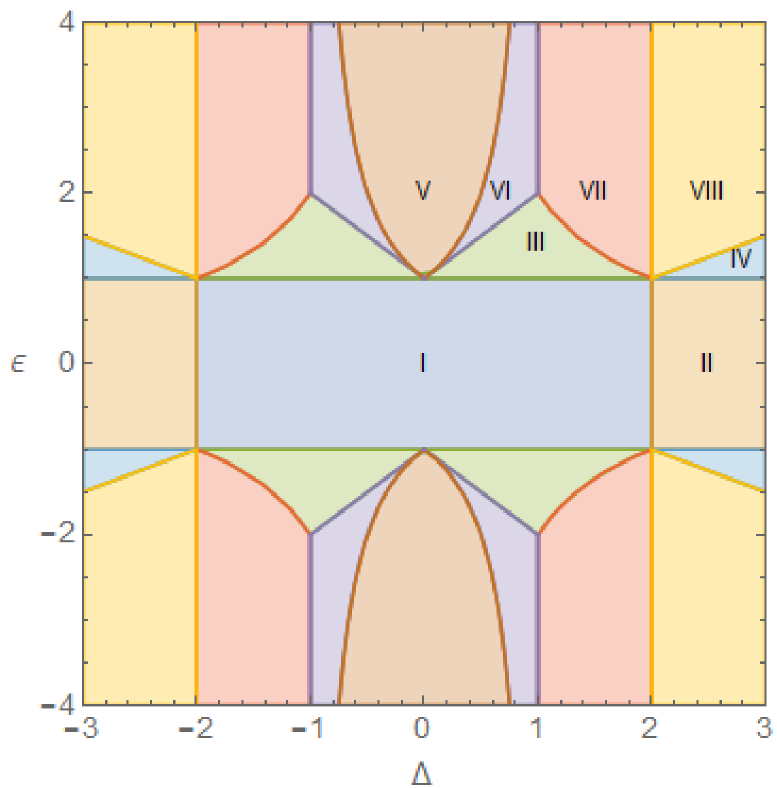

$$
\begin{aligned}
\beta_{C,\mathrm{I}} &= -n(4+2|\Delta|)\\
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[ITensor – Intelligent Tensor Library, http://itensor.org]

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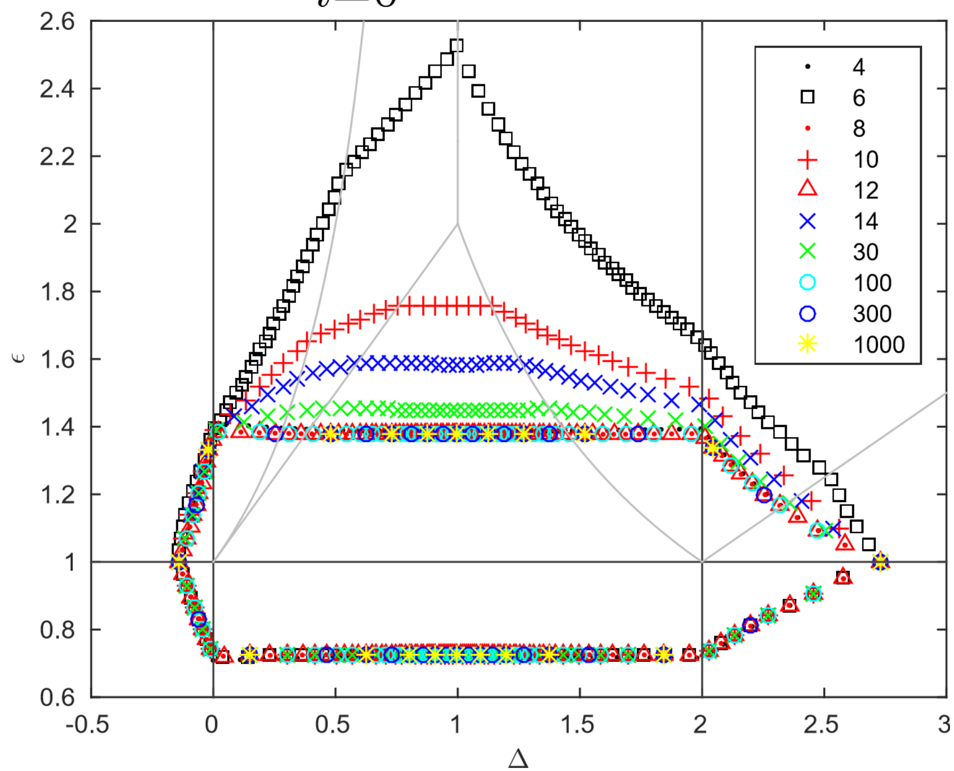

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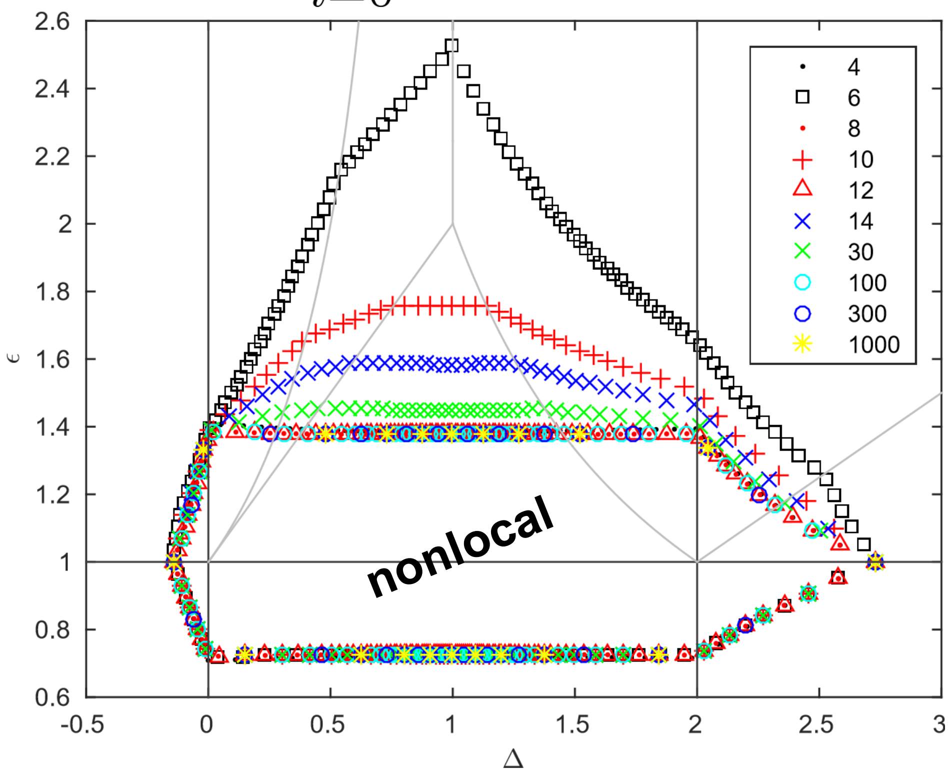

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Thanks for your attention!

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Thanks for your attention!