Semidefinite programming strong converse bounds for quantum channel capacities

Xin Wang

UTS: Centre for Quantum Software and Information

Joint work with **Wei Xie**, **Runyao Duan** (UTS:QSI)

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CENTRE FOR QUANTUM SOFTWARE AND INFORMATION

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UNIVERSITY OF TECHNOLOGY SYDNEY

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Let's combine them!

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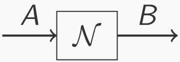

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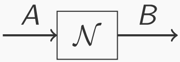

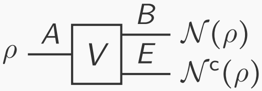

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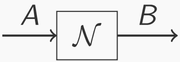

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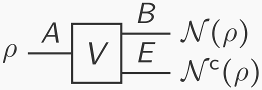

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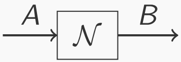

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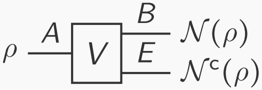

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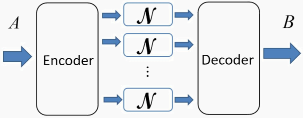
- Capacity is the maximum rate for asymptotically error-free (classical, quantum or private) data transmission using the channel $\mathcal{N}$ many times.



Classical communication via quantum channels

- Classical capacity (Holevo'73, 98; Schumacher & Westmoreland'97):

$$C(\mathcal{N}) = \sup_{k\to\infty} \frac{1}{k}\chi(\mathcal{N}^{\otimes k}),$$

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  - Classical capacity of amplitude damping channel is unknown.

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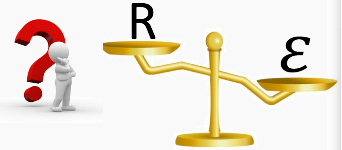

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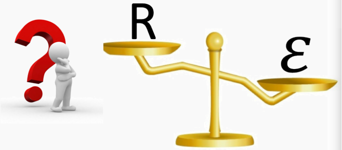

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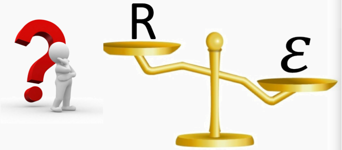

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  - Entanglement-assisted capacity (Bennett, Shor, Smolin, Thapliyal 1999, 2002)

Main question and outline

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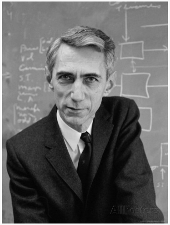
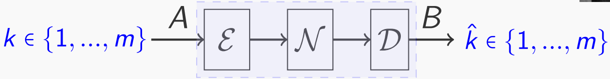
- All these results are given by SDPs.
  - An analytical tool in proof (Watrous' Book)
  - There are efficient algorithms.
  - Implementations: CVX for MATLAB, toolbox QETLAB.

Non-asymptotic communication capability

Optimal success probability and capacity



- (Shannon, 1948) The fundamental problem of communication is that of reproducing at one point, either exactly or approximately, a message selected at another point.



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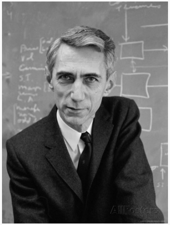

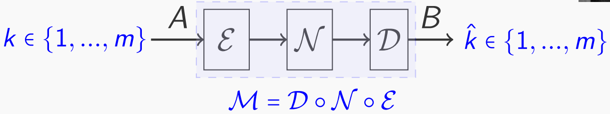

- Optimal success probability

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\begin{aligned}
p_s(\mathcal{N}, m) := & \sup_{\mathcal{E},\mathcal{D}} \frac{1}{m} \sum_{k=1}^{m} p(k = \hat{k}) \\
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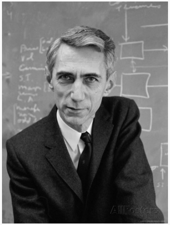

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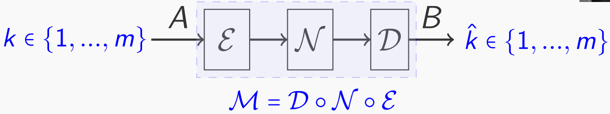

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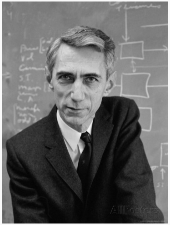

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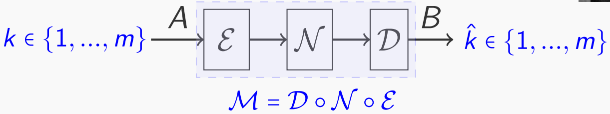

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- Question: how to solve or estimate $p_s(\mathcal{N}, m)$?

General codes

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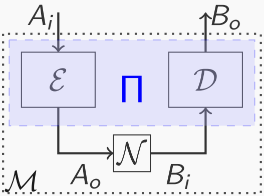
- Also see causal operations (Beckman, Gottesman, Nielsen, Preskill'01; Eggeling, Schlingemann, Werner'02, Piani, Horodecki et al.'06).



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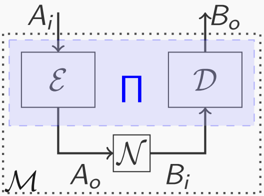

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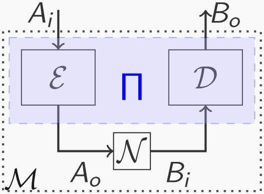

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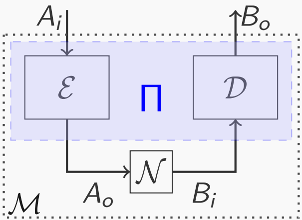
- Classical (Cubitt, Leung, Matthews, Winter'11;Matthews'12)
- A hierarchy of codes by adding constraints on Π, e.g., Positive-partial-transpose preserving (PPT) constraint (Rains'01; Leung & Matthews'16).

Optimal success probability



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- Optimal success probability of Ω codes ($\Omega = \mathrm{NS}$ or $\mathrm{NS} \cap \mathrm{PPT}$ in this talk)

$$p_{s,\Omega}(\mathcal{N}, m) = \sup_{\Pi \in \Omega} \frac{1}{m} \sum_{k=1}^{m} \operatorname{Tr}[\mathcal{M}(|k\rangle\langle k|)|k\rangle\langle k|],\ \mathcal{M} \text{ given by } \mathcal{N}, \Pi.$$

Result 1: Optimal success probability for NS/PPT codes

Theorem

For any $\mathcal{N}$, the optimal success probability to transmit m messages assisted by NS∩PPT codes is given by the following SDP:

$$
\begin{aligned}
p_{s,\mathrm{NS}\cap\mathrm{PPT}}(\mathcal{N}, m) = \max \operatorname{Tr} J_{\mathcal{N}} F_{AB} \\
\text{s.t. } & 0 \le F_{AB} \le \rho_A \otimes \mathbb{1}_B, \operatorname{Tr} \rho_A = 1, \\
& \operatorname{Tr}_A F_{AB} = \mathbb{1}_B / m, \\
& 0 \le F_{AB}^{T_B} \le \rho_A \otimes \mathbb{1}_B \ (\mathrm{PPT}),
\end{aligned}
$$

where $J_{\mathcal{N}}$ is the Choi-Jamiołkowski matrix of $\mathcal{N}$.
When assisted by NS codes, one can remove PPT constraint to obtain

$$
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- Target:
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- Recall $J_{\mathcal{M}} = \sum_{ij} |i\rangle\langle j|_{A_i'} \otimes \mathcal{M}(|i\rangle\langle j|_{A_i})$ and let $V = \sum_{\mathrm{k}=1}^{m} |kk\rangle\langle kk|$

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Key: $$\frac{1}{m} \sum_{k=1}^{m} \operatorname{Tr}[\mathcal{M}(|k\rangle\langle k|)|k\rangle\langle k|] = \frac{1}{m} \operatorname{Tr}[J_{\mathcal{M}} V_{A_i B_o}]. \qquad (2)$$

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- Combining Eqs. (1), (2), (3), we have
$$p_{s,\Omega}(\mathcal{N}, m) = \max_{\Pi \in \Omega} \mathrm{Tr}[(J_{\mathcal{N}}^T \otimes \mathbb{1}_{A_i B_o}) J_{\Pi} (\mathbb{1}_{A_o B_i} \otimes V_{A_i B_o})]/m,$$

Sketch of proof

- Target:

$$p_{s,\Omega}(\mathcal{N}, m) = \sup_{\Pi\in\Omega} \frac{1}{m}\sum_{k=1}^{m} \mathrm{Tr}[\mathcal{M}(|k\rangle\langle k|)|k\rangle\langle k|], \qquad (1)$$

- Recall $J_{\mathcal{M}} = \sum_{ij} |i\rangle\langle j|_{A_i'} \otimes \mathcal{M}(|i\rangle\langle j|_{A_i})$ and let $V = \sum_{k=1}^{m} |kk\rangle\langle kk|$

Key: $$\frac{1}{m}\sum_{k=1}^{m} \mathrm{Tr}[\mathcal{M}(|k\rangle\langle k|)|k\rangle\langle k|] = \frac{1}{m}\mathrm{Tr}[J_{\mathcal{M}} V_{A_iB_o}]. \qquad (2)$$

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- Impose the NS and PPT constraints of Π to obtain the SDP.
- Exploit the permutation invariance of $V_{A_iB_o}$ to simplify SDP.

Example: assess the preformance of AD channel

- For amplitude damping channel $\mathcal{N}_\gamma^{AD}(\rho) = \sum_{i=0}^{1} E_i \rho E_i^\dagger$ with $E_0 = |0\rangle\langle 0| + \sqrt{1-\gamma}|1\rangle\langle 1|$ and $E_1 = \sqrt{\gamma}|0\rangle\langle 1|$,

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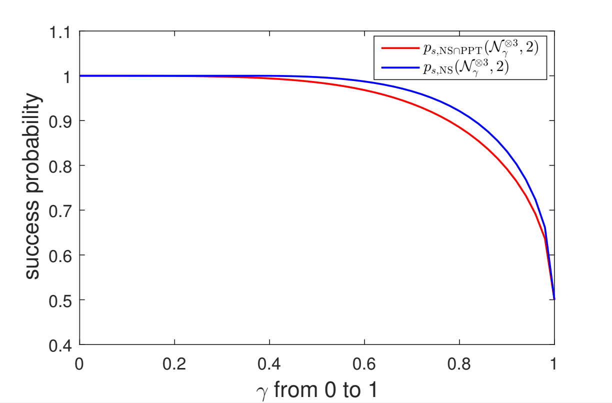
- if we use the channel 3 times, the optimal success probabilty to transmit 1 bit is given as follows:



Result 2: One-shot capacities

- One-shot ϵ-error capacity assisted with Ω-codes:

$$C_{\Omega}^{(1)}(\mathcal{N},\epsilon) := \sup\{\log\lambda : 1 - p_{s,\Omega}(\mathcal{N},\lambda) \le \epsilon\}.$$

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Theorem

For given channel $\mathcal{N}$ and error threshold ϵ,

$$\begin{aligned} C^{(1)}_{\mathrm{NS}\cap\mathrm{PPT}}(\mathcal{N},\epsilon) = -\log\min\eta \ \textit{s.t.} \quad & 0 \le F_{AB} \le \rho_A \otimes \mathbb{1}_B, \operatorname{Tr}\rho_A = 1, \\ & \operatorname{Tr}_A F_{AB} = \eta\mathbb{1}_B, \operatorname{Tr} J_{\mathcal{N}} F_{AB} \ge 1-\epsilon, \\ & 0 \le F_{AB}^{T_B} \le \rho_A \otimes \mathbb{1}_B \ (\mathrm{PPT}), \end{aligned}$$

To obtain $C^{(1)}_{\mathrm{NS}}(\mathcal{N},\epsilon)$, one only needs to remove the PPT constraint:

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- Study zero-error capacity by setting $\epsilon = 0$, e.g., $C_{\mathrm{NS}}^{(1)}(\mathcal{N},0)$ recovers the one-shot NS assisted zero-error capacity in (Duan & Winter'16).

Comparsion with previous converse bounds

- Converse for classical channel (Polyanskiy, Poor, Verdú 2010) and classical-quantum channel (Wang & Renner 2010).
- (Matthews & Wehner 2014) shows SDP converse bounds

$$C_{\mathrm{E}}^{(1)}(\mathcal{N},\epsilon) \leq \max_{\rho_A} \min_{\sigma_B} D_H^{\epsilon}((id_{A'} \otimes \mathcal{N})(\rho_{A'A}) \| \rho_{A'} \otimes \sigma_B),$$

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  where D_H^{ϵ} and $D_{H,PPT}^{\epsilon}$ are hypothesis testing relative entropies.
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- (Datta & Hsieh'13) gives converse for $C_{\mathrm{E}}^{(1)}(\mathcal{N}, \epsilon)$ (hard to compute).
- One-shot ϵ-error capacities can provide better efficiently computable converse bounds:

$$C_{\mathrm{E}}^{(1)}(\mathcal{N}, \epsilon) \leq C_{\mathrm{NS}}^{(1)}(\mathcal{N}, \epsilon) \leq \max_{\rho_A} \min_{\sigma_B} D_H^{\epsilon}((id_{A'} \otimes \mathcal{N})(\rho_{A'A}) \| \rho_{A'} \otimes \sigma_B),$$

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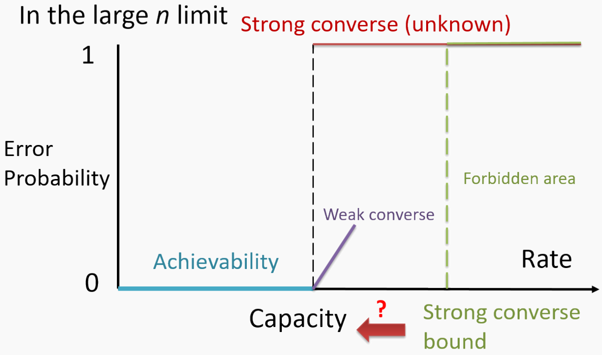
  The blue inequalities can be strict for amplitude damping channels.

Asymptotic communication capability

Weak vs Strong Converse



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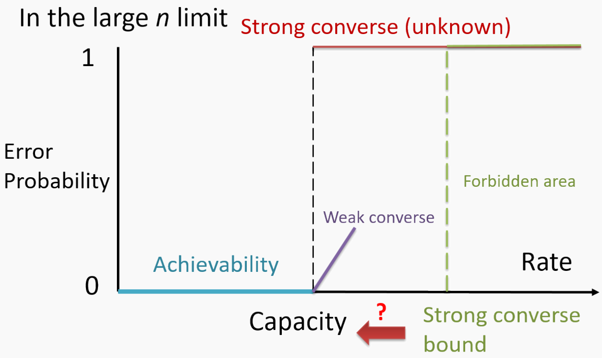

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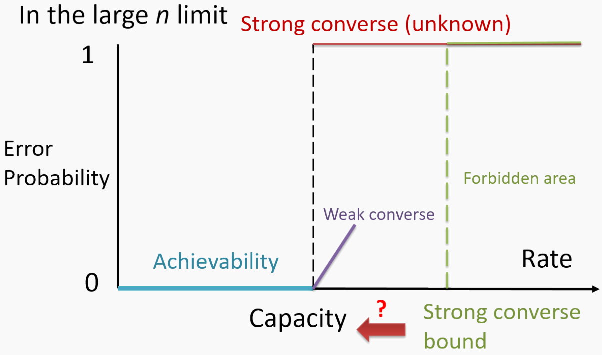

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- A strong converse bound: $p_{succ} \to 0$ as n increases if the rate exceeds this bound.
- If the capacity of a channel is also its strong converse bound, then the strong converse property holds.

Result 3: Strong converse bound for classical capacity

- Known strong converse bound: the entanglement-assisted capacity (Bennett, Shor, Smolin, Thapliyal 1999, 2002)

Theorem (SDP strong converse bound for C)

For any quantum channel $\mathcal{N}$,

$$C(\mathcal{N}) \le C_\beta(\mathcal{N}) = \log \min \operatorname{Tr} S_B$$

$$\text{s.t.} - R_{AB} \le J_{\mathcal{N}}^{T_B} \le R_{AB},$$

$$-\mathbb{1}_A \otimes S_B \le R_{AB}^{T_B} \le \mathbb{1}_A \otimes S_B.$$

And $p_{succ} \to 0$ when the rate exceeds $C_\beta(\mathcal{N})$.

Properties:

- A relaxed bound: $C(\mathcal{N}) \le C_\beta(\mathcal{N}) \le \log d_B \| J_{\mathcal{N}}^{T_B} \|_\infty$.
- For qudit noiseless channel I_d, $C(I_d) = C_\beta(I_d) = \log d$.
- $C_\beta(\mathcal{N}_1 \otimes \mathcal{N}_2) = C_\beta(\mathcal{N}_1) + C_\beta(\mathcal{N}_2)$ for any $\mathcal{N}_1$ and $\mathcal{N}_2$.

Sketch of proof

- Subadditive bounds on p_s (Tool: duality of SDP)

$$p_{s,\text{NS}\cap\text{PPT}}(\mathcal{N}^{\otimes n}, 2^{rn}) \le p_s^+(\mathcal{N}^{\otimes n}, 2^{rn}) \le p_s^+(\mathcal{N}, 2^r)^n, \quad (4)$$

where

$$p_s^+(\mathcal{N}, m) = \min\ \operatorname{Tr} Z_B \text{ s.t. } \begin{aligned} &-R_{AB} \le J_{\mathcal{N}}^{T_B} \le R_{AB}, \\ &-m\mathbb{1}_A \otimes Z_B \le R_{AB}^{T_B} \le m\mathbb{1}_A \otimes Z_B. \end{aligned} \quad (5)$$

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- For any $r > C_\beta(\mathcal{N})$, one can prove that $p_s^+(\mathcal{N}, 2^r) < 1$. Thus,

$$p_{s,\mathrm{NS}\cap\mathrm{PPT}}(\mathcal{N}^{\otimes n}, 2^{rn}) \leq p_s^+(\mathcal{N}, 2^r)^n \to 0, \text{ (when } n \text{ increases)}$$

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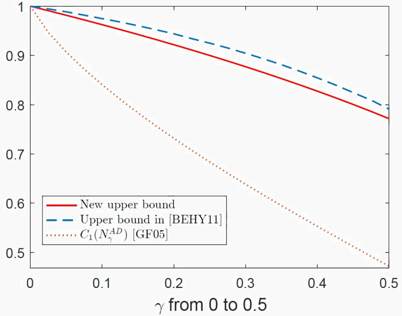

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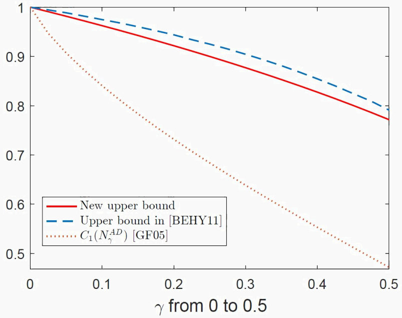

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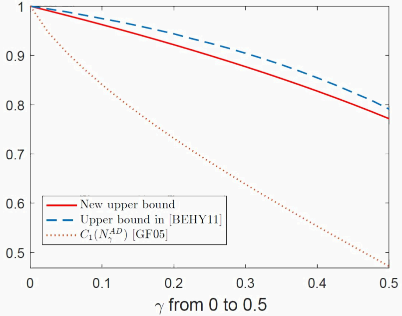

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- Note that $C_{\mathrm{E}}(\mathcal{N}_\gamma^{AD}) \geq 1$ when $\gamma \leq 0.5$.
- Problem: how to further improve the lower bound or upper bound?

Application 2: Strong converse property for new channels

- Previous known channels:
  - classical-quantum channels (Ogawa, Nagaoka'99; Winter'99)
  - particular covariant quantum channels (Koenig and Wehner'09)
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- In particular,

$$Q(\mathcal{N}_\alpha) < 1 = P(\mathcal{N}_\alpha) = C(\mathcal{N}_\alpha) = \frac{1}{2} C_E(\mathcal{N}_\alpha).$$

Quantum capacity

- Quantum capacity is established by (Lloyd, Shor, Devetak 97-05) & (Barnum, Nielsen, Schumacher 96-00)

$$Q(\mathcal{N}) = \lim_{m\to\infty} \frac{1}{m} I_c(\mathcal{N}^{\otimes m}).$$

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SDP strong converse bound for quantum capacity

Theorem (SDP strong converse bound for Q)

For any quantum channel $\mathcal{N}$,

$$Q(\mathcal{N}) \leq Q_\Gamma(\mathcal{N}) = \log \max \operatorname{Tr} J_{\mathcal{N}} R_{AB} \quad \text{s.t.} \quad R_{AB}, \rho_A \geq 0, \operatorname{Tr} \rho_A = 1, \quad -\rho_A \otimes \mathbb{1}_B \leq R_{AB}^{T_B} \leq \rho_A \otimes \mathbb{1}_B.$$

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- For noiseless quantum channel $\mathcal{I}_d$, $Q(\mathcal{I}_d) = Q_\Gamma(\mathcal{I}_d) = \log_2 d$.
- $Q_\Gamma(\mathcal{N} \otimes \mathcal{M}) = Q_\Gamma(\mathcal{M}) + Q_\Gamma(\mathcal{N})$ (by utilizing SDP duality).

Comparison with other bounds

- Partial Transposition bound (Holevo & Werner'01, Muller-Hermes, Reeb, Wolf'16)

$$Q(\mathcal{N}) \leq Q_{\Theta}(\mathcal{N}) = \log_2 \| J_{\mathcal{N}}^{T_B} \|_{cb},$$

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- Example: $\mathcal{N}_r = \sum_i E_i \cdot E_i^\dagger$ with $E_0 = |0\rangle\langle 0| + \sqrt{r}|1\rangle\langle 1|$ and $E_1 = \sqrt{1-r}|0\rangle\langle 1| + |1\rangle\langle 2|$.
- Solid line: SDP bound Q_Γ
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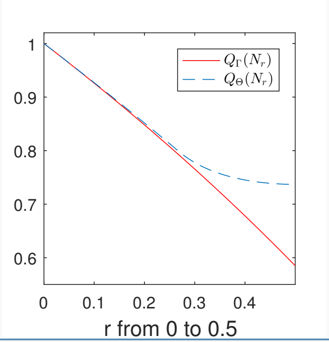

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  - Optimal success probability of communication is given by SDP
  - One-shot ϵ-error capacity is given by SDP
- Limits for asymptotic classical/quantum communication
  - SDP strong converse bound for classical capacity
    - Improved upper bound for $C(\mathcal{N}^{AD})$
    - Strong converse property for new class of quantum channels
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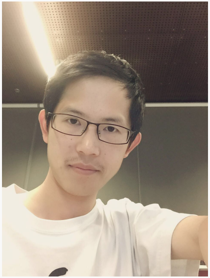
arXiv:1610.06381 & 1601.06888



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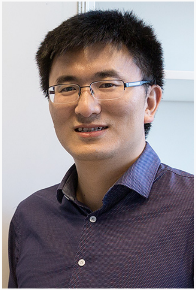

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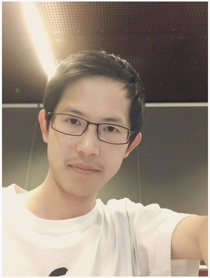

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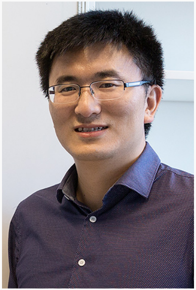

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Thank you for your attention!